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Civil Services Main Examination Previous Solved Papers : Mechanical Engg. (Paper-II)

First Edition: 2017
Second Edition: 2018

Preface

B. Singh (Ex. IES)

Civil Service is considered as the most prestigious job in India and it has become a preferred destination by all engineers. In order to reach this estimable position every aspirant has to take arduous journey of Civil Services Examination (CSE). Focused approach and strong determination are the pre-requisites for this journey. Besides this, a good book also comes in the list of essential commodity of this odyssey.

I feel extremely glad to launch the revised edition of such a book which will not only make CSE plain sailing, but also with 100% clarity in concepts.

MADE EASY team has prepared this book with utmost care and thorough study of all previous years papers of CSE. The book aims to provide complete solution to all previous years questions with accuracy.

On doing a detailed analysis of previous years CSE question papers, it came to light that a good percentage of questions have been asked in Engineering Services, Indian Forest Service and State Services exams. Hence, this book is a one stop shop for all CSE, ESE and other competitive exam aspirants.

I would like to acknowledge efforts of entire MADE EASY team who worked day and night to solve previous years papers in a limited time frame and I hope this book will prove to be an essential tool to succeed in competitive exams and my desire to serve student fraternity by providing best study material and quality guidance will get accomplished.

With Best Wishes
B. Singh (Ex. IES)
CMD, MADE EASY Group

Previous Years Solved Papers of

Civil Services Main Examination

Mechanical Engineering : Paper-II

CONTENTS

1 Thermodynamics

1. Basic Concepts, Heat and Work

Q.1 A rigid insulated tank of 3 m^3 volume is divided into 2 compartments. One compartment of volume 1 m^3 contains an ideal gas at 0.1 MPa and 300 K while the second compartment of volume 2 m^3 contains the same gas at 1 MPa and 1000 K. If the partition between the two compartments is ruptured, calculate the final temperature and pressure of the gas.

[CSE (Mains) 2002 : 20 Marks]

Solution:

Consider the gas contained in 2 compartments A and B of the rigid tank.

Assumption: Ideal gas behaviour of gas in both compartment.

Let the final temperature and pressure of gas after partition is removed be T and P, respectively.

$$m_A = \frac{PV}{RT} = \frac{0.1 \times 1}{R \times 300} = \frac{1}{3000R}$$

$$m_B = \frac{PV}{RT} = \frac{1 \times 2}{R \times 1000} = \frac{1}{500R}$$

$$\Rightarrow \quad \frac{m_A}{m_B} = \frac{1}{6}$$

$$\Rightarrow \quad m_B = 6\, m_A$$

1 m^3	2 m^3
$P_{1,A}$ = 0.1 MPa	$P_{1,B}$ = 1 MPa
$T_{1,A}$ = 300 K	$T_{1,B}$ = 100 K
A	B

Comparing final and initial states,

$$R = \frac{P_{1,A}V_A}{m_A \cdot T_{1,A}} = \frac{P(V_A + V_B)}{(m_A + m_B)T}$$

$$\Rightarrow \quad \frac{0.1 \times 1}{m_A \cdot 300} = \frac{P \cdot 3}{7m_A \cdot T}$$

$$\Rightarrow \quad \frac{P}{T} = \frac{7}{9000}$$

Heat lost by one compartment = Heat gained by other compartment

$$\therefore \quad m_A C_P (T - 300) = m_B C_V (1000 - T)$$

$$\Rightarrow \quad T - 300 = 6000 - 6T$$

$$\Rightarrow \quad T = 900 \text{ K}$$

$$\therefore \quad P = \frac{7 \times T}{9000} = \frac{7}{9000} \times 900 = 0.7 \text{ MPa}$$

Q.2 A mass of air initially at 760 kPa and 250°C occupies 0.026 m³. The air is expanded at constant pressure to 0.07 m³. A polytropic process with $n = 1.52$ is then carried out followed by an isothermal process and the cycle is thus completed. Assuming all the processes to be reversible,

(i) show all the processes on *PV* and *TS* planes

(ii) compute the heat received and rejected in the cycle

(iii) calculate the efficiency of the cycle.

[CSE (Mains) 2002 : 30 Marks]

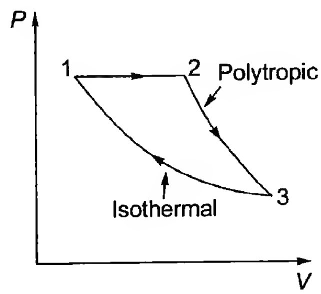
Solution:

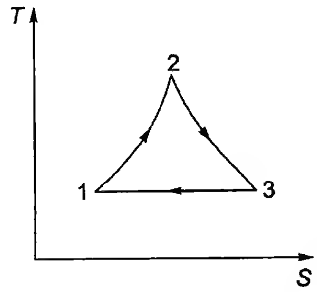
Processes can be represented on P-V and T-S diagram as shown in figure.

State 1: $P_1 = 760$ kPa , $T_1 = 250°C = 523$ k and $V_1 = 0.026$ m³

$$m = \frac{P_1V_1}{RT_1} = \frac{760 \times 0.026}{0.287 \times 523}$$

$$m = 0.1316 \text{ kg}$$





State 2: $P_2 = P_1 = 760$ kPa, $V_2 = 0.07$ m³

$$\frac{P_2V_2}{T_2} = \frac{P_1V_1}{T_1}$$

$$\Rightarrow \qquad T_2 = \frac{V_2}{V_1}\cdot T_1 = 1408.08 \text{ K}$$

State 3: $T_3 = T_1 = 523$ K

Process 1-2: Heat exchanged $= Q_{1-2} = mC_P(T_2 - T_1)$

$$= m\cdot C_P(T_2 - T_1) = 0.1316 \times 1.005 \times (1408.08 - 523) = 117.098 \text{ kJ}$$

Work done $= P_1(V_2 - V_1) = 760 \times 10^3 \times (0.07 - 0.026)$ J $= 33.44$ kJ

Process 2-3: Work done in polytropic process,

$$W_{2\text{-}3} = \frac{P_2V_2 - P_3V_3}{n-1} = \frac{mR(T_2 - T_3)}{n-1} = \frac{0.1316 \times 0.287 \times (1408.08 - 523)}{1.52 - 1}$$

$$= 64.31 \text{ kJ}$$

Heat transfer in a polytropic process $= Q_{2\text{-}3} = -W\cdot\left(\frac{n-\gamma}{\gamma - 1}\right) = -64.31\left(\frac{1.52 - 1.4}{1.4 - 1}\right) = -19.29$ kJ

Process 3-1: Work done in isothermal process,

$$W_{2\text{-}3} = mRT\ln\left(\frac{V_1}{V_3}\right) = mRT\ln\left(\frac{V_1}{V_3}\right)$$

For polytropic process 2-3 $\Rightarrow TV^{n-1} =$ constant

$$\Rightarrow \qquad T_2V_2^{n-1} = T_2V_2^{n-1}$$

$$\Rightarrow \qquad V_3 = V_2\cdot\left(\frac{T_2}{T_3}\right)^{\frac{1}{n-1}} = 0.07 \times \left(\frac{1408.08}{523}\right)^{\frac{1}{1.52-1}} = 0.47 \text{ m}^3$$

$$\therefore \qquad W_{3\text{-}1} = 0.1316 \times 0.287 \times 523 \times \ln\left(\frac{0.026}{0.47}\right) = -57.18 \text{ kJ}$$

For an ideal gas internal energy change for an isothermal process = 0.

∴ From 1st law of thermodynamics,

$$Q = \Delta U + W = 0 + W$$

$$\Rightarrow \quad Q_{3-1} = W_{3-1} = -57.18 \text{ kJ}$$

∴ Total heat transfer in the cycle

$$\text{Heat received} = Q_{1-2} = 117.098 \text{ kJ}$$

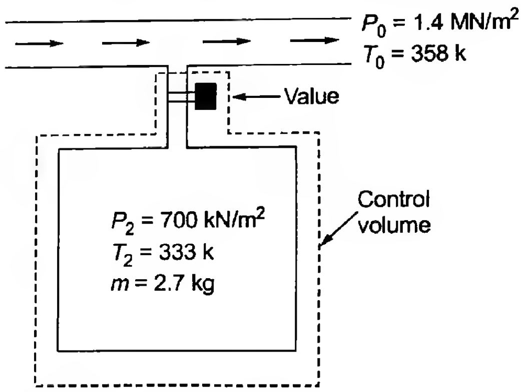
$$\text{Heat rejected} = Q_{3-1} + Q_{2-3} = 76.47 \text{ kJ}$$

$$\text{Total work done} = W_{2-1} + W_{2-3} + W_{3-1} = 40.57 \text{ kJ}$$

$$\therefore \quad \text{Efficiency of cycle} = \frac{W}{Q_{in}} = \frac{40.57}{117.098} = 0.3465 = 34.65\%$$

Q.3 **A pressure vessel is connected, via a valve, to a gas main in which gas is maintained at a constant pressure and temperature of 1.4 MN/m² and 85°C respectively. The pressure vessels is initially evacuated. The valve is opened and a mass of 2.7 kg of gas passes into the pressure vessel. The valve is closed and the pressure and temperature of the gas in the pressure vessel are then 700 kN/m² and 60°C, respectively. Determine the heat transfer to or from the gas in the vessel. Determine the volume of gas before transfer.**
For the gas, take C_P = 0.88 kJ/kg K, C_V = 0.67 kJ/kg K. Neglect the velocity of the gas in the main.
[CSE (Mains) 2004 : 30 Marks]

Solution:



Consider the pressure vessel and main connected by a valve as shown in figure.
Gas flows in side the vessel initially evacuated. Consider control volume as shown in figure.
Applying conservation of energy for variable flow process of this control volume,

$$\text{Rate of change} = \frac{dE_V}{d\tau}$$

$$= \text{Rate of inflow of energy} - \text{Rate of outflow of energy}$$

$$\Rightarrow \quad \frac{dE_V}{d\tau} = h_p \cdot \frac{dm}{d\tau} + \frac{dQ}{d\tau}$$

$$\Rightarrow \quad \frac{du}{d\tau} = h_p \frac{dm}{d\tau} + \frac{dQ}{d\tau} \quad \text{(since velocity of pipe is negligible)}$$

Integrating, we get, $m_2 u_2 - m_1 u_1 = h_p (m_2 - m_1) + Q$

Initial mass of gas in control volume, $m_1 = 0$

Final mass of gas in control volume, $m_2 = 2.7$ kg

∴ Heat transfer to or from pressure vessel

$$Q = m_2 u_2 - m_2 h_p = m_2 [C_V T_2 - C_P T_p] = 2.7 [0.67 \times 333 - 0.88 \times 358]$$

$$Q = -248.211 \text{ kJ}$$

∴ 248.21 kJ of heat is lost from the pressure vessel. Assume initial volume of gas in pipe before transfer is V_p.
Since the gas can be assumed to follow ideal gas behaviour–

$$\frac{P_P V_P}{T_P} = \frac{P_2 V_2}{T_2} = mR = m(C_P - C_V)$$

$$\Rightarrow \quad V_P = \frac{358 \times 2.7 \times (0.88 - 0.67)}{1.4 \times 10^3} = 0.145 \text{ m}^3$$

Hence, volume of gas before filling is 0.145 m³.

Q.4 A fluid, contained in a horizontal cylinder fitted with a frictionless leakproof piston, is continuously agitated by means of stirrer passing through the cylinder cover. The cylinder diameter is 0.40 m. During the stirring process lasting 10 minute, the piston slowly moves out a distance of 0.485 m against the atmosphere. The net work done by the fluid during the process is 2 kJ. The speed of the electric motor driving the stirrer is 840 rpm. Determine the torque in the shaft and the power output of the motor.

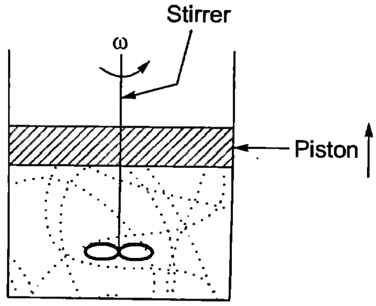
[CSE (Mains) 2007 : 20 Marks]

Solution:

Consider the system of fluid contained in the leakproof piston along with the stirrer. Work is being done by the piston on the fluid by stirring, through electric motor.

As a result of this fluid moves out against atmospheric pressure and does work.

Work done by fluid against atmospheric pressure,



$$W_{atm} = \int PdV = P_{atm} \times (V_2 - V_1)$$

$$= (1.01325 \times 10^5 \text{ Pa}) \times \frac{\pi D^2}{4}(\Delta x) \text{ m}^3$$

$$= 1.01325 \times 10^5 \times \frac{\pi \times 0.4^2}{4} \times 0.485$$

$$W_{atm} = 6.175 \text{ kJ}$$

$$\text{Net work done by the fluid} = W_{net} = W_{stirrer} + W_{atm} = 2 \text{ kJ}$$

$$\therefore \quad W_{stirrer} = 2 - 6.175 = -4.175 \text{ kJ}$$

Sign is negative, since this work is done on the system.

$$\text{Speed of rotation of motor} = W = 840 \text{ rpm} = \frac{2\pi \times 840}{60} \text{ rad/s} = 87.965 \text{ rad/s}$$

$$\text{Power output of shaft} = \frac{W_{stirrer}}{t} = \frac{-4.175 \text{ kJ}}{10 \text{ min}} = 6.96 \text{ watt}$$

Assume torque of motor as τ Nm.

$$\therefore \quad \text{Power of motor} = \tau\omega = 6.96 \text{ watt}$$

$$\Rightarrow \quad \tau = \frac{6.96}{87.965} = 7.91 \times 10^{-2} \text{ Nm}$$

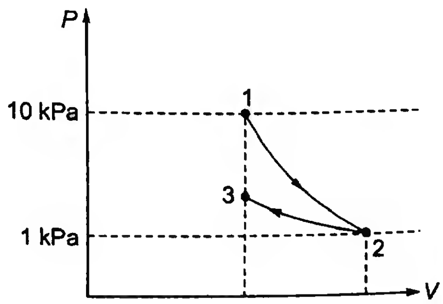
Q.5 (i) 0.5 m^3 of gas at 10 kPa and 130°C expands adiabatically to 1 kPa. It is then isothermally compressed to its original volume. C_p = 1.005 kJ/kg-K and C_v = 0.718 kJ/kg-K. Represent these processes on P-V diagram. Find final temperature and pressure of gas.

(ii) For compression work to be minimum, what should be process of compression? Is it used in practice?

[CSE (Mains) 2010 : 20 Marks]

Solution:

Consider the isothermal and adiabatic process as represented on P-V diagram in figure.



The gas ratio of specific heat capacities

$$= \frac{C_P}{C_V} = \gamma = \frac{1.005}{0.718} \simeq 1.4$$

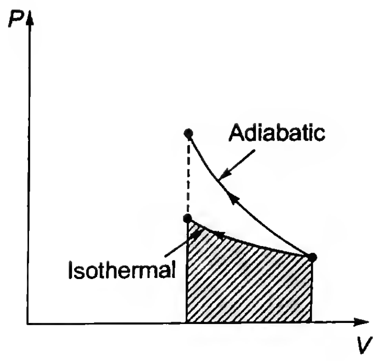
For process 1-2, Adiabatic expansion (reversible)

$$P^{1-\gamma} T^{\gamma} = \text{Constant}$$

$\therefore$ $$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = 403\times\left(\frac{1}{10}\right)^{\frac{1.4-1}{1.4}} = 208.73 \text{ K}$$

Also $$PV^{\gamma} = \text{Constant}$$

$\Rightarrow$ $$V_2 = V_1\left(\frac{P_2}{P_1}\right)^{1/\gamma} = 0.5\times\left(\frac{10}{1}\right)^{1/1.1} = 2.59 \text{ m}^3$$



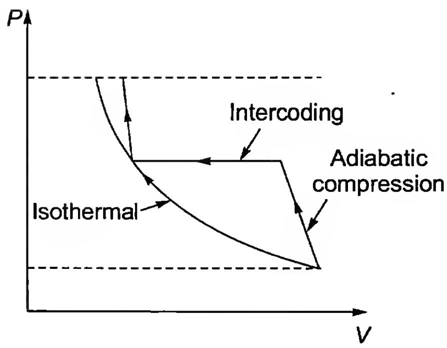
For process 2-3, Isothermal compression

$\therefore$ $$T_3 = T_2 = 208.73 \text{ K}$$

$$PV = \text{Constant}$$

$\Rightarrow$ $$P_3 = \frac{P_2V_2}{V_3} = \frac{P_2V_2}{V_1} = \frac{1\times 2.59}{0.5} = 5.18 \text{ kPa}$$

As can be seen in figure, slope of an isothermal process is less than that of an isentropic/adiabatic process.
Hence area under the curve, which is equal to work done is minimum in case of an isothermal process.
$\therefore$ Isothermal process should be used in compression
In Practise, for compression involving high compression ratios, adiabatic process with intercooling is employed. This method closely approaches an isothermal process.



Q.6 In the event of failure of heaters in a spacecraft, heat is lost by radiation at the rate of 100 kJ/hr while electronic instruments generate 75 kJ/hr inside the spacecraft. Initially the air inside the spacecraft is at 1 bar, 25°C with a volume of 10 m. How long it will take to reach air temperature of 0°C?

[CSE (Mains) 2012 : 12 Marks]

Solution:

Given: $\dot{Q}$ = Net heat loss rate = (100 – 75) = 25 kJ/hr, P_1 = 1 bar, T_1 = 25°C, V = 10 m^3

As the process is isochoric i.e., $V = 0$

$\therefore$ Total heat required, $Q = m\,C_V\,\Delta T = m \times 0.718 \times 25$

$$m = \frac{P_1V_1}{RT_1} = \frac{100\times 10}{0.287\times 298} = 11.692$$

$$Q = 209.877 \text{ kJ}$$

$$\text{Time} = \frac{Q}{\dot{Q}} = \frac{209.877}{25}$$

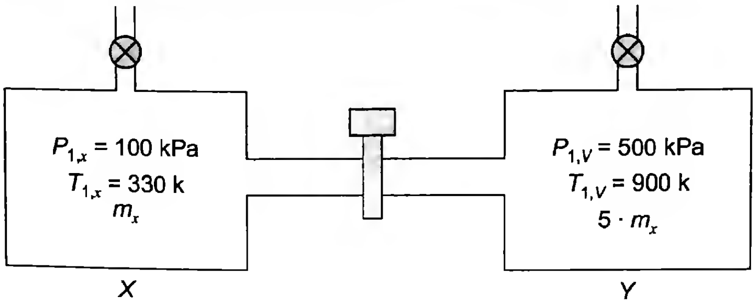
= 8 hour 23 min 42.38 sec

Q.7 A certain amount of gas is filled in a tank X until its pressure is 100 kPa and temperature is 330 K. In another tank Y, 5 times the weight of gas in X is filled raising the pressure to 500 kPa and temperature 900 K. Both tanks X and Y are now connected through a tube having a valve which is closed. Assuming the gas is ideal and if the valve is opened till equilibrium state is achieved, find the ratio of the volumes of both tanks, equilibrium temperature and pressure. The tanks are insulated. For the gas, take: R = 0.296 kJ/kg K and C_v = 0.75 kJ/kg K.

[CSE (Mains) 2014 : 20 Marks]

Solution:

Assume final equilibrium pressure and temperature as P and T respectively.



Since gas in both compartments has been assumed to be an ideal gas, using ideal gas equation $PV = mRT$, we get

$$R = \frac{P_{1,x} \cdot V_x}{m_x T_{1,x}} = \frac{P_{1,y} \cdot V_y}{5m_x \cdot T_{1,y}}$$

$$\Rightarrow \quad \frac{100 \times V_x}{330} = \frac{500 \cdot V_y}{5 \cdot 900} \Rightarrow \frac{V_x}{V_y} = \frac{11}{30}$$

After the valve has been opened and equilibrium achieved, final volume for gas

$$V_x + V_y = V_x + \frac{30\,V_x}{11} = \frac{41}{11}V_x$$

$$\text{Total mass} = m_x + 5\,m_x = 60\,m_x$$

$$R = \frac{P_{1,x} \cdot V_x}{m_x \cdot T_{1,x}} = \frac{P \cdot (41V_x/11)}{6\,m_x \cdot T}$$

$$\Rightarrow \quad \frac{41P}{66T} = \frac{100}{330} \Rightarrow \frac{P}{T} = \frac{20}{11}$$

Heat exchange at constant volume takes place between the gas in compartment X and Y.

$\therefore$ Heat lost by Y = Heat gained by tank X

$$\Rightarrow \quad 5 \cdot m_x \cdot C_v(900 - T) = m_x \cdot C_v(T - 330)$$

$$\Rightarrow \quad T = 805\text{ K}$$

Also, we have $\quad \dfrac{P}{T} = \dfrac{20}{41}$

$$\Rightarrow \quad P = \frac{20}{41} \cdot 805 = 392.7\text{ kPa}$$

$\therefore$ Final temperature and pressure are 805 K and 392.7 kPa respectively.

2. First Law of Thermodynamics

Q.8 The ratio of heat transfer to work transfer in the process of an air compressor reciprocating type is 1 : 4. If the compression follows PV^n = constant, what is the value of n? Derive the equation that you use. In such a compression process the work required is 200 kJ/kg and the specific heat at constant volume is 0.75 kJ/kg K. What rise of temperature is expected at the end of compression process?

[CSE (Mains) 2001 : 20 Marks]

Solution:

Consider an air compressor reciprocating type as a closed system in which working fluid/gas is undergoing process $PV^n = K$.

$$\text{Work done in the process} = \int PdV = \int K \cdot V^{-n}dV = K \cdot \left[\frac{V^{-n+1}}{-n+1}\right]_{V_1}^{V_2}$$

$$W = \frac{P_1V_1 - P_2V_2}{n-1} = \frac{mR(T_1 - T_2)}{n-1}$$

From 1st law of thermodynamics,

$$dQ = dU + dW$$

$\Rightarrow$ $$Q = mC_v(T_2 - T_1) + W \quad \text{(Since for an ideal gas } U = C_vT)$$

$$= \frac{mR}{\gamma - 1}(T_2 - T_1) - \frac{mR(T_2 - T_1)}{n-1} = \frac{mR(T_2 - T_1)}{n-1}\left[\frac{n-\gamma}{\gamma-1}\right] = -W \cdot \left[\frac{n-\gamma}{\gamma-1}\right]$$

$\therefore$ Ratio of heat transfer to work transfer $= \dfrac{Q}{W} = -\left[\dfrac{n-\gamma}{\gamma-1}\right]$

Neglecting sign, $$\frac{Q}{W} = \frac{n-\gamma}{\gamma-1} = \frac{1}{4}$$

$\Rightarrow$ $$n = 1.5$$

Given: Work required in such a compression process,

$$W = 200 \text{ kJ/kg}$$

$$W = \frac{mR[T_1 - T_2]}{n-1}$$

$\Rightarrow$ $$\frac{W}{m} = w = -200 = \left(\frac{R}{\gamma-1}\right) \cdot \left[\frac{\gamma-1}{n-1}\right](T_1 - T_2) \quad \text{[*work transfer is negative]}$$

$$-200 = 0.75 \times \frac{0.4}{0.5}(T_1 - T_2) \Rightarrow \text{temp. rise} = 333.33 \text{ K}$$

3. Second Law of Thermodynamics

Q.9 **A reversible heat engine absorbs heat energy from a source of hot gases whose temperature is falling linearly from T_1 to T_2. The engine rejects heat to a constant temperature sink at T_0. Assuming that any reversible cycle can be considered to be equivalent to an infinite number of Carnot cycles, prove that the maximum efficiency attainable for this heat engine is:**

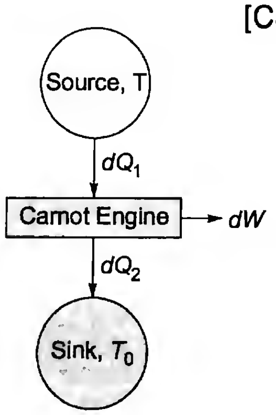
$$1 - \frac{T_0}{T_1 - T_2}\ln\frac{T_1}{T_2}.$$

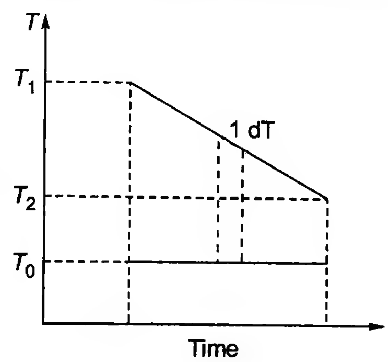
[CSE (Mains) 2002 : 20 Marks]

Solution:

Consider the Carnot engine as shown above.

Assume at the given instant engine source is at a temperature T. For the temperature to change by dT,





Heat extracted from source by the engine

$= dQ_1 = \dot{m}C_p dT$

Efficiency of Carnot engine at this instant, $\eta_T = \left(1 - \dfrac{T_0}{T}\right)$

$\therefore$ Work done by Carnot engine, $dW = \eta_T \cdot dQ_1 = \left(1 - \dfrac{T_0}{T}\right)\dot{m}C_p dT$

∴ Total work done by Carnot engine when temperature of source changes from T_1 to T_2,

$$= \int_{T_1}^{T_2} \left(1 - \frac{T_0}{T}\right) \dot{m} C_p dT$$

$$\therefore \quad W_{max} = \dot{m} C_p \left[(T_2 - T_1) - T_0 \ln\left(\frac{T_2}{T_1}\right) \right]$$

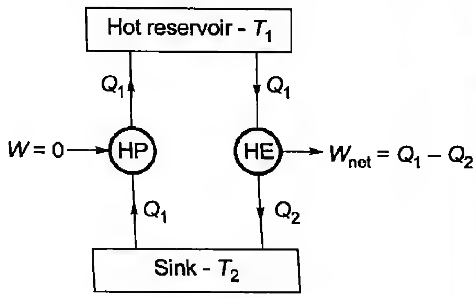
$$\text{Total heat extracted from source, } Q_1 = \int_{T_1}^{T_2} \dot{m} C_p dT = \dot{m} C_p (T_2 - T_1)$$

$$\therefore \quad \text{Minimum efficiency, } \eta = \frac{W_{max}}{Q} = \frac{\dot{m} C_p [(T_2 - T_1) - T_0 \ln(T_2 - T_1)]}{\dot{m} C_p (T_2 - T_1)} = 1 - \frac{T_0}{T_1 - T_2} \ln\left(\frac{T_1}{T_2}\right)$$

Q.10 Give the Kelvin-Planck's statement and Clausius statement of second law of thermodynamics and show that violation of either statement implies violation of the other. A Carnot engine I operates between two reservoirs at temperatures of 2000 K and *T* K. Another Carnot engine II operates between the reservoirs at *T* K and 300 K. If both the engines have the same efficiency, determine the value of *T*.

[CSE (Mains) 2002 : 20 Marks]

Solution:

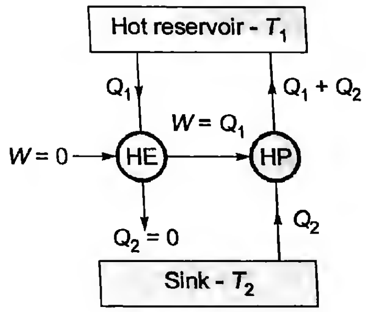


Kelvin-Planck's Statement: It is impossible for a heat engine working on a cycle to produce work continuously by exchanging heat with only one reservoir.

Clausius Statement: It is impossible to construct a device which will produce no effect other than transfer heat from a cooler body to a hotter body.

Equivalence of the two statements: Consider a H.P. as shown in figure which violates Clausius statement.

If this heat pump is operated along with a heat engine as shown in figure, net work, $W_{net} = Q_1 - Q_2$ is produced while heat is being exchanged only with sink. This is a violation of Kelvin Planck's statements.

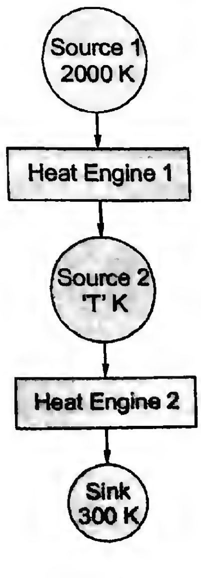


Consider a heat engine as shown in figure which violates Kelvin-Planck's statement.

This heat engine is used to power a heat pump which transfers heat Q_2 from sink and pumps heat $(Q_1 + Q_2)$ to source.

In effect this heat pump transfers heat Q_2 from sink to source without any external work, which is a violation of Clausius statement.

Consider the schematic for heat engines I and II operating in series.



For heat engine I

$$\text{Efficiency,} \quad \eta_I = \left(1 - \frac{T}{2000}\right)$$

For heat engine II,

$$\text{Efficiency,} \quad \eta_{II} = 1 - \frac{300}{T}$$

Since efficiency of both engines is same,

$$\eta_I = \eta_{II}$$

$$\Rightarrow \quad 1 - \frac{T}{2000} = 1 - \frac{300}{T}$$

$$\Rightarrow \quad T = \sqrt{300 \times 2000} = 774.6 \text{ K}$$

Q.11 A solar powered refrigeration system is run by heat transfer Q_H from solar collector at temperature 400 K and rejects heat Q_A at 300 K. It receives heat Q_C from cold space at 200 K. Assuming the cycle to be reversible, and using 1st law of thermodynamics and Clausius inequality, find the ratio Q_C/Q_H.

[CSE (Mains) 2003 : 20 Marks]

Solution:

Consider reversible refrigeration cycle as shown in figure exchanging heat with three sources - A, C and H.

From 1st law of thermodynamics,

$$Q_C + Q_H + Q_A = 0 \quad \text{... (i)}$$

From Clausius inequality, $\oint \frac{dQ}{T} = 0$ for a reversible cycle

Solving this integral for given refrigerator

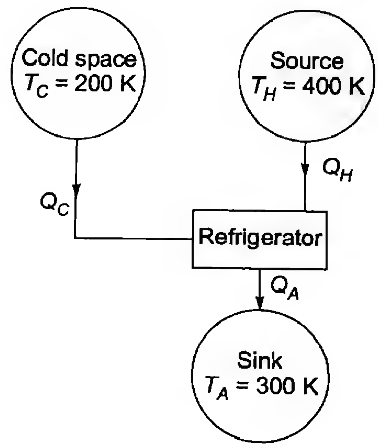
$$\frac{Q_H}{T_H} + \frac{Q_C}{T_C} + \frac{Q_A}{T_A} = 0$$

$$\Rightarrow \quad \frac{Q_H}{400} + \frac{Q_C}{200} + \frac{Q_A}{300} = 0 \quad \text{... (ii)}$$

From (i), we have,

$$Q_A = -(Q_C + Q_H)$$

Substituting this value in equation (ii), we get,



$$\frac{Q_H}{400} + \frac{Q_C}{200} - \frac{Q_C + Q_H}{300} = 0\text{s}$$

$$\Rightarrow \quad \frac{1}{400} + \frac{Q_C}{Q_H} \cdot \frac{1}{200} - \frac{1}{800} \cdot \frac{Q_C}{Q_H} - \frac{1}{300} = 0$$

$$\Rightarrow \quad \frac{Q_C}{Q_H}\left(\frac{1}{200} - \frac{1}{300}\right) = \frac{1}{300} - \frac{1}{400} = \frac{1}{12}$$

$$\Rightarrow \quad \frac{Q_C}{Q_H} = \frac{1}{2}$$

Q.12 Two reversible heat engines A and B are arranged in series. Heat engine A rejects heat directly to B. Engine A receives 300 kJ of heat at a temperature of 427°C from a high temperature source while engine B rejects heat to a cold sink at 7°C. If the work output of A is two times that of B, find

(i) Intermediate temperature between A and B.

(ii) Efficiency of each engine.

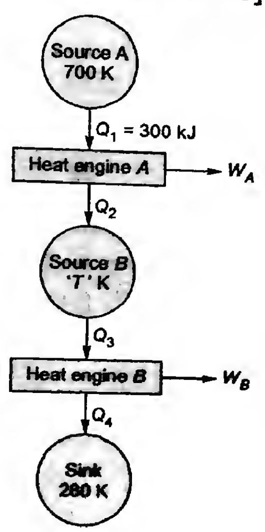
(iii) Heat rejected by engine A and received by engine B.

(iv) Heat rejected to the sink.

[CSE (Mains) 2003 : 30 Marks]

Solution:

Consider two heat engines A and B acting in series as shown in figure.



$$\text{Efficiency of engine } A,\ \eta_A = \left(1 - \frac{T}{700}\right)$$

Where, T - Temperature of intermediate between A and B.

$$\text{Heat received by } A,\ Q_1 = 300 \text{ kJ}$$

$$\text{Work output of engine } A,\ W_A = Q_1\left(1 - \frac{T}{700}\right)$$

$$W_A = 300\left(1 - \frac{T}{700}\right)$$

$$\text{Efficiency of engine } B,\ \eta_B = \left(1 - \frac{280}{T}\right)$$

Heat rejected by engine A, $Q_2 = Q_1 - W_A$

$$\Rightarrow \qquad Q_2 = 300 - 300\left(1 - \frac{T}{700}\right) = \frac{300 \cdot T}{700} = \frac{3}{7} \text{ kJ}$$

$$\text{Work output of engine, } B = \left(1 - \frac{280}{T}\right) \cdot \frac{3T}{7}$$

According to question, $\qquad W_A = 2 \cdot W_B$

$$\Rightarrow \qquad 300\left(1 - \frac{I}{700}\right) = 2 \cdot \frac{3T}{7}\left(1 - \frac{280}{T}\right)$$

$$\Rightarrow \qquad T = 420 \text{ K}$$

Efficiency of each engine, $\qquad \eta_A = 1 - \dfrac{420}{700} = 0.40 = 40\%$

$$\eta_B = 1 - \frac{280}{420} = 0.33 = 33.33\%$$

Heat rejected by engine A and received by engine B

$$= Q_3 = Q_2 = \frac{3T}{7} \text{ kJ} = 180 \text{ kJ}$$

$$\text{Heat rejected to the sink} = Q_4 = Q_3 - \frac{T_4}{T_3} = \frac{180 \times 280}{420}$$

$$= 120 \text{ kJ}$$

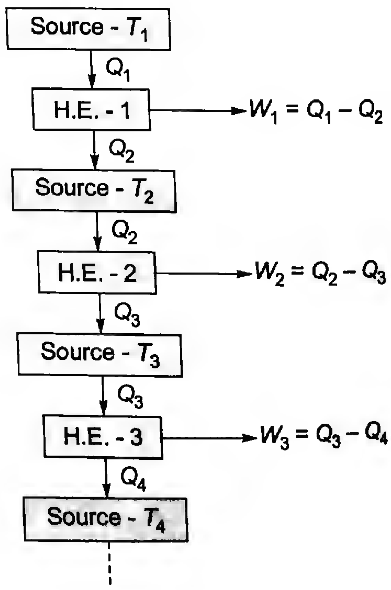
Q.13 Using concept of Second Law of thermodynamics show that it is impossible to reach absolute zero temperature.

[CSE (Mains) 2005 : 20 Marks]

Solution:

Consider a series of that engine starting from source temperature T_1, T_2..... and so on.

By definition of thermodynamic scale of temperature, we get



$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

$$\Rightarrow \qquad \frac{Q_1 - Q_2}{Q_2} = \frac{T_1 - T_2}{T_2}$$

Assume, $\qquad T_1 - T_2 = T_2 - T_3 = T_3 - T_4 \ldots\ldots$

$$\Rightarrow \qquad W_1 = W_2 = W_3 = W_4$$

Consider an infinite number of such heat engines placed in series such that total amount of work produced.

$$W_{total} = W_1 + W_2 + W_3 + W_4 \ldots\ldots \text{ approaches } Q_1.$$

In the limiting case, when required number of such heat engines have been placed, $Q_1 = W_{total}$. This is a violation of second law of thermodynamics since last heat engine rejects no heat to sink and produces work by exchanging heat with only one reservoir.

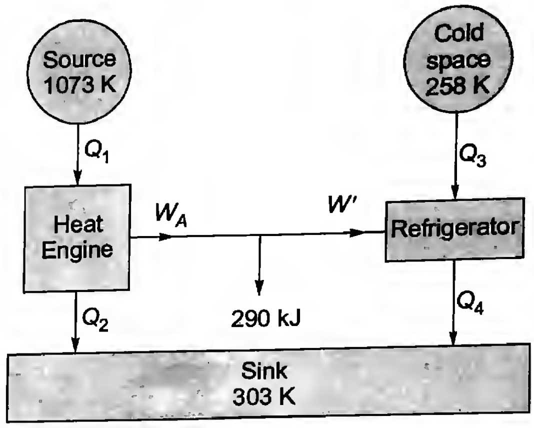
In this limiting case, temperature of sink becomes equal to absolute zero. Since this is a violation of second law of thermodynamics, absolute zero can never be achieved.

Q.14 **A reversible heat engine operating between thermal reversible reservoir at 800°C and 30°C drives a reversible refrigerator which refrigerates a space at –15°C and delivers heat to a thermal reservoir at 30°C. The heat input to the heat engine is 1900 kJ and there is a net work output from the combined plant (heat engine and refrigerator) of 290 kJ. Determine the heat transfer to the 30°C thermal reservoir.**

[CSE (Mains) 2005 : 30 Marks]

Solution:

Consider heat engine and refrigerator operating between source at 1073 K and sink at 303 K.



$$\text{Efficiency of Carnot heat engine} = 1-\frac{303}{1073} = 0.7176$$

Heat extracted by heat engine, $Q_1 = 1900$ kJ

$\therefore$ Work done by heat engine, $W_A = \eta . Q_1 = 0.7176 \times 1900$

$$= 1363.47 \text{ kJ}$$

Work transferred to refrigerator $= W_A - W_{net}$

$$= 1363.47 - 290$$

$\Rightarrow$ $W' = 1073.47$ kJ

$$\text{COP}_{\text{refrigerator}} = \frac{Q_3}{W'} = \frac{258}{303-258} = 5.73$$

$\Rightarrow$ $Q_3 = 6154.56$ kJ

$\therefore$ Heat transferred to 303 K reservoir by refrigerator $= Q_3 + W'$

$$Q_4 = 7228.03 \text{ kJ}$$

Heat transferred to 303 K reservoir by heat engine, Q_2

$$Q_2 = Q_1 - W_A = 1900 - 1363.47 = 536.53 \text{ kJ}$$

$\therefore$ Total head transferred to 303 K reservoir

$$Q_2 + Q_4 = 7764.56 \text{ kJ}$$

Q.15 **Consider a Carnot cycle heat engine operating in the outer space. Heat can be rejected from this engine only by thermal radiation, which is proportional to the radiator area and the fourth power of the absolute temperature of the radiator (T_1). Show that for a given engine work output and given temperature of the higher temperature reservoir (T_H), the radiator area will be a minimum when the ratio $T_1 / T_H = 3/4$.**

[CSE (Mains) 2007 : 20 Marks]

Solution:

Consider the Carnot engine operating in space as shown in figure.

Heat rejected in space by the engine,

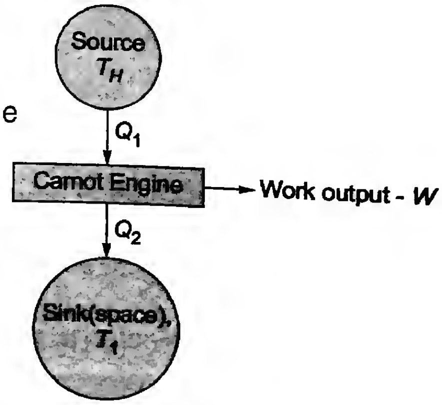
$$Q_2 \propto A \cdot T_1^4$$

$\Rightarrow$ $Q_2 = kA\, T_1^4$

where K - constant

A - radiator area

T_1 - temperature of radiator of engine



Hence, heat engine operates between sources at T_H and radiator at T_1.

Heat transfer to space, $Q_2 = Q_1 - w$

$\Rightarrow$ $kAT_1^4 = Q_1 - w$

Also, from thermodynamic definition of temperature,

$$\frac{Q_2}{Q_1} = \frac{T_1}{T_H} \Rightarrow Q_1 = \frac{Q_2 \cdot T_H}{T_1}$$

$$\therefore \quad kAT_1^4 = \frac{Q_2 T_H}{T_1} - w = kAT_1^3 T_H - w$$

$$\Rightarrow \quad A\left(\frac{T_H}{T_1} - 1\right) = \frac{w}{kT_1^4}$$

$$\Rightarrow \quad A = \frac{w}{K}\left[\frac{1}{T_1^3(T_H - T_1)}\right]$$

Differentiating A with respect to T_1, we get

$$\frac{dA}{dT_1} = \frac{w}{K}\left[\frac{-3T_H T_1^2 + 4T_1^3}{\left[T_1^3(T_H - T_1)\right]^2}\right] \quad \text{(Work done and source temperature are constant)}$$

For area of radiator to be minimum,

$$\frac{dA}{dT_1} = 0$$

$$\Rightarrow \quad -3T_H T_1^2 + 4T_1^3 = 0$$

$$\Rightarrow \quad \frac{T_1}{T_H} = \frac{3}{4} \quad \text{Hence, proved.}$$

Q.16 A Carnot engine operates between source temperature of 500 K and sink temperature of 300 K. It produces work utilizing the heat of 10 kJ from the source at 500 K. The work produced by this engine is utilized by a Carnot refrigerator operating between refrigerator temperature of 200 K and sink temperature of 300 K. Represent schematically these engine and refrigerator operations. Find:

(i) work produced by the Carnot engine.

(ii) refrigerating effect produced at 200 K by the Carnot refrigerator.

(iii) total heat rejected to the sink at 300 K.

(iv) by how much the refrigerator temperature be increased to get double the refrigerating effect as per (ii) above?

(v) total heat rejected to the sink at 300 K when the refrigerator operates as per the temperature for above.

[CSE (Mains) 2010 : 20 Marks]

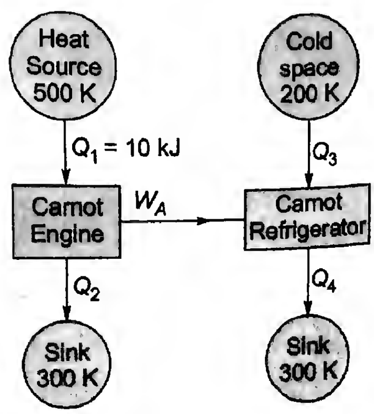
Solution:

Consider Carnot engine and Carnot refrigeration as shown in figure.

$$\text{Efficiency of Carnot engine, } \eta_C = 1 - \frac{T_{min}}{T_{max}} = 1 - \frac{300}{500} = 0.40$$

$$\therefore \quad \text{Work done by Carnot engine, } W = \eta_C \cdot Q_1 = 0.4 \times 10 = 4 \text{ kJ}$$

This work is utilized by Carnot refrigerator in coding the space at 200 k.



$$\therefore \quad COP_{refrigeration} = \frac{Q_3}{W} = \frac{T_3}{T_5 - T_3} = \frac{200}{300 - 200}$$

$$\Rightarrow \quad Q_3 = 2 \times W = 2 \times 4 = 8 \text{ kJ}$$

$\therefore$ Refrigeration effect produced is 8 kJ.

$$Q_2 = Q_1 - W = 10 - 4 = 6 \text{ kJ}$$

Also,

$$Q_4 = Q_3 + W = 8 + 4 = 12 \text{ kJ}$$

$\therefore$ Total heat rejected to sink by both heat engine and refrigerator is

$$= Q_2 + Q_4 = 6 + 12 = 18 \text{ kJ}$$

Refrigeration effect required = 2 × 8 = 16 kJ

$$COP_R = \frac{T_3}{300 - T_3} = \frac{16}{4}$$

$$\Rightarrow \quad T_3 = \frac{16 \times 300}{20} = 240 \text{ K}$$

Temperature should be increase by 40 K,

$$240 - 200 = 40 \text{ K}$$

Now heat rejected by refrigerator to surrounding

$$= Q_3' + W = 16 \text{ kJ} + 4 \text{ kJ} = 20 \text{ kJ}$$

∴ Total heat rejected to surroundings = 20 + 6 = 26 kJ

Q.17 Air enters a steady flow adiabatic turbine at 1600 K and exhaust to atmosphere at 1000 K, P_{atm} = 1 bar. If the second law efficiency is 85%, what is the turbine inlet pressure?
What is irreversibility during expansion process?
Given, surrounding temperature is 25°C.
Properties of air are given in table.

T, (K)	h, (kJ/kg)	s, (kJ/kg-K)
1000	1046	8.6905
1600	1757	8.1349

[CSE (Mains) 2012 : 12 Marks]

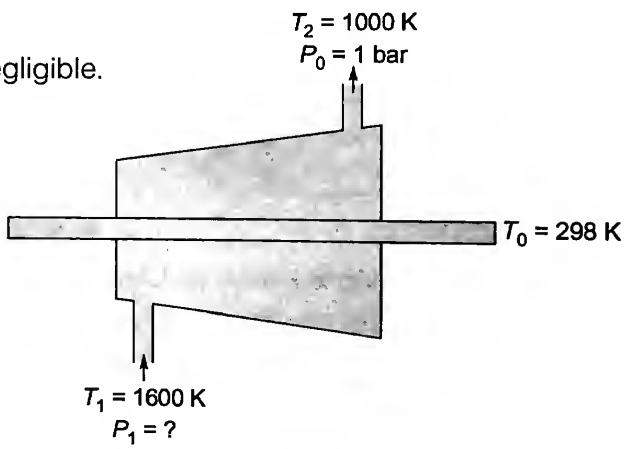
Solution:

Consider the expansion of air in turbine as shown in figure.

Assuming kinetic and potential energy changes of air are negligible.

Change in availability of air between state 1 and state 2,

$$X_1 - X_2 = (h_1 - h_2) - T_0 (S_1 - S_2)$$
$$= (1757 - 1046) - 298\,(8.1349 - 8.6905)$$
$$= 876.57 \text{ kJ/kg}$$



This is the maximum work that can be obtained from the turbine operating between the two states.

We know, $\eta = \dfrac{W_{actual}}{W_{max}} = 0.85$

$$\Rightarrow \quad W_{actual} = W_{max} \times 0.85 = 876.57 \times 0.85 \text{ kJ/kg} = 745.0845 \text{ kJ/kg}$$

Irreversibility during expansion process = $W_{max} - W_{actual}$ = 131.49 kJ/kg

Change in entropy during expansion process

$$\Delta S = C_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1} = 1.005\ln\frac{1000}{1600} - R\ln\frac{1}{P_1}$$

From Gouy Stodola theorem, we know

$$\text{Irreversibility, } I = T_0 \Delta S_{total}$$

$$\Rightarrow \quad \Delta S_{total} = \frac{I}{T_0} = \frac{131.49}{298} = 0.4412 \text{ kJ/K}$$

$$\therefore \quad \Delta S = 1.005\ln\frac{1000}{1600} - 0.287\ln\frac{1}{P_1} = 0.4412$$

$$\Rightarrow \quad P_1 = 24.12 \text{ bar}$$

Hence, inlet pressure of turbine is 24.12 bar.

Q.18 Prove that the cyclic integral of ratio between heat transfer and temperature of any thermodynamic process is less than or equal to zero.

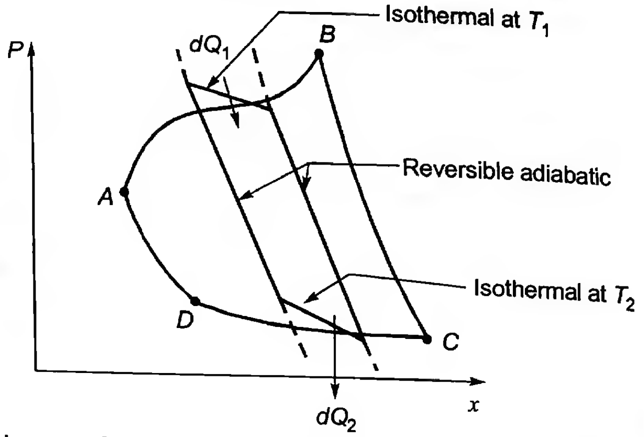
[CSE (Mains) 2013 : 20 Marks]

Solution:

Consider any general process cycle *ABCD* as shown in figure.

Assume that this cycle is made up of small cycles as shown in figure consisting of two isothermal and two adiabatic processes.

This cycle takes in heat dQ_1 at temperature T_1 and rejects heat dQ_2 at temperature T_2.



Efficiency of this cycle will be, $\eta = 1 - \dfrac{dQ_2}{dQ_1}$

Efficiency of such a cycle will be less than or equal to efficiency of a Carnot cycle operating between same temperature.

$$\therefore \qquad \eta < \eta_{Carnot}$$

$$\Rightarrow \qquad 1 - \frac{dQ_2}{dQ_1} \le 1 - \frac{T_2}{T_1}$$

$$\Rightarrow \qquad -\frac{dQ_2}{T_2} \le -\frac{dQ_1}{T_1} \quad \Rightarrow \quad \left(-\frac{dQ_2}{T_2}\right) + \frac{dQ_1}{T_1} \le 0$$

Since dQ_2 is the heat being rejected, its sign should be negative.

$$\Rightarrow \qquad \frac{dQ_1}{T_1} + \frac{-dQ_2}{T_2} \le 0$$

$$\Rightarrow \qquad \oint \frac{dQ}{T} \le 0$$

The inequating is known as Clausius inequality and it shows that cyclic integral of ratio of heat transfer and temperature of any thermodynamic process is less than or equal to zero.

4. Entropy

Q.19 Calculate the decrease in available energy when 25 kg of water at 97°C is mixed with 35 kg of water at 47°C, the pressure being constant and temperature of surroundings is 25°C. Specific heat of water is $C_{p,w}$ = 4.2 kJ/kgK

[CSE (Mains) 2003 : 20 Marks]

Solution:

Assume final temperature of water is 'T' K.

$$\text{Heat exchanged} = m_A C_{p,w}(370 - T) = m_B C_{p,w}(T - 320)$$

$$\Rightarrow \qquad 25\,(370 - T) = 35\,(T - 320)$$

$$\Rightarrow \qquad T = 340.83 \text{ K}$$

Change in entropy for 25 kg water during the process

$$\Delta S_1 = m_A\left[C_{p,w} \ln\frac{T_2}{T_1} - R \ln\frac{P_2}{P_1}\right]$$

Since pressure remains constant during the process, $P_2 = P_1$.

$$\therefore \qquad \Delta S_1 = 25 \times 4.2 \times \ln\frac{340.83}{370} = -8.62 \text{ kJ/K}$$

Change in entropy for 35 kg water during the process,

$$\Delta S_2 = m_B\left[C_{p,w}\ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}\right] = 35\times 4.2\times\ln\frac{340.83}{320} = 9.27 \text{ kJ/K}$$

Net entropy change during the process, $\Delta S = \Delta S_1 + \Delta S_2 = -8.62 + 9.27 = 0.6502$ kJ/K

We know, decrease in available energy in a process = $T_0\, S_{gen}$

Since no heat transfer to or from surrounding takes place, $\Delta S = S_{gen} = 0.6502$ kJ/K

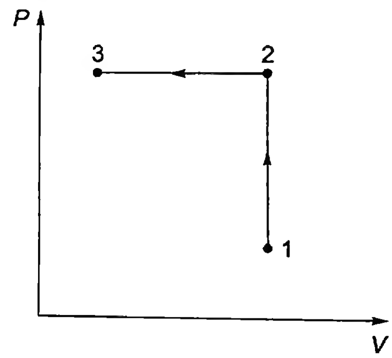
∴ Decrease in available energy of system, $T_0\Delta S = 298\times 0.6502 = 193.77$ kJ

Q.20 Air at 17°C and 1.1 bar occupies 0.05 m³. The air is first heated at constant volume until the pressure is 4.4 bar and then cooled at constant pressure back to original temperature. Calculate
(i) the net heat transfer to or from the air and
(ii) the net entropy change.
Given that for air C_p = 1.005 and C_v = 0.718 kJ/kg-K.

[CSE (Mains) 2003 : 30 Marks]

Solution:

Processes undergoes by the gas are depicted in the P-V diagram as shown in figure.



State 1: P_1 = 1.1 bar, V_1 = 0.05 m³, T_1 = 17°C = 290 K

State 2: $V_2 = V_1$ = 0.05 m³, P_2 = 4.4 bar

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\Rightarrow \quad T_2 = \frac{P_2}{P_2}\cdot T_1 = \frac{4.4}{1.1}\times 290 = 1160 \text{ k}$$

State 3: $P_3 = P_2$ = 4.4 bar, $T_3 = T_1$ = 290 k

$$\frac{P_3V_3}{T_3} = \frac{P_2V_2}{T_2}$$

$$\Rightarrow \quad V_3 = V_2\cdot\frac{T_3}{T_2} = 0.0125 \text{ m}^3$$

$$R \text{ for air} = C_p - C_v = 1.005 - 0.718 = 0.287 \text{ kJ/kgK}$$

$$\therefore \quad \text{Mass of gas in system} = \frac{PV}{RT} = \frac{1.1\times 10^5\times 0.05}{0.287\times 290\times 10^3} = 0.0661 \text{ kg}$$

$$\text{Heat transfer for step 1–2}, Q_{1\text{-}2} = mC_v(T_2 - T_1) = 0.0661\times 0.718\times(1160 - 290) = 41.28 \text{ kJ}$$

$$\text{Heat transfer for step 2–3}, Q_{2\text{-}3} = mC_p(T_3 - T_2) = 0.066\times 1.005\times(290 - 1160) = -57.79 \text{ kJ}$$

$$\text{Heat transfer in the process} = Q_{1\text{-}2} + Q_{2\text{-}3} = -16.51 \text{ kJ}$$

$$\text{Entropy change in process 1–2}, \Delta S_1 = m\left[C_p\ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}\right]$$

$$= 0.066\times\left[1.005\times\ln\frac{1160}{290} - 0.287\ln\frac{4.4}{1.1}\right] = 0.0658 \text{ kJ/K}$$

$$\text{Entropy change in process 2-3}, \Delta S_2 = m\left[C_p\ln\frac{T_3}{T_2} - R\ln\frac{P_3}{P_2}\right]$$

$$= 0.066\times 1.005\times\ln\left(\frac{290}{1160}\right) = -0.0921 \text{ kJ/K}$$

∴ Net entropy change in entire process, $\Delta S_1 + \Delta S_2 = -26.3$ J/K

Q.21 A 0.8 kg metal bar kept initially at 1500°C is removed suddenly from an oven and quenched by immersing it in a closed tank containing 12 kg of water kept initially at 400°C. The metal and water can be modeled as incompressible and the specific heat of water and metal are 4.18 kJ/kg-K and 0.5 kJ/kg-K respectively. The heat transfer from the tank may be neglected. Work out the following:

(i) Draw the system and system boundary and list assumptions made

(ii) The final temperature of metal bar and water

(iii) The entropy produced

[CSE (Mains) 2006 : 20 Marks]

Solution:

Assumptions:

(i) Water and metal bar together are considered as a system.

(ii) No heat loss occurs from the system boundary to surrounding.

(iii) Heat transfer by radiation to surrounding is negligible.

Assume final temperature of metal bar and water as 'T_f' K.

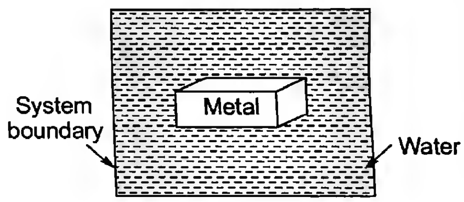
Amount of heat transferred from metal bar for a small change dT in its temperature, $dq = m_m C_{p,m}\, dT$

$\therefore$ Entropy change associated, $dS = \dfrac{dq}{T}$

$$\Rightarrow \quad dS = \frac{m_m C_{p,m} dT}{T}$$

Integrating both sides,

$$\int dS = \Delta S_m = \int_{1773}^{T_f} \frac{m_m C_{p,m} dT}{T}$$



$$\Rightarrow \quad \Delta S_m = m_m C_{p,m} \ln\left(\frac{T_f}{1773}\right)$$

Similarly, the entropy change for water $= \Delta S_\omega m_\omega C_{p,\omega} \ln\left(\dfrac{I_f}{673}\right)$

Heat lost by metal bar = Heat gained by water

$$m_m C_{p,m} (T_f - 1773) = m_\omega C_{p,\omega} (673 - T_f)$$

$$\Rightarrow \quad 0.8 \times 0.5 \times (1773 - T_f) = 12 \times 4.18 \times (T_f - 673)$$

$$\Rightarrow \quad T_f = 681.702 \text{ K}$$

$\therefore$ Entropy change for metal bar, $\Delta S_m = 0.8 \times 0.5 \times \ln\left(\dfrac{681.702}{1773}\right) = -0.383$ kJ/K

Entropy change for water, $\Delta S_\omega = 12 \times 4.18 \times \ln\left(\dfrac{681.702}{673}\right) = 0.563$ kJ/K

$\therefore$ Total entropy change of system,

$$\Delta S_m + \Delta S_\omega = 0.18 \text{ kJ/K}$$

We know that, $\Delta S + \int \dfrac{dQ}{T} = S_{gen}$

Since, no heat is exchanged with surrounding.

$$\int \frac{dQ}{T} = 0$$

$\therefore$ Entropy generated $= S_{gen} = \Delta S = 0.18$ kJ/K

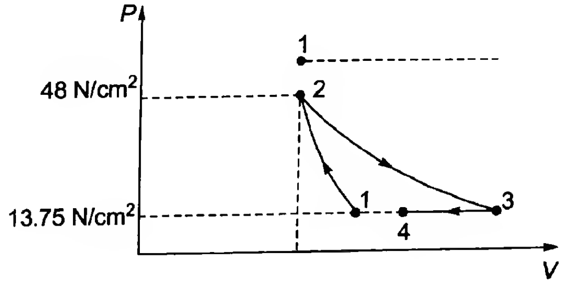
Q.22 **2 kg of air is first compressed from state 1 at 13.75 N/cm² and 5°C to state 2 at 48 N/cm² and 283°C. It is then throttled to state 3 until its pressure is again 13.75 N/cm². Finally it is cooled at constant pressure to state 4 until its volume becomes 50% of that before the cooling process. Determine the net change in entropy.**

$R = 0.291$ Nm/gK; $C_p = 1.004$ kJ/kgK

[CSE (Mains) 2009 : 20 Marks]

Solution:

Consider the various processes as shown in P-V diagram.



Process 1-2:

We know

$$Tds = dh - Vdp$$

$$\Rightarrow \quad dS = \frac{C_P dT}{T} - \frac{R}{P}dP$$

$$\Rightarrow \quad \Delta S = C_p \ln\frac{T_2}{T_1} - R\ln\frac{P_f}{P_i}$$

$$\Delta S_{1\text{-}2} = C_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1} = 1.004 \times \ln\frac{556}{278} - 0.291\ln\frac{48}{13.75} = 0.3328 \text{ kJ/kgK}$$

Process 2-3: Throttling process ($\Delta h = 0$)

for an ideal gas, in throttling process,

$$\Delta h = C_p dT = 0$$

$$\Rightarrow \quad dT = 0$$

$$\therefore \quad \Delta S_{2\text{-}3} = -R\ln\frac{P_3}{P_2} = -0.291 \times \ln\left(\frac{13.75}{48}\right) = 0.3637 \text{ kJ/kgK}$$

Process 3-4: Constant pressure compression

$$\frac{P_3V_3}{T_3} = \frac{P_4V_4}{T_4}$$

$$\Rightarrow \quad T_4 = T_3 \cdot \frac{V_4}{V_3} = \frac{T_3}{2} = \frac{T_2}{2} = 278 \text{ k}$$

(since $P_3 = P_4$ and $T_3 = T_2$)

$$\Delta S_{3\text{-}4} = C_p \ln\frac{T_4}{T_3} - R\ln\frac{P_4}{P_3} = 1.004 \ln\frac{278}{556} = -0.6959 \text{ kJ/kgK}$$

$\therefore$ Net change in entropy of gas $= m\Delta S_{net}$

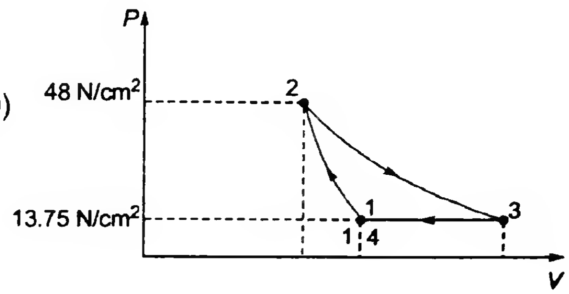
$$= m(\Delta S_{1\text{-}2} + \Delta S_{2\text{-}3} + \Delta S_{3\text{-}4})$$

$$= 2 \times (0.3328 + 0.3637 - 0.6959)$$

$$= 0 \text{ kJ/K}$$

Therefore we can conclude that point 1 and 4 are same.

Hence the graph can be redrawn as shown:



Q.23 **Derive an expression for entropy (S) as given below**

$$dS = \left(\frac{\delta Q}{T}\right)_{rev}$$

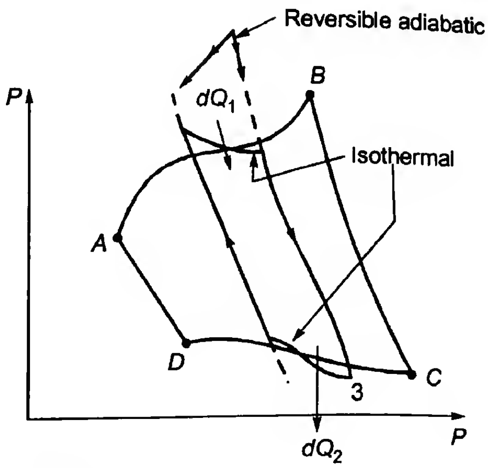
for a closed system undergoing a reversible process.

[CSE (Mains) 2013 : 10 Marks]

Solution:

Consider a general thermodynamic process $ABCD$.

Let this process be broken down into smaller cycle - example 1234 consisting of reversible adiabatic and isothermal process.



Efficiency of this reversible cycle, $\eta_{Carnot} = 1 - \frac{T_2}{T_1}$

or,
$$\eta = 1 - \frac{dQ_2}{dQ_1} = 1 - \frac{T_2}{T_1}$$

$$\Rightarrow \qquad \frac{dQ_1}{T_1} + \left(\frac{-dQ_2}{T_2}\right) = 0$$

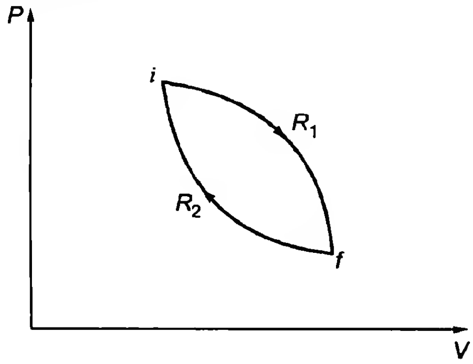
Since dQ_2 is heat rejected, it is taken as negative.

Similarly rest of the process $ABCD$ can also be broken down into similar cycles, so heat –

$$\frac{dQ_1}{T_1} + \frac{dQ_2}{T_2} + \frac{dQ_3}{T_3} + \frac{dQ_4}{T_4} + \ldots\ldots\ldots\ldots = 0$$

$$\Rightarrow \qquad \sum \frac{dQ_i}{T_i} = 0 \quad \Rightarrow \quad \oint \frac{dQ}{T} = 0$$

Now consider a cycle composed of two reversible process - R_1 and R_2 for cycle –



$$\oint \frac{dQ}{T} = 0$$

$$\Rightarrow \qquad \underset{R_1}{\int_i^f} \frac{dQ}{T} + \underset{R_2}{\int_f^i} \frac{dQ}{T} = 0$$

$$\Rightarrow \qquad \underset{R_1}{\int_i^f} \frac{dQ}{T} = \underset{R_2}{\int_i^f} \frac{dQ}{T}$$

Hence $\frac{dQ}{T}$ represents a property since it only depends on initial and final states. This property is known as entropy and is defined as:

For a reversible process - heat exchanged dQ_{rev} at temperature T.

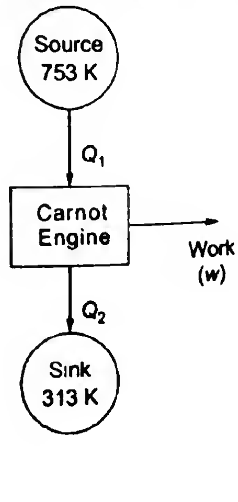
$$\therefore \qquad \text{Entropy change} = dS = \frac{dQ_{rev.}}{T}$$

Q.24 In a Carnot cycle, heat is received at 480°C and rejected at 40°C. The entropy of the sink increase, by 0.0785 kJ/kg K per cycle. Determine the work done per cycle.

[CSE (Mains) 2014 : 10 Marks]

Solution:

Consider the Carnot engine as shown in figure exchanging heat between source (480°C/753 K) and sink (40°C / 313 K).



$$\text{Efficiency of Carnot engine} = 1 - \frac{T_1}{T_2} = -\frac{313}{753}$$

$$\eta = 0.5843$$

Entropy of sink increases due to rejection of heat (Q_2) to sink, which takes place at constant temperature 313 K.

$$\therefore \qquad \text{Increase in entropy of sink, } \Delta S = \int \frac{dQ}{T} = \frac{Q_2}{T} = 0.0785 \text{ kJ/kg K}$$

$$\therefore \qquad Q_2 = 24.5705 \text{ kJ/kg per cycle}$$

We know, efficiency of Carnot cycle, $\eta = \frac{W}{Q_1} = \frac{W}{W+Q_2}$

$$\Rightarrow \quad 0.5843 = \frac{W}{24.5705 + W}$$

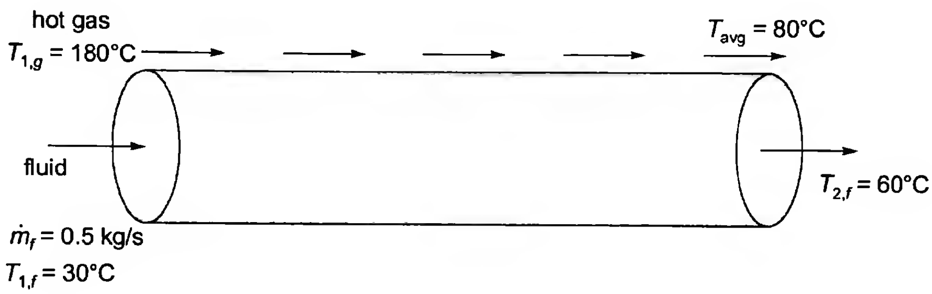
$\Rightarrow$ Work done per cycle = 34.54 kJ/kg

Q.25 A fluid flowing in a tube at the rate of 0.5 kg/s is heated from 30°C to 60°C by hot gases entering at a temperature of 180°C and leaving at 80°C. The specific heats of the fluid and gases are 4.186 kJ/kg K and 1.08 kJ/kg K. Calculate the change in entropy and increase in unavailable energy for ambient temperature of surrounding of 20°C.

[CSE (Mains) 2014 : 20 Marks]

Solution:

Consider heat exchange between the fluid and hot gases as shown in figure.



Heat exchange rate between the two fluids $= \dot{m}_f\, c_{p,f} \cdot (T_{f,2} - T_{f,1})$

$$= 0.5 \times 4.186 \times (30) = 62.79 \text{ kJ/s}$$

$$\text{Heat lost by hot gases} = \dot{m}_g \cdot c_{p,g}(T_{1,g} - T_{2,g}) = 62.79$$

$$\Rightarrow \quad \dot{m}_g = \frac{62.79}{1.08 \times 100} = 0.5814 \text{ kg/s}$$

Entropy change for the fluid

Assume fluid is present at a temperature T and dQ is the heat required for change in temperature by dT of the fluid.

$$\text{Entropy change, } dS = \frac{dQ}{T} = \dot{m}_f \frac{C_{p,f}\, dT}{T}$$

$$\Rightarrow \quad \Delta S_f = \int dS = \int_{303}^{333} \frac{0.5 \times 4.186\, dT}{T} = 0.5 \times 4.186 \cdot \ln\frac{333}{303}$$

$$= 0.1976 \text{ kJ/s K}$$

Similarly, for the gas,

$$\Delta S_g = m_g \cdot c_{p,g} \ln\frac{T_{f,g}}{T_{t,g}} = 0.5814 \times 1.08 \times \ln\frac{353}{453} = -0.1567 \text{ kJ/ K}$$

$\therefore$ Total entropy change, $\Delta S = \Delta S_f + \Delta S_g = (-0.1567 + 0.1976)$ kJ/s K

$$= 40.98 \text{ J/s K} = 41 \text{ J/s K}$$

Increase in unavailable energy for a process $= T_0\, \Delta S = 293 \times 41$

$$= 12.008 \text{ kJ/s} = 12 \text{ kJ/s}$$

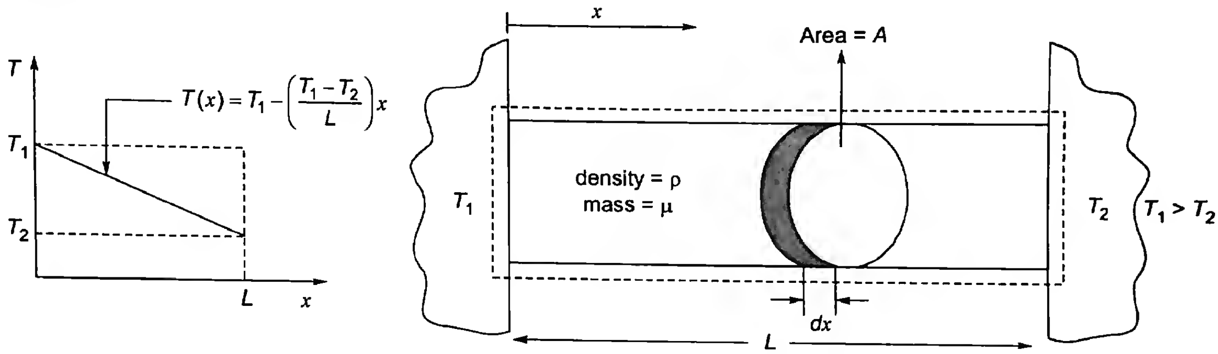
Q.26 A cylindrical rod of length L insulated on its lateral surface is initially in contact at one end with a wall at temperature T_1 and at the other end with a wall at lower temperature T_2. The temperature within the rod initially varies linearly with position x according to $T(x) = T_1 - \dfrac{T_1 - T_2}{L}x$. The rod is insulated on its ends and eventually comes to a final equilibrium state where the temperature is T_f. Evaluate T_f in terms of T_1 and T_2, and show that the amount of entropy generated is

$$S_{gen} = mc\left[1 + \ln T_f + \frac{T_2}{T_1 - T_2}\ln T_2 - \frac{T_1}{T_1 - T_2}\ln T_1\right]$$ where, c is the specific heat of the rod.

[CSE (Mains) 2016 : 20 Marks]

Solution:

Given: Temperature of rod varies linearly $T(x) = T_1 - \dfrac{T_1 - T_2}{2}x$. The rod is then insulated on its end to reach final equilibrium state of temperature T_F



Taking a small strip in the rod and calculating its change in internal energy.

$$du = dm \cdot C \cdot (T_f - T) = (\rho A dx) \cdot C \cdot (T_f - T)$$
$$du = \rho AC\,(T_f - T)dx$$

Now integrating the above to get the change of internal energy for the rod

$$\Delta U = \int_0^L dU = \int_0^L \rho AC(T_f - T)dx = \rho AC\int_0^L\left(T_F - T_1 + \frac{T_1 - T_2}{L}x\right)dx$$

$$\Delta U = \rho AC\left[(T_F - T_1)x + \frac{T_1 - T_2}{L} \times \frac{x^2}{2}\right]_0^L = \rho ALC\left[(T_F - T_1) + \frac{T_1 - T_2}{2}\right] = m \cdot c\left[T_F - \frac{T_1 + T_2}{2}\right] \{m = \rho AL\}$$

According to 1st law of thermodynamics (for the rod)

$$\Delta U = Q - W \text{ and since } Q = 0 \text{ and } w = 0 \qquad \therefore \Delta U = 0$$

Now, $$S_{gen} = \int_0^L dS = \int_0^L \rho AC\ln\left(\frac{T_F}{T}\right)dx = \rho Ac\int_0^L[\ln T_F - \ln T]dx$$

$$S_{gen} = \rho AC\left[L\ln(T_f) - \int_0^L \ln(T) \cdot dx\right] \qquad \text{...(i)}$$

Since $$T = T_1 - \frac{T_1 - T_2}{L}x \;; \quad \therefore \quad dT = -\frac{T_1 - T_2}{L}dx \quad \therefore dx = \frac{-L}{T_1 - T_2}dT$$

So, $$\int_0^L \ln(T)dx = \int_{T_1}^{T_2} \ln(T)\frac{-L}{T_1 - T_2}dT = \frac{L}{T_1 - T_2}\int_{T_2}^{T_1} \ln T \cdot dT = \frac{L}{T_1 - T_2}[T\ln T - T]_{T_2}^{T_1}$$

$$\int_0^L \ln(T)dx = L\left[\frac{T_1 \ln T_1}{T_1 - T_2} - \frac{T_2 \ln T_2}{T_1 - T_2} - 1\right] \quad ...(ii)$$

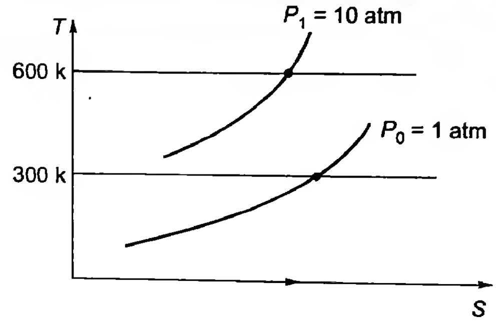
Substituting equation (ii) in (i) we get,

$$S_{gen} = \rho ALC\left[\ln T_F - \frac{T_1 \ln T_1}{T_1 - T_2} + \frac{T_2 \ln T_2}{T_1 - T_2} + 1\right] = m \cdot C\left[1 + \ln T_F + \frac{T_2}{T_1 - T_2}\ln T_2 - \frac{T_1}{T_1 - T_2}\ln T_1\right]$$

5. Availability

Q.27 What is the available energy i.e., the maximum amount of work that can be obtained from 1 kg of air at state point 1 in the figure. The dead state is also marked '0' in the figure?

[CSE (Mains) 2001 : 20 Marks]



Solution:

Assumptions:

- Air is contained inside a closed system, with no heat transfer from or to the surroundings during the process.
- KE and PE changes are negligible.
- Air follows ideal gas behaviour.

State - 1: P_1 = 10 atm, T_1 = 600 K, Mass of air, m = 1 kg
Dead state-0: P_0 = 1 atm, T_0 = 300 K

Volume of air at state 1, $$V_1 = \frac{mRT_1}{P_1} = \frac{1 \times 0.287 \times 600}{10 \times 101.325} = 0.17 \text{ m}^3$$

Volume of air at state 0, $$V_0 = \frac{mRT_0}{P_0} = \frac{1 \times 0.287 \times 300}{1 \times 101.325} = 0.8497 \text{ m}^3 \simeq 0.85 \text{ m}^3$$

Difference between entropy values at states '0' and '1':

$$S_1 - S_0 = \left[C_p \ln\frac{T_1}{T_0} - R\ln\frac{P_1}{P_0}\right] = 1.005\ln\frac{600}{300} - 0.287\ln\frac{10}{1} = 0.0358 \text{ kJ/kgK}$$

∴ Available energy of gas at state 1 with respect to dead state '0'.

$$\begin{aligned}\phi_1 - \phi_0 &= m[(u_1 - u_0) - T_0(S_1 - S_0)] + P_0(V_1 - V_0) \\ &= m[C_v(T_1 - T_0) - T_0(S_1 - S_0)] + P_0(V_1 - V_0) \\ &= 1\left[\frac{0.287}{1.4-1}(600-300) - 300 \times 0.0358\right] + 101.325 \times (0.17 - 0.85) = \mathbf{135.61 \text{ kJ}}\end{aligned}$$

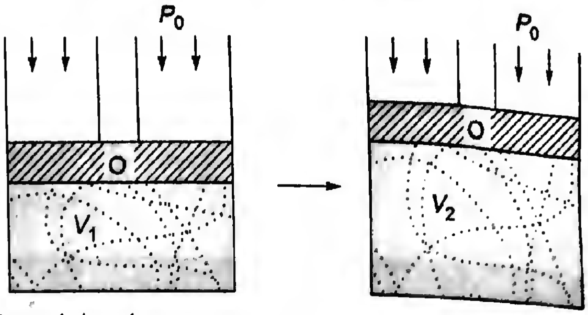
Q.28 **Define availability. Explain the difference between useful work and the maximum useful work done in the context of availability of a closed system. Heat flows through a wall at the rate of 3 × 10^5 kJ/h. The temperature of the two faces of the wall are 327°C and 207°C. If the surrounding are at 270°C, what is the loss in available energy?**

[CSE (Mains) 2002 : 30 Marks]

Solution:

Availability of a system is defined as the maximum work potential of the system at the given state. Availability is hence the maximum amount of work which a system can perform when it comes to equilibrium (temperature pressure and chemical) with respect to surroundings.

Consider a closed system as shown in figure undergoing a volume change from V_1 to V_2 against atmospheric pressure and doing work W in the process.



Work done by system = W

Work done by system in expansion against atmospheric pressure

$$P_0 = P_0(V_2 - V_1)$$
$$= W_{surr.}$$

This work done against surrounding is not available as useful work by the system.

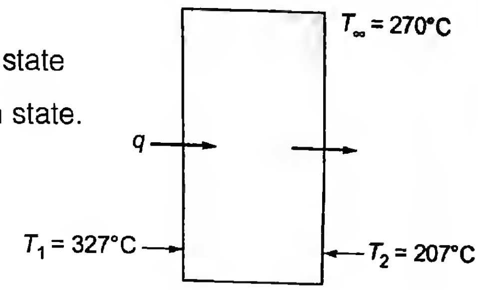
Hence, Net useful work = $W - W_{surr.} = W - P_0(V_2 - V_1)$

Maximum useful work that can be obtained from a closed system is defined as the availability of the closed system.

Availability of a closed system, $X = (e - e_0) - T_0(s - s_0)$

Where, e = Energy of system at given state

δ = Entropy of system at given state.



Consider the well as shown in figure

Heat being transferred through the wall,

$$q = 3 \times 10^5 \text{ kJ/h} = 83.33 \text{ kW}$$

Rate of entropy generation in the system

$$\Delta S = -\frac{Q_1}{T_1} + \frac{Q_1}{T_2} = 83.33\left[-\frac{1}{600} + \frac{1}{480}\right] = 34.72 \text{ J/ks}$$

Los in available energy = $T_0 \Delta S = 543 \times 34.72 = 18.853$ kJ/s = 18.853 kW

Q.29 6 kg of air at 600 K and 5.0 bar is enclosed in a closed system.

(i) Determine the availability of the system if the surrounding pressure and temperature are 1.0 bar and 300 K.

(ii) If the air is cooled at constant pressure to the atmospheric condition, determine the availability and effectiveness.

For air take, C_p = 1.005 kJ/kg.K

C_v = 0.718 kJ/kg-K and R = 0.287 kJ/kgK.

[CSE (Mains) 2005 : 30 Marks]

Solution:

Given: $m = 6$ kg, $P = 5$ bar, $T = 600$ k, $P_0 = 1$ bar, $T_0 = 300$ K

Assumption:

Air behave, as an ideal gas.

Changes in KE and PE are negligible.

$$\text{Volume of air at given state, } V = \frac{mRT}{P} = \frac{6 \times 0.287 \times 600}{5 \times 10^2} = 2.0664 \text{ m}^3$$

Volume of air at surrounding conditions,

$$V_0 = \frac{mRT_0}{P_0} = \frac{6 \times 0.287 \times 300}{1 \times 10^2} = 5.166 \text{ m}^3$$

Entropy change when gas moves from current state to surrounding state

$$\Delta S = S - S_0 = m\left[C_p \ln\frac{T}{T_0} - R\ln\frac{P}{P_0}\right] = 6\left[1.005\ln\frac{600}{300} - 0.287\ln\frac{5}{1}\right]$$

$$= 1.408 \text{ kJ/K}$$

Availability of the system at the given state $= \phi - \phi_0$

$$= (U + P_0V - T_0S) - (U_0 + P_0V_0 - T_0S_0)$$

$$= (U - U_0) + P_0(V - V_0) - T_0(S - S_0)$$

$$= mC_V(T - T_0) + P_0(V - V_0) - T_0 . \Delta S$$

$$= 6 \times 0.718 \times (600 - 300) + 100 \times (2.0664 - 5.166) - 300 \times 1.408$$

$$= 560.04 \text{ kJ}$$

Now air is cooled from this state to surrounding conditions at constant pressure.

Heat transferred at constant pressure from air for a small dT change in temperature, $dq = mC_p dT$

Maximum work (reversible) done possible with this heat transfer

$$= \eta_{Carnot} \cdot dq = \left(1 - \frac{T_0}{T}\right) mC_p dT$$

$\therefore$ Availability of air in this process = Maximum reversible work possible

$$= \int_T^{T_0} \left(1 - \frac{T_0}{T}\right) mC_p dT = \int_{600}^{300} \left(1 - \frac{T_0}{T}\right) \cdot (6 \times 1.005) dT$$

$$= 555.1 \text{ kJ}$$

Q.30 Define 'Availability' with regard to system. What is the other term by which this property is also referred to? Also derive an expression for "A" (the availability) for a reversible cycle in which heat 'Q' is withdrawn. The cycle works between temperature T and T_0.

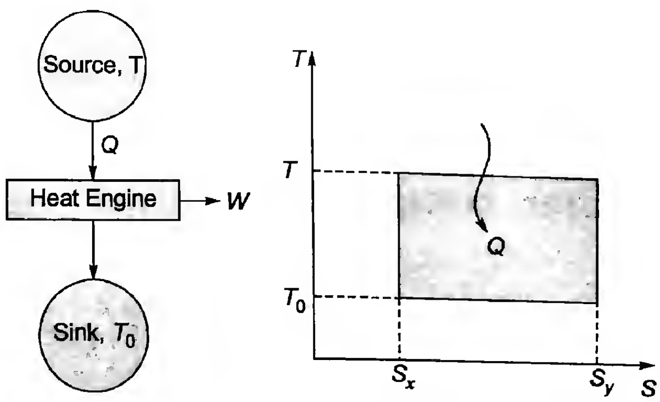
[CSE (Mains) 2009 : 20 Marks]

Solution:

Availability of a system is the maximum useful work potential of a system at the given state. Availability is equal to the maximum work obtainable from the system when it moves from the current state to a state of complete equilibrium with the surroundings (also known as dead state) - temperature, pressure and chemical equilibrium.

Consider a Carnot cycle as shown in figure which extracts heat from source at temperature T and rejects it to surrounding at temperature T_0.

Work 'W' is produced by the Carnot engine during the process. This is the maximum work which can be obtained by any cycle (reversible work) and hence, as per the definition of availability.



$$\text{Availability, } A = W_{max} = W_{Carnot}$$

$$\text{Efficiency of Carnot engine} = 1 - \frac{T_0}{T}$$

$\therefore$ Work obtained, $W = \eta_C \times Q \Rightarrow W = \left(1 - \frac{T_0}{T}\right)Q$

$\therefore$ Availability, $W = \left(1 - \frac{T_0}{T}\right)Q$

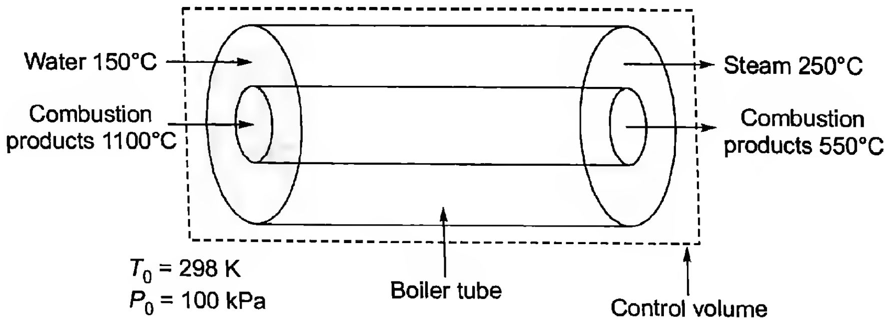
Q.31 The temperature of product of combustion in a boiler decreases from 1100°C to 550°C while the pressure remains constant at 0.1 MPa. Water at 0.8 MPa, 150°C is converted into steam at 0.8 MPa, 250°C with the surroundings is at 100 kPa and 25°C. Sketch the control volume depicting the terminal and process conditions and show on a T-s diagram the processes. Calculate the following:

(i) change in availability of water on unit mass of water basis.
(ii) change in availability of product of combustion per kg of water.
(iii) process irreversibility per unit mass of water.
(iv) second law efficiency, and
(v) entropy generated per kg of water.

Take average specific heat of product of combustion as 1.09 kJ/kg-K. Specify enthalpy and specific entropy of water at 0.8 MPa, 150°C are respectively, 632.2 kJ/kg and 1.8418 kJ/kg-K and that of steam at 0.8 MPa, 250°C are respectively 2950 kJ/kg and 7.0389 kJ/kg-K.

[CSE (Mains) 2010 : 20 Marks]

Solution:



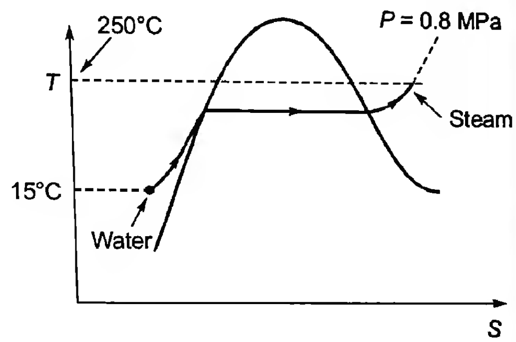
(i) Change in availability of water on unit mass of water basis

$$= X_2 - X_1 = (h_2 - h_1) - T_0 (S_2 - S_1)$$

[Assuming *KE* and *PE* change are negligible]

$$= (2950 - 632.2) - 298 (7.0389 - 1.8418)$$

$$= 769.06 \text{ kJ/kg}$$



(ii) Heat exchanged between combustion products and water in boiler

$$= (mC_p \Delta T)_{\text{combustion products}} = m_w (\Delta h)_{\text{water}}$$

$$\Rightarrow \quad m \cdot 1.09 \cdot 550 = m_w \cdot (2950 - 632.2)$$

$$\Rightarrow \quad m = m_w \cdot 3.866$$

For 1 kg water, $m = 3.866$ kg

Availability of combustion products at 1100°C = Maximum reversible work that can be obtained when gases achieve surrounding conditions from current state

$$\therefore \quad X_{1100°C} = W_{max} = \int_{1373}^{298} \left(1 - \frac{298}{T}\right) - (mC_p \cdot dT) = -3.866 \times 1.09 \cdot \int_{1373}^{298} \left(1 - \frac{298}{T}\right) dT = 2611.62 \text{ kJ}$$

$$X_{550°C} = W_{max} = \int_{823}^{298} \left(1 - \frac{298}{T}\right)(-mC_p dT) = 936.645 \text{ kJ}$$

$\therefore$ Change in availability of combustion products per kg

$$\text{Water} = X_{1100°C} - X_{550°C} = 1674.98 \text{ kJ}$$

This implies a reduction in availability of combustion products.

(iii) Process irreversibility per unit mass of water

= Availability supplied by combustion products – Availability recovered by water

= (1674.98 – 769.06 kJ) = **905.92 kJ**

(iv) Second law efficiency,

$$\eta_{II} = \frac{\text{Exergy recovered}}{\text{Energy supplied}} = \frac{769.06}{1674.98} = 45.91\%$$

(v) We know, from Gouy Stodola theorem:

Irreversibility in a process

$$= I = T_0 \, \Delta S_{total}$$

∴ Total entropy generated,

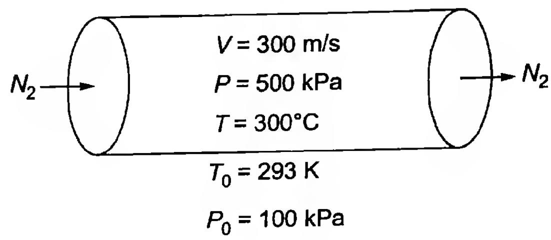
$$\Delta S_{total} = \frac{I}{T_0} = \frac{905.92}{298} = 3.04 \text{ kJ/K per kg of water}$$

Q.32 Nitrogen flows in a pipe with velocity 300 m/s at 500 kPa, 300°C. What is availability with respect to an ambient atmosphere at 100 kPa and 20°C?

[CSE (Mains) 2012 : 12 Marks]

Solution:

Consider flow of Nitrogen as shown in pipe.



Availability of Nitrogen for flow in given conditions,

$$X = (h - h_0) - T_0 (S - S_0) + \frac{V^2}{2} + gz$$

Assuming pipe is at the same level of elevation as ground, $z = 0$

for nitrogen,molecular mass = 28 gram/mol.

∴ Gas constant for N_2 = $R = \frac{\bar{R}}{M} = \frac{8.314}{28}$ = 0.2969 kJ/kg K ; γ for N_2 (since it is diatomic) = 1.4

$$\therefore \quad C_p = \frac{\gamma R}{\gamma - 1} = 1.03925 \text{ kJ/kgK}$$

$$S - S_0 = C_p \ln\frac{T}{T_0} - R\ln\frac{P}{P_0} = 1.03925 \times \ln\left(\frac{573}{293}\right) - 0.2969\ln\left(\frac{500}{100}\right) = 0.2192 \text{ kJ/kg K}$$

∴ Availability of N_2 flowing,

$$X = (h - h_0) - T_0(S - S_0) + \frac{V^2}{2} = C_p(T - T_0) - T_0(S - S_0) + \frac{V^2}{2}$$

$$= 1039.25 \times (573 - 293) - 293 \times (219.2) + \frac{300^2}{2} = 271.764 \text{ kJ/kg}$$

Q.33 Define availability of a closed and steady-flow system. Atmospheric air is compressed steadily from 100 kPa, 27°C to 500 kPa,117°C, by a compressor that is cooled only by atmospheric air. Neglecting kinetic energy changes, determine the minimum work required per kg of air compressed.

[CSE (Mains) 2013 : 15 Marks]

Solution:

Availability of a system can be defined as the maximum work that can be obtained when a system arrives at the dead state or environmental conditions from the current state.

For a closed system, availability of a system is defined as below–

$$\text{Availability, } X = (U - u_0) + P_0 (V - v_0) - T_0 (S - S_0) + \frac{v^2}{2} + gz$$

where, P_0, V_0, T_0 denote ambient conditions.

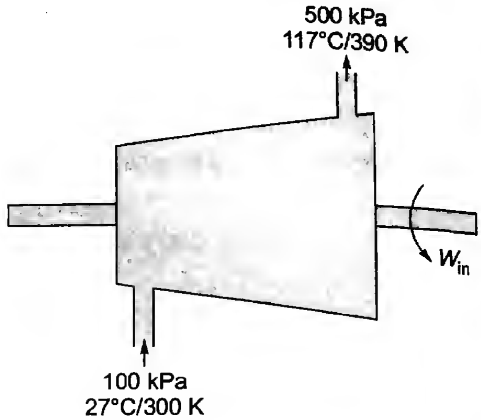
For a steady flow system, availability also includes work potential of flow work, along with that of closed system as above.

Work potential of flow work= $PV - P_0 V = (P - P_0) v$

∴ Availability of steady flow system

$$X = (h - h_0) - T_0(S - S_0) + \frac{V^2}{2} + gz$$

Consider steady flow compressor as shown in figure



Assuming ambient conditions as,

$$P_0 = 100 \text{ kPa}$$

$$T_0 = 298 \text{ K}$$

Increase in availability of air in this steady flow process will be equal to the minimum compression work required.

Neglecting changes in kinetic and potential energy,

$$\text{Change in availability} = X_2 - X_1 = (h_2 - h_1) - T_0(S_2 - S_1)$$

$$\text{For air, } C_p = 1.005 \text{ kJ/kg K}, \quad R = 0.287 \text{ kJ/kg K}$$

Change in entropy of air in the process,

$$S_2 - S_1 = C_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1} = 1.005 \times \ln\frac{390}{300} - 0.287\ln\frac{500}{100} = -0.1982 \text{ kJ/kg K}$$

$$\therefore \quad \text{Change in availability} = \Delta X = (h_2 - h_1) - T_0(S_2 - S_1) = C_p(T_2 - T_1) - T_0\Delta S$$

$$= 1.005 \times (390 - 300) - 298 \times (-0.1982) \doteq 149.52 \text{ kJ/kg}$$

∴ Minimum work required by compressor,

$$W_{min} = \Delta X = 149.52 \text{ kJ/kg}$$

6. Gases and Mixture

Q.34 A perfectly insulated chamber is divided in two parts by a diaphragm. 1.0 kg of oxygen is stored in one part while 7.0 kg of hydrogen is stored in another part. Both the gases are at the same temperature and pressure of 450 K and 1.0 bar respectively. They are mixed together by removing the diaphragm. Determine the loss in availability after mixing if the surrounding temperature is 290 K. The value of universal gas constant

R_0 = 8314 J/kg-mol-K.

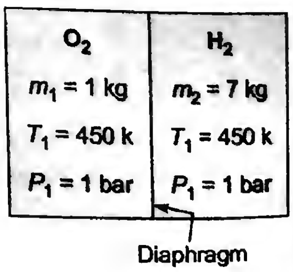
[CSE (Mains) 2004 : 30 Marks]

Solution:

Consider the two gases in insulated chamber before diaphragm between them is removed.

$$\text{Universal gas constant, } R_0 = 8314 \text{ J/kg-mol-K}$$

$$\text{Gas constant for oxygen, } R_{O_2} = \frac{R_0}{M} = \frac{8314}{32} = 0.2598 \text{ kJ/kg K}$$



$$\text{Gas constant for hydrogen, } R_{H_2} = \frac{R_0}{M} = \frac{8314}{2} = 4.157 \text{ kJ/kg K}$$

Ratio of specific heats for O_2 (diatomic),

$$\gamma = 1.4$$

Ratio of specific heats for H_2 (diatomic),

$$\gamma = 1.4$$

$$\therefore \quad C_{v,O_2} = \frac{R_{O_2}}{\gamma - 1} = 0.6495 \text{ kJ/kg K} \; ; \; C_{v,H_2} = \frac{R_{H_2}}{\gamma - 1} = 0.6495 \text{ kJ/kg K}$$

$$\text{Volume of } O_2 \text{ compartment, } V_{O_2} = \frac{m_1 R_{O_2} T_1}{P_1} = \frac{1 \times 0.2598 \times 450}{1 \times 10^2} = 1.1691 \text{ m}^3$$

$$\text{Volume of } H_2 \text{ compartment, } V_{H_2} = \frac{m_2 R_{H_2} T_1}{P_1} = \frac{7 \times 4.157 \times 450}{1 \times 10^2} = 130.95 \text{ m}^3$$

Total volume of both compartment, $V = 132.12 \text{ m}^3$

Pressure and temperature of both gases remains constant after mixing two.

$$\text{Entropy change for } O_2,\ \Delta S_1 = m_1\left[C_v \ln\frac{T_2}{T_1} + R\ln\frac{V_2}{V_1}\right]$$

$$= 1 \times \left[0.2598 \ln\left(\frac{132.12}{1.1691}\right)\right] \text{ kJ/K} = 1.228 \text{ kJ/K}$$

$$\text{Entropy change for } H_2,\ \Delta S_2 = m_2\left[C_v \ln\frac{T_2}{T_1} + R\ln\frac{V_2}{V_1}\right]$$

$$= 7 \times \left[4.157 \ln\frac{132.12}{130.95}\right] = 0.2588 \text{ kJ/K}$$

$\therefore$ Total entropy change $= 1.228 + 0.2588$

$$\Delta S = 1.487 \text{ kJ/K}$$

Loss in availability post mixing $= T_0 \Delta S = 290 \times 1.487 =$ **431.23 kJ**

7. Thermodynamic Relations

Q.35 Develop the Clapeyron equation for the pure substance changing the phase. Hence find the enthalpy of evaporation for R-22 at –10°C and compare the same with the tabulated value. What is the percentage error involved?

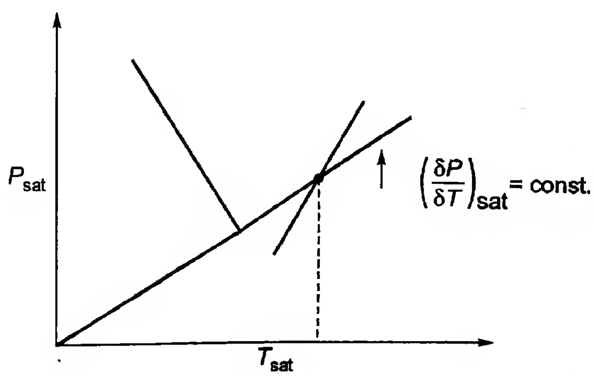
Properties of R-22

T_s, °C	P_s, kPa	V_f, litres/kg	V_g, m³/kg	h_{fs}, kJ/kg
–20	244.72	0.7409	0.0929	220.331
–10	354.16	0.7587	0.0654	213.136
0	497.41	0.7783	0.0472	205.369

[CSE (Mains) 2001 : 30 Marks]

Solution:

Consider variation of saturation pressure with saturation temperature.
During a phase change process, saturation pressure which varies only with saturation temperature.



$$\therefore \quad P_{sat} = \int(T_{sat})$$

$$\Rightarrow \quad \left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{dS}{dV}\right)_{sat}$$

From Maxwell's equation, we have—

$$\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$$

$$\therefore \quad \left(\frac{dP}{dT}\right)_{sat} = \frac{S_g - S_f}{V_g - V_f} \quad \text{... (i)}$$

Also, $\quad TdS = dh - vdp$

During phase change process, $dp = 0$ (pressure remains constant at P_{sat}),

$$\therefore \qquad TdS = dh \Rightarrow S_g - S_f = \frac{h_{fg}}{T} \qquad \text{... (ii)}$$

$\therefore$ From (i) and (ii), we get:

$$\left(\frac{\partial P}{\partial T}\right)_{sat} = \frac{h_{fg}}{TV_{fg}}$$

This equation is known as Clapeyron equation. Calculating h_{fg} using Claperon equation

$$\left(\frac{\partial P}{\partial T}\right)_{sat\,@\,-10°C} \cong \left(\frac{\Delta P}{\Delta T}\right)_{sat\,@\,-10°C} = \frac{P_{sat@20°C} - P_{sat@0°C}}{20} = \frac{497.41 - 244.72}{20} = 12.6345 \text{ kPa/K}$$

$$h_{fg} = \left(\frac{\partial P}{\partial T}\right)_{sat} \times T \times V_{fg} = 12.6345 \times 263 \times (0.0654 - 0.0007587) = 214.8 \text{ kJ/kg}$$

Tabulated value of the $h_{fg\,@\,-10°C} = 213.136$ kJ/kg

Error % = 0.78%

Hence, value of h_{fg} obtained is very close to the tabulated value.

Q.36 Prove the following for an ideal gas:

$$dS = \frac{dV}{V} + C_v \frac{dP}{P}$$

Using this result show that for an ideal gas undergoing an isentropic change of state with constant specific heat PV^{γ} = constant.

[CSE (Mains) 2004 : 20 Marks]

Solution:

For an ideal gas, internal energy and enthalpy are functions of temperature alone.

$$\therefore \qquad dh = C_p dT \quad \text{and} \quad du = C_v dT$$

Also, ideal gas equation holds, $PV = RT$

From 1st law of thermodynamics,

$$q = dU + W = du + pdV$$

also,

$$q = TdS = du + PdV \qquad \text{... (i)}$$

$$h = u + pv$$

$$\Rightarrow \qquad dh = du + pdV + vdp$$

$$\therefore \qquad TdS = dh - vdp \qquad \text{... (ii)}$$

From (i)

$$TdS = C_v dT + pdV \qquad \text{... (iii)}$$

From (ii)

$$TdS = C_p dT - vdp \qquad \text{... (iv)}$$

Combining (iii) and (iv),

$$dT = \frac{TdS - PdV}{C_v} = \frac{TdS + vdp}{C_p}$$

$$\Rightarrow \qquad TC_p dS - PC_p dV = TC_v dS + VC_v dP$$

$$\Rightarrow \qquad TdS(C_p - C_v) = TC_p dV + VC_v dP$$

We know,

$$C_p - C_v = R$$

$$\Rightarrow \qquad RTdS = PC_p dV + VC_v dP$$

$$\Rightarrow \qquad PVdS = PC_p dV + VC_v dP$$

$$\Rightarrow \qquad dS = C_p \frac{dV}{V} + C_v \frac{dP}{P}$$ Hence, proved

For isentropic process of an ideal gas,

$$dS = 0$$

$\therefore$ $$dS = C_p \frac{dV}{V} + C_v \frac{dP}{P} = 0$$

$\Rightarrow$ $$\frac{dP}{dV} = -\frac{C_p}{C_v} \cdot \frac{P}{V} = -\gamma \cdot \frac{P}{V}$$

where γ-ratio of specific heats of ideal gas.

$\therefore$ $$\frac{dP}{P} = -\gamma \frac{dV}{V}$$

Integrating both sides, we get

$$\ln P = -\gamma \ln V + \ln C, \quad \text{where } C \text{ is a constant} = \ln (V^{-\gamma} \cdot C)$$

$\Rightarrow$ $$P = V^{-\gamma} \cdot C$$

$\Rightarrow$ $$PV^{\gamma} = C$$

where C is a constant.

Q.37 Using Maxwell's relations, show that for a pure substance,

$$TdS = C_p\, dT - T v \beta\, dp = C_v dT + T\frac{\beta}{K} dv = \frac{KC_v dp}{\beta} + \frac{C_p}{\beta_v} dv$$

where β is the coefficient of cubical expansion, K is coefficient of compressibility and C_p, C_v are specific heats at constant pressure and constant volume respectively.

[CSE (Mains) 2005 : 20 Marks]

Solution:

For a pure substance, for an internally reversible process, we have –

$$TdS = dh - vdp \qquad \text{... (i)}$$

Coefficient of cubical expansion, $\beta = \frac{1}{v}\left(\frac{dV}{dT}\right)_P$

From (i), for a constant pressure process $dp = 0$

$\therefore$ $$TdS = C_p\, dT - vdp$$

$\Rightarrow$ $$C_p = T\left(\frac{\partial S}{\partial T}\right)_P$$

Considering entropy 'S' as a function of T and P. The,

$$dS = \left(\frac{\partial S}{\partial T}\right)_P dT + \left(\frac{\partial S}{\partial P}\right)_T dP \qquad \text{... (ii)}$$

From Maxwell's relations, we get,

$$\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P = -\beta \cdot v \text{ (from definition of } \beta)$$

Substituting above and value of C_p in eqn. (ii) above

$$dS = \frac{C_p}{T} dT + (-\beta \cdot v) dP$$

$\Rightarrow$ $$TdS = C_p\, dT - Tv\beta\, dP \quad \text{Hence, proved.}$$

By definition of coefficient of compressibility,

$$K = \frac{-1}{V}\left(\frac{\partial V}{\partial P}\right)_T$$

Consider entropy as a function of 'T' and 'V'. Then

$$dS = \left(\frac{\partial S}{\partial T}\right)_V dT + \left(\frac{\partial S}{\partial V}\right)_T dV \quad \text{... (iii)}$$

From Maxwell's equation,

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$$

From TdS relations

$$TdS = du + pdv = C_v\, dT + pdv$$

For a constant volume process, $dV = 0$

$$\Rightarrow \quad C_v = T\left(\frac{\partial S}{\partial T}\right)_V$$

Also since P, V and T are independent variables, applying cyclic relation, we get

$$\left(\frac{\partial V}{\partial T}\right)_P \cdot \left(\frac{\partial T}{\partial P}\right)_V \cdot \left(\frac{\partial P}{\partial V}\right)_V = -1$$

$$\Rightarrow \quad \frac{\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P}{\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T} = -\left(\frac{\partial P}{\partial T}\right)_V \quad \text{[Using reciprocity relation, } \left(\frac{\partial V}{\partial P}\right)_T = \left(\frac{\partial P}{\partial V}\right)_T \text{]}$$

$$\Rightarrow \quad \frac{\beta}{K} = \left(\frac{\partial P}{\partial T}\right)_V \quad \text{... (iv)}$$

Substituting value of C_v in equation (iii), we get

$$dS = \frac{C_v dT}{T} + \left(\frac{\partial P}{\partial T}\right)_V dV = \frac{C_v dT}{T} + \frac{\beta}{K} dV$$

$$\Rightarrow \quad TdS = C_v dT + \frac{T\beta}{K} dV \qquad \text{Hence, proved.}$$

Considering entropy as a function of 'P' and 'V',

$$dS = \left(\frac{\partial S}{\partial P}\right)_V dP + \left(\frac{\partial S}{\partial V}\right)_P dV \quad \text{... (v)}$$

From definition of K. C_v and β, we get

$$\frac{KC_v}{\beta} = \frac{\frac{-1}{V}\cdot\left(\frac{\partial V}{\partial P}\right)_T}{\frac{1}{V}\cdot\left(\frac{\partial V}{\partial T}\right)_P} \cdot \left(\frac{\partial S}{\partial T}\right)_V = -\left(\frac{\partial T}{\partial P}\right)_V \cdot T\left(\frac{\partial S}{\partial T}\right)_V = -T\frac{\left(\frac{\partial S}{\partial T}\right)_V}{\left(\frac{\partial P}{\partial T}\right)_V}$$

From Maxwell's relations, we get

$$\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$$

From reciprocity relation, $$\left(\frac{\partial S}{\partial V}\right)_T = \frac{1}{\left(\frac{\partial V}{\partial S}\right)_T}$$

$$\therefore \quad \frac{KC_v}{\beta} = -T\cdot\left(\frac{\partial S}{\partial T}\right)_V \cdot \left(\frac{\partial V}{\partial S}\right)_T = -T\frac{-1}{\left(\frac{\partial T}{\partial V}\right)_S} = \frac{T}{\left(\frac{\partial T}{\partial V}\right)_S}$$

From Maxwell's relations, we get,

$$\left(\frac{\partial T}{\partial V}\right)_S = \left(\frac{\partial P}{\partial S}\right)_V$$

$$\therefore \quad \frac{KC_V}{\beta} = \frac{T}{\left(\frac{\partial P}{\partial S}\right)_V} = T\left(\frac{\partial S}{\partial P}\right)_V \Rightarrow \left(\frac{\partial S}{\partial P}\right)_V = \frac{KC_V}{\beta T} \quad \text{... (vi)}$$

Similarly,

$$\frac{C_p}{v\beta} = \frac{T\left(\frac{\partial S}{\partial T}\right)_P}{\left(\frac{\partial V}{\partial T}\right)_P}$$

From Maxwell's relations, we get,

$$\left(\frac{\partial V}{\partial T}\right)_P = -\left(\frac{\partial S}{\partial P}\right)_T = \frac{-1}{\left(\frac{\partial P}{\partial S}\right)_T}$$

$$\therefore \quad \frac{C_p}{v\beta} = -T\cdot\left(\frac{\partial S}{\partial T}\right)_P\cdot\left(\frac{\partial P}{\partial S}\right)_T$$

For three independent variables S, T, P from cyclic relation,

$$\left(\frac{\partial S}{\partial T}\right)_P\left(\frac{\partial T}{\partial P}\right)_S\left(\frac{\partial P}{\partial S}\right)_T = -1$$

$$\Rightarrow \quad \left(\frac{\partial S}{\partial T}\right)_P\left(\frac{\partial P}{\partial S}\right)_T = \frac{-1}{\left(\frac{\partial T}{\partial P}\right)_S}$$

From Maxwell's relations,

$$\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P$$

$$\therefore \quad \left(\frac{\partial S}{\partial T}\right)_P\cdot\left(\frac{\partial P}{\partial S}\right)_T = \frac{-1}{\left(\frac{\partial V}{\partial S}\right)_P} = -\left(\frac{\partial S}{\partial V}\right)_P$$

$$\therefore \quad \frac{C_p}{\beta\cdot V} = -T\times -\left(\frac{\partial S}{\partial V}\right)_P = T\left(\frac{\partial S}{\partial V}\right)_P \Rightarrow \left(\frac{\partial S}{\partial V}\right)_P = \frac{1}{T}\frac{C_p}{\beta V} \quad \text{... (vii)}$$

From (v), (vi) and (vii), we get,

$$dS = \frac{KC_V}{\beta T}dP + \frac{1}{T}\frac{C_p}{\beta V}dV$$

$$\Rightarrow \quad TdS = \frac{KC_V}{\beta T}dP + \frac{C_p}{\beta V}dV \quad \text{Hence, proved.}$$

Q.38 With the help of Maxwell's relation of thermodynamics, prove that Joule-Thomson coefficient, μ_J of a gas is given by the following expression:

$$\mu_J = \left(\frac{\partial T}{\partial p}\right)_h = \frac{T^2}{C_p}\left[\frac{\partial}{\partial T}\left(\frac{v}{T}\right)_p\right]$$

[CSE (Mains) 2006 : 20 Marks]

Solution:

By definition of Joule Thomson coefficient,

$$\mu_J = \left(\frac{\partial T}{\partial p}\right)_h \quad \text{... (i)}$$

Consider entropy S as a function of T and P,

$$dS = \left(\frac{\partial S}{\partial T}\right)_p + \left(\frac{\partial S}{\partial P}\right)_T dp$$

Also from TdS relations

$$dh = TdS + vdp = T\left(\frac{\partial S}{\partial T}\right)_p dT + \left[v + T\left(\frac{\partial S}{\partial P}\right)_T\right]dp$$

We know,

$$C_p = T\left(\frac{\partial S}{\partial T}\right)_p$$

Also from Maxwell's relations,

$$\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_p$$

Hence, we get

$$dh = C_p dT + \left[v - \left(\frac{\partial v}{\partial T}\right)_p \cdot T\right]dp$$

For a process with constant enthalpy, $dh = 0$,

$$\Rightarrow \quad \left(\frac{\partial T}{\partial P}\right)_h = \frac{-1}{C_P}\left[v - T \cdot \left(\frac{\partial v}{\partial T}\right)_P\right] \quad \text{... (ii)}$$

$$\frac{\partial}{\partial T}\left(\frac{v}{T}\right)_P = v \cdot \frac{-1}{T^2} + \frac{1}{T}\left(\frac{\partial v}{\partial T}\right)_P = \frac{1}{T^2} \times \left[-v + T\left(\frac{\partial v}{\partial T}\right)_P\right] \quad \text{... (iii)}$$

From (i), (ii) and (iii), we get

$$\mu_J = \left(\frac{\partial T}{\partial P}\right)_h = \frac{1}{C_p} \cdot T^2\left[\frac{\partial}{\partial T}\left(\frac{v}{T}\right)_P\right]$$

$$\Rightarrow \quad \mu_J = \left(\frac{\partial T}{\partial P}\right)_h = \frac{T^2}{C_p}\left[\frac{\partial}{\partial T}\left(\frac{v}{T}\right)_P\right] \quad \text{hence, proved}$$

Q.39 The equation of state of a certain gas is $v = \frac{RT}{P} + \frac{K}{RT}$ (where K is a constant). Show that the change in temperature during throttling (enthalpy before and after remains the same) of such a gas from an initial pressure P_1 to a final pressure P_2 is given by

$$\frac{T_2^2 - T_1^2}{4K} = \frac{P_1 - P_2}{C_p R}$$

where, C_p is assumed constant during the process 1-2.

[CSE (Mains) 2008 : 20 Marks]

Solution:

Consider entropy 'S' as a function of 'T' and 'S',

$$ds = \left(\frac{\partial S}{\partial T}\right)_P dT + \left(\frac{\partial S}{\partial P}\right)_T dP$$

Substituting this value of ds in TdS equation

$$TdS = dh - vdp$$

$\Rightarrow \qquad dh = TdS + vdp = \left[T\left(\frac{\partial S}{\partial T}\right)_P\right]dT + \left[T\left(\frac{\partial S}{\partial P}\right)_T + v\right]dP \qquad \text{... (i)}$

Consider enthalpy h as a function of T and P. Then

$$dh = \left[\left(\frac{\partial h}{\partial T}\right)_P\right]dT + \left[\left(\frac{\partial h}{\partial P}\right)_T\right]dP$$

$$= C_p dT + \left(\frac{\partial h}{\partial T}\right)_T dP \quad \text{[By definition of } C_p\text{]} \qquad \text{... (ii)}$$

From Maxwell's equation

$$\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P \quad \Rightarrow \quad \left(\frac{\partial h}{\partial P}\right)_T = V - T\left(\frac{\partial V}{\partial T}\right)_P$$

From (i) and (ii)

$\therefore \qquad dh = C_p dT + \left[V - T\left(\frac{\partial V}{\partial T}\right)_P\right]dP$

In this case, enthalpy is constant.

$\therefore \qquad dh = 0$

$$\left(\frac{\partial V}{\partial T}\right)_P = \frac{\partial}{\partial T}\left(\frac{RT}{P} + \frac{k}{RT}\right)_P = \frac{R}{P} - \frac{k}{RT^2}$$

$\therefore \qquad V - T\left(\frac{\partial V}{\partial T}\right)_P = V - \frac{RT}{P} + \frac{k}{RT} = \frac{k}{RT} + \frac{k}{RT} = \frac{2k}{RT}$

$\therefore \qquad dh = C_P dT + \left[V - T\left(\frac{\partial V}{\partial T}\right)_P\right]dP = 0$

$\Rightarrow \qquad -C_p\, dT = \frac{2k}{RT}dP$

$\Rightarrow \qquad -T \cdot C_p dT = \frac{2k}{R}dP$

Integrating both sides with in proper limits,

$$-C_p \int_{T_1}^{T_2} T \cdot dT = \frac{2K}{R}\int_{P_1}^{P_2} 1 \cdot dP$$

$$\frac{-C_p}{2}[T^2]_{T_1}^{T_2} = \frac{2k}{R}[P]_{P_1}^{P_2}$$

$\Rightarrow \qquad \frac{C_p(T_2^2 - T_1^2)}{4} = \frac{2k}{R}(P_1 - P_2)$

$\Rightarrow \qquad \frac{T_2^2 - T_2^2}{4k} = \frac{P_1 - P_2}{C_p R} \qquad$ Hence, proved.

Q.40 Define the Joule-Thomson coefficient and prove that for an ideal gas, the value of Joule-Thomson coefficient tends to zero.

[CSE (Mains) 2009 : 10 Marks]

Solution:

[Refer Question Number 38]

From Maxwell's relations, we know

$$\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P$$

$\therefore$ $$dh = C_p dT + \left[V - \left(\frac{\partial V}{\partial T}\right)_P \cdot T\right] dP$$

For $$dh = 0,\ C_p dT + \left[V - \left(\frac{\partial V}{\partial T}\right)_P \cdot T\right] dP$$

$$\mu_{JT} = \left(\frac{\partial T}{\partial P}\right)_h = \frac{1}{C_P}\left[\left(\frac{\partial V}{\partial T}\right)_P \cdot T - V\right]$$

For an ideal gas,

$$PV = RT \quad \Rightarrow \quad V = RT/P$$

$\Rightarrow$ $$\left(\frac{\partial V}{\partial P}\right)_P = \frac{R}{P}$$

$\therefore$ $$V - \left(\frac{\partial V}{\partial T}\right)_P \cdot T = V - \frac{RT}{P} = V - V = 0$$

Hence for an ideal gas

$$\mu_{JT} = \frac{-1}{C_p}\left[V - \left(\frac{\partial V}{\partial T}\right)_P \cdot T\right] = 0$$

Q.41 Derive equations for the change in internal energy and entropy of a gas which obeys the Van der Walls equation of state.

[CSE (Mains) 2009 : 15 Marks]

Solution:

$$dU = C_v dT + \left[T\left(\frac{\partial P}{\partial T}\right)_V - P\right] dV$$

From Van der Waals equation,

$$\left(P + \frac{a}{V^2}\right)(v - b) = RT$$

$\Rightarrow$ $$P + \frac{a}{V^2} = \frac{RT}{v - b}$$

$\Rightarrow$ $$P = \frac{RT}{v - b} - \frac{a}{V^2}$$

$$\left(\frac{\partial P}{\partial T}\right)_V = \frac{R}{v - b}$$

$\therefore$ $$T\left(\frac{\partial P}{\partial T}\right)_V - P = \frac{RT}{v - b} - P = \frac{a}{v^2}$$

$\therefore$ $$dU = C_v dT + \frac{a}{v^2} dV$$

Assuming C_v stays constant for the gas over given range of temperature.
Integrating within proper limits, we get

$$U_2 - U_1 = C_v(T_2 - T_1) + \left(\frac{1}{v_1} - \frac{1}{v_2}\right)$$

This is the required expression for internal energy change for a gas following Van der Waals equation
Expressing entropy S as function of T and V,

$$dS = \left(\frac{\partial S}{\partial V}\right)_T dV + \left(\frac{\partial S}{\partial T}\right)_v dT$$

By definition $$C_v = T\left(\frac{\partial S}{\partial T}\right)_v$$

From Maxwell's relations, we get

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$$

$$\Rightarrow \quad dS = \left(\frac{\partial P}{\partial T}\right)_V dV + \frac{C_v dT}{T}$$

For a gas obeying Van der Waals equation

$$P = \frac{RT}{v-b} - \frac{a}{v^2}$$

$$\therefore \quad \left(\frac{\partial P}{\partial T}\right)_V = \frac{R}{v-b}$$

$$\therefore \quad dS = \left(\frac{R}{v-b}\right)dV + \frac{C_v dT}{T}$$

Assuming C_v as constant and integrating, we get

$$S_2 - S_1 = R\ln\left(\frac{v_2 - b}{v_1 - b}\right) + C_v \ln\left(\frac{T_2}{T_1}\right)$$

This is the required expansion for entropy change.

Q.42 Derive $\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P$, $\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$, $\left(\frac{\partial V}{\partial T}\right)_P = \left(\frac{\partial S}{\partial P}\right)_T$ from the first principles. Explain any assumptions needed here.

[CSE (Mains) 2012 : 20 Marks]

Solution:

Assumptions:

- Relation involving entropy calculations (T-dS relations) are valid only for an internally reversible process. However, results from these equations are valid for irreversible processes as well.
- Only boundary work or ($P * dV$) work has been considered.

From 1st law of thermodynamics, we have

$$dQ = dU + \delta W = dU + PdV$$

By definition of entropy change, we get

$$dS = \frac{dQ}{T}$$

$$\Rightarrow \quad dQ = TdS$$

$$\therefore \quad TdS = dU + PdV \quad \text{... (i)}$$

From definition of enthalpy, we get

$$h = u + pv$$

$$\Rightarrow \quad dh = du + pdv + vdp$$

$$\therefore \quad TdS = (dh - pdv - vdp) + pdv$$

$$\Rightarrow \quad TdS = dh - vdp \quad \text{... (ii)}$$

Helmholtz function (a) and Gibb's function (g) are defined as below

$$a = u - Ts$$

$$g = h - Ts$$

Differentiating above two functions, we get

$$da = du - TdS - SdT$$

From (1), we get

$$da = -SdT - pdv \quad \text{... (iii)}$$

$$dg = dh - TdS - SdT$$

From (2), we get

$$dg = vdp - SdT \quad \text{... (iv)}$$

Since enthalpy (h), internal energy (u), Helmholtz function (a) and Gibbs function (g) are properties of system and hence continuous point function with exact differentials, we get

From (2)
$$dh = TdS + vdp$$

$\therefore$
$$\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P$$

From (3)
$$da = -PdV - SdT$$

$\Rightarrow$
$$\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$$

From (4)
$$dg = vdp - SdT$$

$\Rightarrow$
$$\left(\frac{\partial V}{\partial T}\right)_P = \left(\frac{\partial S}{\partial P}\right)_T$$

Hence, proved.

Q.43 Hot gases enter the blades of a gas turbine with a velocity of 550 m/s and leave with a velocity of 120 m/s. There is an increase in the enthalpy of the gases in the blade passages to the extent of 5.1 kJ/kg. The rate of gas flow is 98 kg/min. Determine the power produced.

[CSE (Mains) 2013 : 15 Marks]

Solution:

Given: V_1 = 550 m/sec, V_2 = 120 m/sec, Δh = increase in enthalpy = $h_2 - h_1$ = 5.1 kJ/kg,

$\dot{m}_g$ = 98 kg/min = 1.63 kg/sec

Assuming the flow of hot gases through the gas turbine to be adiabatic process.

$$\dot{Q} = 0$$

$\because$
$$\dot{Q} - \dot{W} = \dot{m}\left[(h_2 - h_1) + \left(\frac{V_2^2 - V_1^2}{2}\right)\right]$$

$$\dot{W} = \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right)\right] = 1.63\left[(-5.1) + \left(\frac{550^2 - 120^2}{2000}\right)\right]$$

$$\dot{W} = 226.4885 \text{ kW}$$

Power produced is $\dot{W}$ = 226.4885 kW.

Q.44 Show that the slope of a reversible adiabatic process on a temperature vs pressure diagram, when multiplied by c_P is give by $TV\beta$.

[CSE (Mains) 2015 : 10 Marks]

Solution:

Consider three independent variables - P, S and T.

Using cyclic relation for partial differentials of independent variables, we get

$$\left(\frac{\partial P}{\partial S}\right)_T \left(\frac{\partial S}{\partial T}\right)_P \left(\frac{\partial T}{\partial P}\right)_S = -1 \quad \text{... (i)}$$

Slope of temperature vs pressure graph for a reversible adiabatic or isentropic process = $\left(\frac{\partial T}{\partial P}\right)_S$

From (i)

$$\left(\frac{\partial T}{\partial P}\right)_S = \frac{-1}{\left(\frac{\partial P}{\partial S}\right)_T\left(\frac{\partial S}{\partial T}\right)_P} = \frac{\left(\frac{\partial S}{\partial P}\right)_T}{\left(\frac{\partial S}{\partial T}\right)_P} \quad \text{... (ii)}$$

[from reciprocity relation, $-\frac{1}{\left(\frac{\partial P}{\partial S}\right)_T} = \left(\frac{\partial S}{\partial P}\right)_T$]

From Maxwell's relations, we get

$$\left(\frac{\partial S}{\partial P}\right)_T = \left(\frac{\partial V}{\partial T}\right)_P \quad \text{... (iii)}$$

We know,

$$TdS = dh - vdp = C_p\, dT - vdp$$

For a constant pressure process, $dp = 0$

$$\Rightarrow \quad T\partial S = C_P \partial T - 0 = C_P \partial T$$

$$\Rightarrow \quad \left(\frac{\partial S}{\partial T}\right)_P = \frac{C_p}{T} \quad \text{... (iv)}$$

Solving (ii), (iii) and (iv), we get

$$\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial T}\right)_P \cdot \frac{T}{C_P}$$

We know, volume expansion coefficient,

$$\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P \Rightarrow \left(\frac{\partial V}{\partial T}\right)_P = \beta \cdot V$$

$$\therefore \quad \left(\frac{\partial T}{\partial P}\right)_S = \beta \cdot V \frac{T}{C_P}$$

$$\Rightarrow \quad C_P \times \left(\frac{\partial T}{\partial P}\right)_S = \beta V T$$

Hence, proved.

Q.45 Dry saturated steam at 5 bar enters a convergent-divergent nozzle at a velocity of 100 m/s. The exit pressure is 1.5 bar. The throat and exit areas are 1280 mm² and 1600 mm² respectively. Assuming isentropic flow upto the throat and taking the critical pressure ratio as 0.58, estimate the mass flow rate. If the nozzle efficiency is 0.973, determine the exit condition of steam dryness fraction. Show the process on T-S and h-s diagrams.

Properties of Steam

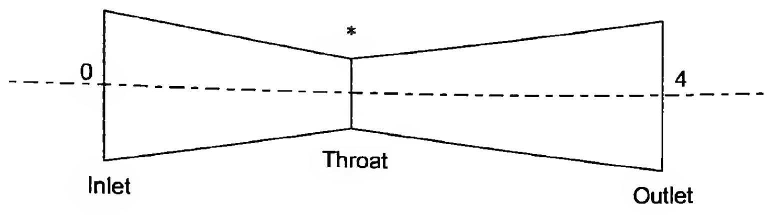
P (bar)	Enthalpy (kJ/kg)		Entropy (kJ/kg K)		Volume (m³/kg)	
	h_f	h_{fg}	s_f	s_{fg}	v_f	s_{fg}
5.0	640.23	2108.5	1.8607	4.9606	0.00109	0.3708
2.9	556	2168	1.660	5.344	0.00107	0.6253
1.5	467.11	2226.5	1.4336	5.7897	0.00105	1.158

[CSE (Mains) 2015 : 20 Marks]

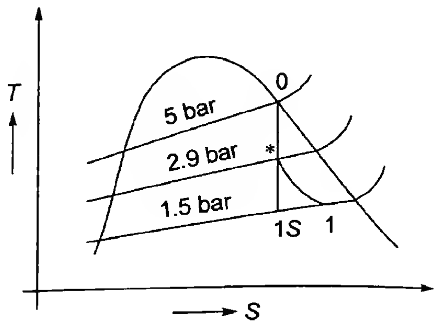
Solution:

Given: $V_1 = 100$ m/sec, $P_1 = 5$ bar, $P_2 = 1.5$ bar, $A^* = 1280$ mm², $A_2 = 1600$ mm²

$$\frac{P^*}{P_1} = 0.58 \quad \Rightarrow \quad P^* = 0.58 \times 5 = 2.9 \text{ bar}$$



Nozzle



From table given:

$$h_0 = 2108.5 + 640.23 = 2748.73 \text{ kJ/kg}$$

$$S_0 = 4.9606 + 1.8607 = 6.821 \text{ kJ/kg-K}$$

So,

$$S^* = 6.8213 = 1.660 + x^*(5.344)$$

$$x^* = 0.966$$

$$h^* = 556 + 0.966\,(2168) = 2650.28 \text{ kJ/kg}$$

Energy equation will give

$$h_0 + \frac{V_0^2}{2} = h^* + \frac{V^{*2}}{2}$$

$$2748.73 + \frac{(100)^2}{2000} = 2650.28 + \frac{V^{*2}}{2000}$$

$$\text{Velocity, } V* = 454.84 \text{ m/sec} = (v_f)^* + x^*(v_{fy})$$

$$v^* = 0.00107 + 0.966\,(0.6253) = 0.605 \text{ m}^3\text{/kg}$$

$$\text{Mass flow rate } (\dot{m}) = \frac{A^* V^*}{v^*}$$

$$\dot{m} = \frac{1280 \times 10^{-6} \times 456}{0.605} = 0.965 \text{ kg/sec}$$

$$\text{Nozzle efficiency } (\eta) = 0.973 = \frac{h^* - h_1}{h^* - h_{1s}}$$

$$0.973 = \frac{2650.28 - h_1}{2650.28 - h_{1s}} \quad \text{...(i)}$$

Also,

$$S_0 = S_{1s} = 6.8213 = 1.4336 + x_{1s}(5.7897)$$

$$x_{1s} = 0.93$$

$$h_{1s} = 467.11 + 0.93 \times 2226.5 = 2537.75 \text{ kJ/kg} \quad \text{...(ii)}$$

From (i) and (ii)

$$0.973 = \frac{2650.28 - h_1}{2650.28 - 2537.75}$$

$$h_1 = 2540.8 \text{ kJ/kg}$$

Let x_1 be the exit condition dryness fraction

$$h_1 = (h_f)_{1.5\,\text{bar}} + x_1(h_{fg})_{1.5\,\text{bar}}$$

$$2540.8 = 467.11 + x_1(2226.5)$$

$$x_1 = 0.932$$

■■■■

2 Fluid Mechanics

1. Fluid Kinematics

Q.1 With the help of a neat sketch explain the concept of a flow net. Clearly mention the various assumptions made. Also explain the uses of flow net. **[CSE (Mains) 2012 : 12 Marks]**

Solution:

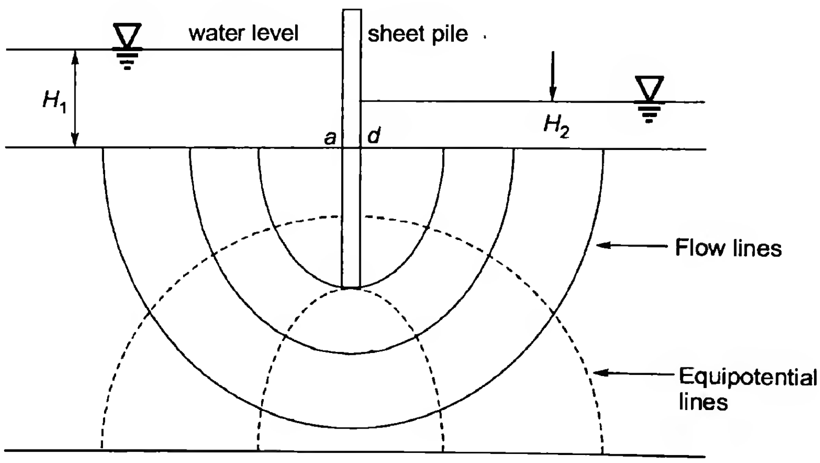
Flow Net: A grid obtained by drawing a series of equipotential lines and stream lines is called a flow net. The flow net is an important tool in analysing two-dimensional irrotational flow problems.

Uses of flow net:

1. Estimation of seepage loss from reservoir: It is possible to use the flow net in the transformed space to calculate the flow underneath the dam.
2. Determination of uplift pressures below dams: From the flow net, the pressure head at any point at the base of the dam can be determined. The uplift pressure distribution along the base can be drawn and then summed up.
3. Checking the possibility of piping beneath dams.

Assumptions:

1. Flow lines and equipotential lines intersect each other at 90 degrees.
2. The areas bounded by the flow lines and equipotential lines form approximate squares.
3. Flow nets must satisfy the boundary conditions of flow field.
4. The potential drop in any two consecutive equipotential lines is same/constant.
5. Flow lines and equipotential lines are smooth curves.
6. Flow lines do show refraction at the interface between two soils having different coefficient of permeability.



Flow Nets : Flow around sheet pile wall.

2. Fluid Dynamics

Q.2 Write Bernoulli's equation and the conditions for which it is valid. If a fluid obeying Bernoulli's equation has elevation, velocity and pressure at a point as 30 m, 50 m/s and 50 bar respectively, calculate the total energy per unit mass of this fluid if its density is 1000 kg/m^3.

[CSE (Mains) 2013 : 10 Marks]

Solution:

Bernoulli's equation is obtained by integrating the Euler's equation of motion as,

$$\int \frac{dp}{\rho} + \int g\,dz + \int v\,dv = \text{constant}$$

$$\frac{p}{\rho g} + z + \frac{v^2}{2g} = \text{constant}$$

where, $\frac{p}{\rho g}$ = Pressure energy per unit weight of fluid or pressure head.

$\frac{v^2}{2g}$ = kinetic head

z = potential head

Assumptions:

Following are the assumptions made in the derivation of the Bernoulli's equation:

1. The flow is incompressible
2. The flow is irrotational
3. The flow is steady
4. The fluid is ideal i.e. viscosity is zero.

Given: $z = 30$ m, $v = 50$ m/sec, $P = 50$ bar $\Rightarrow 50 \times 10^5$ N/m^2 $\rho = 1000$ kg/m^3

Total energy per unit mass of the fluid = Pressure head + Velocity head + potential head

$$= \frac{P}{\rho g} + \frac{V^2}{2g} + z = \left(\frac{50 \times 10^5}{10^3 \times 10}\right) + \frac{(50)^2}{(2 \times 10)} + 30$$

$$= 500 + 125 + 30 = 655 \text{ m}$$

3. Viscous Flow in Pipes

Q.3 The velocity distribution in the fully developed flow region of steady incompressible laminar flow of a fluid in a horizontal pipe is given by $\frac{u}{u_m} = 1 - \left(\frac{r}{R}\right)^2$, where u and u_m are respectively the velocities of any radial distance, r, from the axis and at the axis of the pipe and R is the radius. Show that the kinetic energy correction factor, α and momentum correction factor, β are respectively equal to 2 and 1.33.

[CSE (Mains) 2010 : 20 Marks]

or

Explain what do you mean by kinetic energy correction factor. Show that the kinetic energy correction factor for laminar flow through a circular pipe is 2. Further, explain what will happen to the kinetic energy correction factor when the flow is considered to be turbulent.

[CSE (Mains) 2015 : 20 Marks]

Solution:

Given: $\frac{u}{u_m} = 1 - \left(\frac{r}{R}\right)^2$

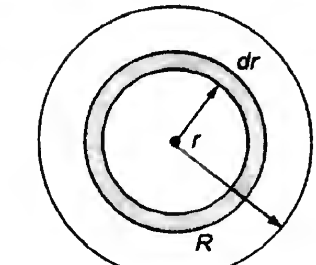
$$\text{Momentum correction factor, } \beta = \frac{\text{Momentum per second based on actual velocity}}{\text{Momentum per seconds based on average velocity}}$$

Considering an elementary area dA in form of a ring at a radius r and width dr,

$$dA = 2\pi r d_r$$

Momentum of fluid through ring per second

$$dM = \rho \times dA \times u^2 = \rho \times (2\pi r d_r)u^2 = 2\pi\rho u^2 r dr$$



Total actual momentum of fluid per second across the section,

$$M = \int_0^R 2\pi\rho u^2 r dr$$

As,
$$u = u_m\left[1-\left(\frac{r}{R}\right)^2\right] = \int_0^R 2\pi\rho u_m^2\left[1-\left(\frac{r}{R}\right)^2\right]^2 r\,dr$$

$$= 2\pi\rho u_m^2\left[\int_0^R r\left(1+\frac{r^4}{R^4}-\frac{2r^2}{R^2}\right)\right]dr = 2\pi\rho u_m^2\left[\int_0^R\left(r+\frac{r^5}{R^4}-\frac{2r^3}{R^2}\right)dr\right]$$

$$= 2\pi\rho u_m^2\left[\frac{R^2}{2}+\frac{R^2}{6}-\frac{2R^2}{4}\right] = 2\pi\rho u_m^2\left[\frac{12R^2+4R^2-12R^2}{24}\right]$$

$$M = \frac{\pi\rho u_m^2 R^2}{3} \qquad \text{...(i)}$$

Momentum of fluid per second based on average velocity $\bar{u}$

$$= \rho A\bar{u}^2$$

As,
$$\bar{u} = \frac{u_m}{2}$$

$$\bar{M} = \rho A\bar{u}^2 = \frac{\rho A u_m^2}{4} = \frac{\rho\pi R^2 u_m^2}{4} \qquad \text{...(ii)}$$

From (i) and (ii)
$$\beta = \frac{(\pi\rho u_m^2 R^2/3)}{(\pi\rho u_m^2 R^2/4)} = \frac{4}{3} = 1.33$$

Similarly
$$\alpha = \frac{\text{K.E./sec based on actual velocity}}{\text{K.E./sec based on average velocity}}$$

$$\frac{K.E.}{\text{sec}} \text{ based on actual velocity} = \frac{1}{2}\times(\rho d\theta)u^2 = \frac{1}{2}\rho(u\times 2\pi r dr)u^2 = \pi\rho r u^3 dr$$

$$\text{Total } K.E/\text{sec} = \int_0^R \pi\rho r u^3 dr$$

$$= \int_0^R \pi\rho u_m^3\left(1-\frac{r^2}{R^2}\right)^3 r\,dr = \int_0^R \pi\rho u_m^3\left[r\left(1-\frac{r^6}{R^6}-\frac{3r^2}{R^2}+\frac{3r^4}{R^4}\right)\right]dr$$

$$= \pi\rho u_m^3\int_0^R\left(r-\frac{r^7}{R^6}-\frac{3r^3}{R^2}+\frac{3r^5}{r^4}\right)dr = \pi\rho u_m^3\left[\frac{R^2}{2}-\frac{R^2}{8}-\frac{3R^2}{4}+\frac{R^2}{2}\right]$$

$$= \pi\rho u_m^3\left[\frac{(12-3-18+12)}{24}R^2\right] = \frac{\pi\rho u_m^3 R^2}{8} \qquad \text{...(iii)}$$

$$K.E/\text{sec based on average velocity} = \frac{1}{2}\rho A\bar{u}^3$$

$$= \frac{1}{2}\rho\times\pi R^2\times\frac{u_m^3}{8} = \frac{\pi\rho u_m^3 R^2}{16} \qquad \text{As } \left(\bar{u} = \frac{u_m}{2}\right)$$

So,
$$\alpha = \frac{\pi\rho u_m^3 R^2/8}{\pi\rho u_m^3 R^2/16}$$
$$\alpha = 2$$

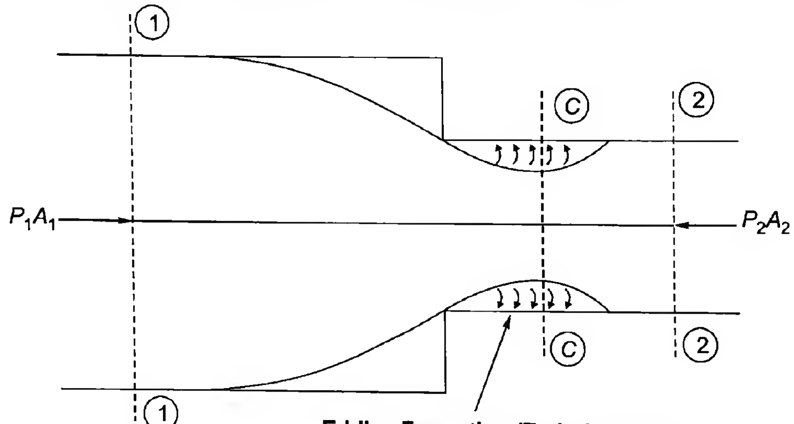
In case of flow is considered to be turbulent, the value of kinetic energy correction factor decreases. Its value lies between 1.04 to 1.11. The reason behind this decrease in value is more uniform flow due to better mixing in case of turbulent flow.

Q.4 Air flows in a circular duct which suddenly contracts in the cross sectional area. Draw the flow sketch and locate the points on the sketch where turbulent flow will occur and calculate the dynamic loss co-efficient. Take the co-efficient of contraction as 0.62.

[CSE (Mains) 2015 : 10 Marks]

Solution:

Consider a liquid flowing in a pipe which has a sudden contraction in area



As the liquid flows from a large pipe to smaller pipe, the area of flows goes on decreasing and becomes minimum at a section C–C as shown. This section C–C is called vena contracta. After section C–C a sudden enlargement of the area takes place. The loss of head due to sudden contraction is actually due to sudden enlargement from vena contracta to smaller pipe.

Let the dynamic loss coefficient be K and coefficient of contraction be C_C.

As,
$$K = \left[\frac{1}{C_C} - 1\right]^2$$

when,
$$C_C = 0.62$$
$$K = \left[\frac{1}{0.62} - 1\right]^2 = 0.375 = 0.375$$

4: Flow over Immersed Bodies

Q.5 What do you mean by boundary layer separation? Briefly explain the various methods of controlling of boundary layer separation.

[CSE (Mains) 2016 : 10 Marks]

Solution:

When a solid boundary is immersed in a flowing fluid, a thin layer of fluid called the boundary layer is formed adjacent to the solid body. In this thin layer of fluid, the velocity varies from zero to free-stream velocity in the direction normal to the solid body.

Along the length of the solid body, the thickness of the boundary layer increases. The fluid layer adjacent to the solid surface has to do work against the surface friction at the expense of its kinetic energy. This loss of the KE is recovered from the immediate fluid layer in contact with the layer adjacent to the solid surface through momentum exchange process. Thus the velocity of the layer goes on decreasing. Along the length of the solid, body, at a certain point a stage may come when the boundary layer will not be able to stick to the solid body, if it cannot provide kinetic energy to overcome the resistance offered by the solid body. In other words, the boundary layer will be separated from the surface. This phenomenon is called the boundary layer separation.

Methods of preventing the separation of boundary layer are:

1. Suction of the slow moving fluid by a suction slot.
2. Supplying additional energy from a blower.
3. Providing a bypass in the slotted wing.
4. Rotating boundary in the direction of flow.
5. Providing small divergence in a diffuser.
6. Providing guide blade in a bend.
7. Providing a trip-wire ring in the laminar region for the flow over a sphere.

5. Dimensional Analysis, Simlitude and Modeling

Q.6 Explain the concept of types of similarities between model and prototype. What do you mean by distorted model? What are its advantages?

It is proposed to design a ship. The proposed ship (prototype) is having a length of 150 m and a wetted surface area of 2000 m^2 with a speed of 40 km/h. A model of 1 : 20 is to be tested in the laboratory at a velocity corresponding to the wave resistance.

The total drag of the model is 50 N. Determine the following:

(i) Wave resistance drag of the model (ii) Wave resistance drag of the prototype

(iii) Friction drag of the prototype

Given: Friction drag, $R_f = \frac{1}{2} C_D \rho A V^2$, where

A = Wetted surface area, C_D = Average friction drag coefficient

$$= \frac{0.074}{(\text{Re})^{1.5}} \text{ for Re} < 2 \times 10^7$$

$$= \frac{0.01}{(\text{Re})^{1.5}} \text{ for Re} > 2 \times 10^7$$

ρ_{model} = 1000 kg/m^3 ν_{model} = 1.110 × 10^{-6} m^2/s

$\rho_{prototype}$ = 1020 kg/m^3 $\nu_{prototype}$ = 1.125 × 10^{-6} m^2/s

[CSE (Mains) 2006 : 15 + 25 = 40 Marks]

Solution:

These are three types of similarities that must exist between the model and the prototype. They are:

1. Geometric similarity 2. Kinematic similarity 3. Dynamic similarity

Geometric Similarity: The geometric similarity is said to exist between the model and the prototype. The ratio of all corresponding linear dimension in the model and prototype are equal.

For geometric similarity between model and the prototype, we have,

$$\left(\frac{L_p}{L_m} = \frac{b_p}{b_m} = \frac{D_p}{D_m} = L_r\right) \text{ where, } L_r \text{ is scale ratio and } \left(\frac{A_p}{A_m} = L_r^2\right)\left(\frac{V_p}{V_m} = L_r^3\right)$$

Kinematic similarity: Kinematic similarity means the similarity of motion between model and prototype. Thus kinematic similarity is said to exist between the model and the prototype if the ratio of the velocity and acceleration at the corresponding points in the model and at the corresponding points in the prototype are same i.e.

$$\left(\frac{V_{p_1}}{V_{m_1}}=\frac{V_{p_2}}{V_{m_2}}=V_r\right) \text{ where, } V_r \text{ is velocity ratio, for acceleration, } \left(\frac{a_{p_1}}{a_{m_1}}=\frac{a_{p_2}}{a_{m_2}}=a_r\right)$$

where a_r is acceleration ratio.

Dynamic similarity: Dynamic similarity means the similarity of forces between the model and prototype. Thus dynamic similarity is said to exist between the model and prototype if the ratios of the corresponding forces acting at the corresponding points are equal. Also the directions of the corresponding forces at the corresponding points should be same.

i.e.
$$\frac{(f_i)_p}{(f_i)_m}=\frac{(f_v)_p}{(f_v)_m}=\frac{(f_g)_p}{(f_g)_m}=f_r \text{ (force ratio)}$$

and f_i, f_{v1} f_g are inertia force, viscous force and gravity force respectively.

Distorted models: A model is said to be distorted if it is not geometrically similar to its prototype. For a distorted model different scale ratios for the linear dimension are adopted. for example, in case of rivers, harbours, reservoirs, etc, two different scale ratios, one for horizontal dimensions and other for vertical dimensions are taken. Thus the models of rivers, harbours and reservoirs will become as distorted models.

Advantage of distorted models:

1. The vertical dimensions of the model can be measured accurately.
2. The cost of the model can be reduced.
3. Turbulent flow in the model can be maintained.

Given: L_r (scale ratio) = 20, L_p = 150 m, A_p = 2000 m^2, v_p = 40 km/h = 11.11 m/sec

Total drag, R_m = 50 N

$$\text{Friction drag, } R_f = \frac{1}{2}C_D\rho A v^2$$

$$(Re)_p = \frac{\rho_p V_p L_p}{\mu_p} \Rightarrow \frac{V_p L_p}{\nu_p} = \frac{11.11\times150}{1.125\times10^{-6}}=148.148\times10^7$$

As $(Re)_p > 2\times10^7$

$$C_D = \frac{0.01}{(\text{Re})^{1/5}}=\frac{0.01}{(148.148\times10^7)^{1/5}} = 1.465\times10^{-4}$$

$$(R_F)_p = \frac{1}{2}\times(1.465\times10^{-4})\times1020\times2000\times(11.11)^2 = 18.445 \text{ kN}$$

As, $$A_m = \frac{A_p}{L_r^2}=\frac{2000}{20\times20}=5 \text{ m}^2$$

and $$L_m = \frac{L_p}{L_r}=\frac{150}{20}=7.5 \text{ m}$$

$$(R_F)_m = \frac{1}{2}(C_D\rho AV^2)_m$$

$$(Re)_m = \frac{\rho_m V_m L_m}{\mu_m}=\frac{V_m L_m}{V_m} = \frac{V_m L_m}{V_m}$$

Also, $(Fe)_m = (Fe)_p$

$$\frac{V_m}{\sqrt{L_m g_m}} = \frac{V_p}{\sqrt{L_p g_p}}$$

Also, $(g_m = g_p)$

$$\frac{V_m}{\sqrt{L_m}} = \frac{V_p}{\sqrt{L_p}}$$

$$V_m = \sqrt{\frac{L_m}{L_p}} V_p = \frac{11.11}{\sqrt{20}} = 2.48 \text{ m/sec}$$

$$(Re)_m = \frac{2.48 \times 7.5}{1.110 \times 10^{-6}} = 1.675 \times 10^7$$

As $(Re)_m < 2 \times 10^7$

$$(C_D)_m = \frac{0.074}{(Re_m)^{1/5}} = \frac{0.074}{(1.675 \times 10^7)^{1/5}} = 2.657 \times 10^{-3}$$

$$(R_F)_m = \frac{1}{2} \times (2.657 \times 10^{-3}) \times 1000 \times 5 \times (2.48)^2 = 40.85 \text{ N}$$

R_w = wave resistance drag

$$(R_w)_m = R_m - (R_F)_m = 50 - 40.85 = 9.15 \text{ N}$$

and

$$(R_w)_p = \left(\frac{\rho_p}{\rho_m}\right) \times \left(\frac{L_p}{L_m}\right)^2 \times \left(\frac{V_p}{V_m}\right)^2 \times (R_w)_m = \left(\frac{1020}{1000}\right) \times (20)^2 \times \left(\frac{11.11}{2.48}\right)^2 \times 9.15$$

$$(R_w)_p = 74.92 \text{ kN}$$

and Total drag $R_p = (R_F)_p + (R_w)_p = 93.36$ kN

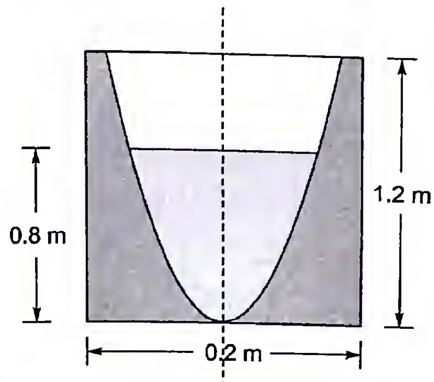
Q.7 A closed cylindrical vessel 0.2 m in diameter and 1.2 m long is filled with water upto a height of 0.8 m from the bottom. Find the speed of the vessel about its vertical axis, when the axial depth of water is zero.

[CSE (Mains) 2012 : 10 Marks]

Solution:

(i) Given: $D = 0.2$ m, $L = 1.2$ m

Let the angular speed, when axial depth of water is zero be ω



$$Z = \frac{\omega^2 r^2}{2g}$$

$$\omega^2 r^2 = 2 \times 1.2 \times 9.81$$
$$= 23.52 \text{ m} \quad ...(i)$$

Volume of air before rotation = Volume of air after paraboloid

$\pi R^2 (1.2 - 0.8)$ = Volume of paraboloid

$$\pi \times 0.1^2 \times 0.4 = \frac{1}{2}(\pi r^2) \times Z = \left(\frac{\pi r^2}{2}\right) \times 1.2$$

$$r^2 = 6.67 \times 10^{-3} \quad ...(ii)$$

From (i) & (ii) $\omega \times (6.67 \times 10^{-3}) = 23.52$ $(\omega = 59.4 \text{ rad/sec})$

and

$$\omega = \frac{2\pi N}{60}$$

$$N = \frac{60 \times 59.4}{2\pi} = 567.22 \text{ rpm}$$

Q.8 Using Buckingham's π theorem method, derive a relation for the efficiency η of a fan which depends on the following parameters:
Mass density ρ, Dynamic viscosity μ, Angular velocity ω, Diameter of the rotor D, Discharge Q.

[CSE (Mains) 2014 : 20 Marks]

Solution:

Given: η is a function of ρ, μ, ω, D, Q

$$\eta = f(\rho, \mu, \omega, D, Q)$$

Total number of variables $n = 6$

and Fundamental dimensions $m = 3$

No. of π-terms $= 6 - 3 = 3$

$\therefore$ $f(\pi_1, \pi_2, \pi_3) = 0$

Choosing D, ω and ρ as repeating variables,

$$\pi_1 = D^{a_1}\omega^{b_1}\rho^{c_1}\cdot\eta$$

$$\pi_2 = D^{a_2}\omega^{b_2}\rho^{c_2}\cdot\mu$$

$$\pi_3 = D^{a_3}\omega^{b_3}\rho^{c3}\cdot Q$$

First π-term $\quad \pi_1 = D^{a_1}\omega^{b_1}\rho^{c_1}\cdot\eta$

$$M^0L^0T^0 = L^{a_1}T^{-b_1}(ML^{-3})^{c_1}\cdot M^0L^0T^0$$

$$0 = c_1 + 0 \Rightarrow c_1 = 0$$

$$0 = a_1 + 0 \Rightarrow a_1 = 0$$

$$0 = -b_1 + 0 \Rightarrow b_1 = 0$$

$$\pi_1 = D^0\,\omega^0\,\rho^0\,\eta$$

$$\pi_1 = \eta$$

Second π-term $\quad \pi_2 = D^{a_2}\cdot\omega^{b_2}\rho^{c_2}\mu$

$$M^0L^0T^0 = L^{a_2}T^{-b_2}(ML^{-3})^{c_2}ML^{-1}T^{-1}$$

$$0 = c_2 + 1 \Rightarrow c_2 = -1 = a_2 - 3c_2 - 1 \Rightarrow a_2 = -3 + 1 = -2$$

$$0 = -b_2 - 1 \Rightarrow b_2 = -1$$

$\therefore$

$$\pi_2 = D^{-2}\,\omega^{-1}\,\rho^{-1}\,\mu$$

$$\pi_2 = \frac{\mu}{D^2\omega\rho}$$

Third π-term

$$\pi_3 = D^{a_3}\omega^{b_3}\rho^{c_3}Q$$

$$M^0L^0T^0 = L^{a_3}T^{-b_3}(ML^{-3})^{c_3}L^{3}T^{-1}$$

$0 = c_3 \Rightarrow c_3 = 0$

$0 = a_3 - 3c_3 + 3 \Rightarrow a_3 = -3$

$0 = -b_3 - 1 \Rightarrow b_3 = -1$

$\therefore$

$$\pi_3 = D^{-3}\omega^{-1}\rho^0 Q = \frac{Q}{D^3\omega}$$

$\therefore$
$$f(\pi_1, \pi_2, \pi_3) = 0$$

$$f\left(\eta, \frac{\mu}{D^2\omega\rho}, \frac{Q}{D^3\omega}\right) = 0$$

or
$$\eta = f\left[\frac{\mu}{D^2\omega\rho}, \frac{Q}{D^3\omega}\right]$$

Q.9 Define distorted model. Explain why models of rivers and harbours are made as distorted models. Write down the merits and demerits of distorted model.

[CSE (Mains) 2016 : 10 Marks]

Solution:

Distorted Models: A model is said to be distorted if it is not geometrically similar to its prototype. For a distorted model different scale ratios for the linear dimensions are adopted. The models of river and harbours are made as distorted models as two different scale ratios, one for the horizontal dimensions and other for vertical dimensions are taken. If for the river and harbours, the horizontal and vertical scale ratios are taken to be same so that the model is undistorted, then the depth of water in the model of the river will be very-very small which may not be measured accurately. Therefore these are made as distorted models.

Advantages:

1. The vertical dimensions of the model can be measured accurately.
2. The cost of the model can be reduced.
3. Turbulent flow in the model can be maintained.

Disadvantages:

1. Due to different scales in the different directions, the velocity and pressure distribution in the model is not same as that in the prototype.
2. Waves are not simulated in distorted models.
3. The results of the distorted model cannot be directly transferred to its prototype.

■■■■

3 Heat Transfer

1. Conduction

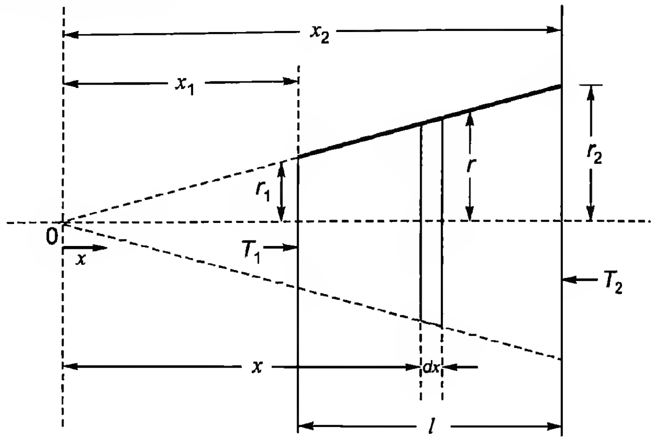
Q.1 Heat is transferred along the axis of a truncated conical cylinder of length l, radius r_1 at the shorter end and radius r_2 at the bigger end. The circumference of the cylinder is completely insulated. Develop an expression to calculate heat transfer along the axis of cylinder. Assume no variation of conductivity with temperature.

[CSE (Mains) 2002 : 20 Marks]

Solution:

Consider a frustum of constant thermal conductivity (k) and apex at 0.

Consider an element of thickness dx at a distance x from the apex.



$$\text{Radius of this element, } r = \left(\frac{r_2 - r_1}{l}\right)x = C \cdot x$$

where,
$$C = \frac{r_2 - r_1}{l}$$

$$\text{Area of this element} = \pi r^2 = \pi C^2 x^2$$

Heat transfer rate through this element,

$$Q = -kA\frac{dT}{dx}$$

$$\Rightarrow \quad \frac{dx}{A} = \frac{-k}{Q} \cdot dT$$

Integrating both sides,

$$\int_{x_1}^{x_2} \frac{dx}{\pi C^2 x^2} = \frac{-k}{Q}\int_{T_1}^{T_2} dT$$

$$\Rightarrow \quad \frac{1}{\pi C^2} \cdot \left[\frac{1}{x_1} - \frac{1}{x_2}\right] = \frac{k}{Q}(T_1 - T_2)$$

Also, $\quad r_1 = Cx_1 \quad$ and $\quad r_2 = Cx_2$

$$\Rightarrow \quad \frac{1}{\pi C^2}\left[\frac{C}{r_1} - \frac{C}{r_2}\right] = \frac{k}{Q}(T_1 - T_2)$$

$$\therefore \quad \text{Heat transfer rate, } Q = \frac{T_1 - T_2}{\dfrac{1}{\pi k C}\left[\dfrac{1}{r_1} - \dfrac{1}{r_2}\right]} = \frac{T_1 - T_2}{\dfrac{l}{\pi k (r_2 - r_1)} \cdot \left[\dfrac{1}{r_1} - \dfrac{1}{r_2}\right]} = \frac{T_1 - T_2}{\left(\dfrac{l}{\pi k r_1 r_2}\right)}$$

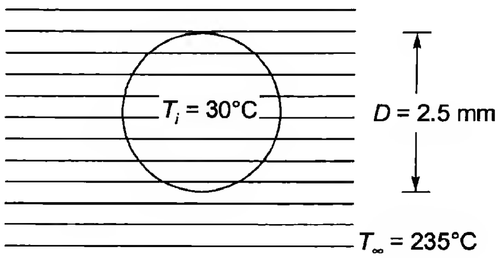
Q.2 A spherical thermocouple of 2.5 mm diameter is used to measure the temperature of air flowing in a pipe. Initially both the thermocouple and the air are at a temperature of 30°C. The air is heated to a temperature of 235°C and maintained at this temperature. Find the time required for the thermocouple to reach 200°C. Also find out the time constant of the thermocouple and comment on the suitability of this thermocouple to measure unsteady state temperature. For thermocouple material take: density = 9000 kg/m³, specific heat = 0.4 kJ/kg K and thermal conductivity = 30 W/mK.
Convective heat transfer coefficient between thermocouple surface and the air is 120 W/m² K.

[CSE (Mains) 2002 : 30 Marks]

Solution:

Consider the spherical thermocouple as shown in figure

Given: $T_i = 30°C$, $T_f = 200°C$, $T_\infty = 235°C$.



Biot number of sphere, $\text{Bi} = \dfrac{h L_c}{k}$

For a sphere, characteristic length, $L_c = \dfrac{V}{A} = \dfrac{R}{3}$

$$\therefore \quad \text{Bi} = \frac{120 \times (0.0025/2)}{30 \times 3} = 1.67 \times 10^{-3}$$

Since Bi < 0.1, entire body can be considered as lumped.

Consider heat exchanged between air and resultant increase in internal energy of sphere, we get

$$mC_P\, dT = h A_s (T_\infty - T)\, dt$$

$$\Rightarrow \quad \frac{dT}{T - T_\infty} = -\frac{h A_s}{\rho V C_P}$$

Integrating both sides, we get

$$\int_{T_i}^{T_f} \frac{dT}{T - T_\infty} = -\frac{h A_s}{\rho V C_P} \int_0^t dt$$

$$\Rightarrow \quad \ln\left(\frac{T_f - T_\infty}{T_i - T_\infty}\right) = \left(-\frac{hA_s}{\rho V C_P}\right) \cdot t = -\left(\frac{t}{\dfrac{\rho V C_P}{hA_s}}\right)$$

$$\Rightarrow \quad \ln\left(\frac{T_f - T_\infty}{T_i - T_\infty}\right) = -\frac{t}{\tau}$$

where, τ = time constant

Time constant in this case, $$\tau = \frac{\rho V C_P}{h A_s} = 9000 \times \left(\frac{0.0025}{3 \times 2}\right) \times \frac{0.4 \times 10^3}{120} = 12.5 \text{ seconds}$$

Time required by sphere to reach 200°C

$$t = -\tau \cdot \ln\left(\frac{T_f - T_\infty}{T_i - T_\infty}\right) = -12.5 \times \ln\left(\frac{200 - 235}{30 - 235}\right) = 22.095 \text{ seconds}$$

For measuring unsteady state temperature, the thermocouple should takes least amount of time to reach environment temperature, T_f. Hence the value of time constant should be as small as possible.

In this case, if the temperature of system changes faster than the time required for thermocouple to reach that temperature, then thermocouple can not be used for temperature measurement. If temperature of air changes from 235°C before 22.1 seconds, it is not a good measuring device.

Not suitable for measuring unsteady state temperature because in unsteady state temperature changes every second while here time constant is as large as 12.5 seconds.

Q.3 A small hemispherical oven is built of two insulating material. The inner layer is of fire brick, 125 mm thick and outer layer is of 85% magnesia, 40 mm thick. The inner surface of the oven is at 800° and the heat transfer coefficient for the outer surface is 10 W/m²-K. The room temperature is 20°C. Calculate the heat loss through hemisphere with the inside radius is 0.6 m. The thermal conductivities of fire brick and 85% magnesia are 0.31 and 0.05 W/m-K respectively. Also calculate the temperature at contact between insulating materials and at the outer surface.

[CSE (Mains), 2003 : 20 Marks]

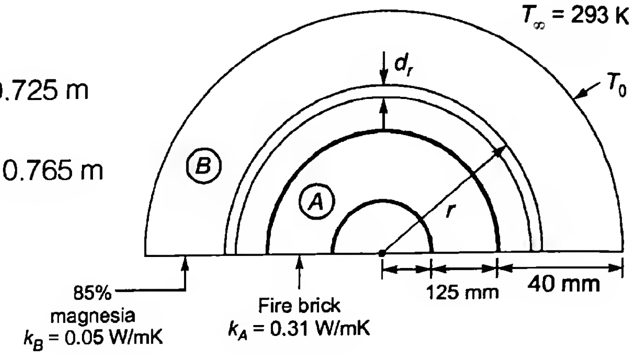
Solution:

Given:

Two different layers of materials A(fire brick) and B(85% magnesia).

Convective heat transfer for outer surface, $h = 10$ W/m²K

$$r_{iA} = 0.6 \text{ m}$$
$$r_{0A} = 0.6 + 0.125 = 0.725 \text{ m}$$
$$r_{iB} = 0.725 \text{ m}$$
$$r_{0B} = 0.725 + 0.04 = 0.765 \text{ m}$$



Assume temperature of outer surface is T_0.

Consider the element of radius r and thickness dr.

Heat transfer through this element, $Q = -KA\dfrac{dT}{dr}$

$$\Rightarrow \quad \frac{dr}{2\pi r^2} = -\frac{K}{Q}dT$$

Integrating, $$\frac{1}{2\pi}\left[\frac{1}{r_i}-\frac{1}{r_0}\right] = \frac{K}{Q}[T_i - T_0]$$

$$\Rightarrow \quad Q = \frac{T_i - T_0}{\dfrac{1}{2k\pi\left(\dfrac{1}{r_i}-\dfrac{1}{r_0}\right)}}$$

∴ Thermal resistance of hemisphere,

$$R = \frac{1}{2\pi K}\left[\frac{1}{r_i}-\frac{1}{r_0}\right]$$

Total thermal resistance, $$R_{net} = R_A + R_B + R_{com} = \frac{1}{2\pi K_A}\left[\frac{1}{r_{iA}}-\frac{1}{r_{0A}}\right] + \frac{1}{2\pi K_B}\left[\frac{1}{r_{iB}}-\frac{1}{r_{0B}}\right] + \frac{1}{hA_0}$$

$$R_{net} = 0.4043 \text{ K/W}$$

∴ Heat transfer between insides of oven and surroundings,

$$Q = \frac{T_i - T_\infty}{R_{net}} = \frac{1073-293}{0.4043} \text{ Watt } = 1.929 \text{ kW} \simeq 2 \text{ kW}$$

Also, temperature at contact of two layer = T_C

Outside temperature = T_0

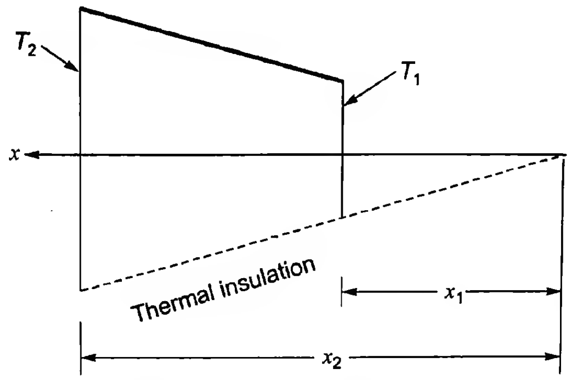
$$Q = \frac{T_0 - T_\infty}{R_{conv}}$$

$$\Rightarrow \quad T_0 = \frac{Q}{hA_0} + T_\infty = 345.47 \text{ K}$$

Also, $$Q = \frac{T_C - T_0}{R_B}$$

$$\Rightarrow \quad T_C = Q \cdot R_B + T_0 = 788.31 \text{ K}$$

Q.4 The diagram shows a truncated conical section fabricated from a material of thermal conductivity K. The circular cross-section of the conical section has the diameter $D = ax$, where a is a constant and x is the axial distance of the section from the apex of the cone. The temperature at the two faces of the conical section (at distance x_1 and x_2 from the apex) are respectively T_1 and T_2 while the lateral surface of the truncated cone is thermally insulated.



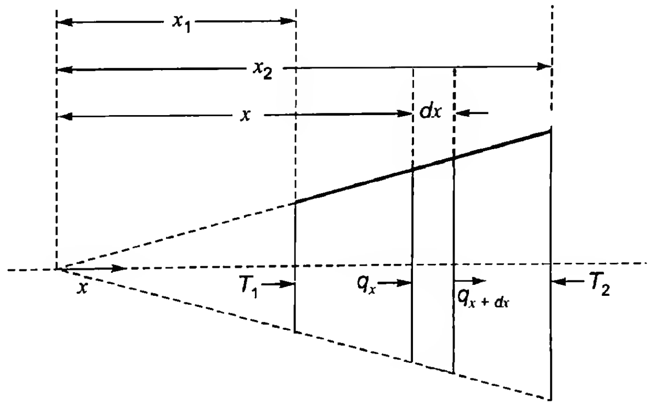
(i) Derive an expression for the temperature distribution $T(x)$ in symbolic form assuming one dimensional steady-state condition. Sketch the temperature distribution.

(ii) Calculate the heat rate q_x through the cone in x-direction.

[CSE (Mains), 2007 : 20 + 40 Marks]

Solution:

Consider the frustum of constant thermal conductivity k as shown in figure.



Consider a section of thickness dx at a distance 'x' from the apex of the cone.

Diameter of this cross section, $D_x = ax$

Area of cross-section of this element,

where, C is a constant.

Assumptions:

1. One dimensional heat conduction.
2. Steady-state: No change in internal energy of frustum.

Energy balance for the section:

Energy in from left face = Energy out from right face

$$\Rightarrow \quad q_x = q_{x+dx}$$

$$\Rightarrow \quad q_{x+dx} - q_x = 0$$

$$\Rightarrow \quad dq_x = 0$$

$$\Rightarrow \qquad \frac{d}{dx}\left(-KA_x\frac{dT}{dx}\right)\cdot dx = 0$$

$$\Rightarrow \qquad \frac{d}{dx}\left(A_x\frac{dT}{dx}\right) = 0$$

Integrating this,

$$A_x\frac{dT}{dx} = C_1$$

$$\Rightarrow \qquad \frac{dT}{dx} = \frac{C_1}{Cx^2}$$

Upon integration, we get

$$T = \frac{C_1}{C}\left(-\frac{1}{x}\right) + C_2$$

Boundary Conditions:

(i) Temperature at $x = x_1$ is T_1

$$T_1 = C_2 - \frac{C_1}{Cx_1} \qquad \text{...(i)}$$

(ii) Temperature at $x = x_2$ is T_2

$$T_2 = C_2 - \frac{C_1}{Cx_2} \qquad \text{...(ii)}$$

Solving (i) and (ii), we get

$$T_1 - T_2 = C_1\left(\frac{1}{Cx_2} - \frac{1}{Cx_1}\right)$$

$$\Rightarrow \qquad C_1 = \frac{C(T_1 - T_2)x_1x_2}{x_1 - x_2}$$

$$C_2 = T_1 + \frac{C_1}{Cx_1} = T_1 + \frac{1}{Cx_1}\cdot\frac{C(T_1 - T_2)x_1x_2}{x_1 - x_2} = \frac{T_1x_1 - T_1x_2 + T_1x_2 - T_2x_2}{x_1 - x_2}$$

$$\Rightarrow \qquad C_2 = \frac{T_1x_1 - T_2x_2}{x_1 - x_2}$$

$$\therefore \qquad T = -\frac{C_1}{C}\frac{1}{x} + C_2 = \frac{(T_2 - T_1)}{x}\cdot\frac{x_1x_2}{x_1 - x_2} + \frac{T_1x_1 - T_2x_2}{(x_1 - x_2)}$$

$$= \frac{T_2x_1x_2 - T_1x_1x_2 + T_1x_1x - T_2x_2x}{x(x_1 - x_2)}$$

$$\therefore \qquad T = \frac{T_1x_1(x - x_2) + T_2x_2(x_1 - x)}{x(x_1 - x_2)}$$

(iii) Heat transfer rate through the section will be

$$Q = -KA_x\cdot\frac{dT}{dx}$$

$$\Rightarrow \qquad \frac{dx}{A_x} = \left(-\frac{K}{Q}\right)\cdot dT$$

Integrating both sides,

$$\int_{x_1}^{x_2}\frac{dx}{Cx^2} = \left(-\frac{K}{Q}\right)\int_{T_1}^{T_2} dT$$

$$\Rightarrow \qquad \frac{1}{C}\left(\frac{1}{x_1}-\frac{1}{x_2}\right) = \frac{K}{Q}(T_1 - T_2)$$

$\Rightarrow$ Heat transfer rate,
$$Q = \frac{T_1 - T_2}{\dfrac{4}{\pi a^2 K}\left(\dfrac{1}{x_1}-\dfrac{1}{x_2}\right)}$$

Q.5 Consider a large plane wall of thickness $L = 0.3$ m, thermal conductivity $k = 2.5$ w/m ·K, and surface area $A = 12\ m^2$. The left side of the wall at $x = 0$ is subjected to a net heat flux of $\dot{q} = 700\ W/m^2$ while the temperature at that surface is maintained at $T_1 = 80°C$. Assuming constant thermal conductivity and no heat generation in the wall:

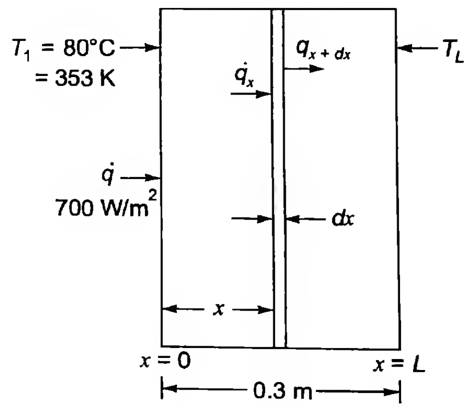
(i) express the differential equation and boundary condition for steady, one-dimensional heat conduction through the wall.

(ii) obtain a relation for the variation of temperature in the wall by solving the differential equation.

(iii) evaluate the temperature of the right surface of the wall.

[CSE (Mains), 2008 : 20 Marks]

Solution:



Consider a section of thickness dx at a distance 'x' m from left face. Assuming constant thermal conductivity, no heat generation and steady state conditions (no changes in internal energy with time) energy balance for this small section can be shown as:

Energy entering left face = Energy leaving right face

$$\Rightarrow \qquad q_x = q_{x+dx}$$

$$\Rightarrow \qquad q_{x+dx} - q_x = 0$$

$$\Rightarrow \qquad \left(\frac{dq_x}{dx}\right)dx = 0$$

$$\Rightarrow \qquad \frac{d}{dx}\left(-kA\frac{dT}{dx}\right)\cdot dx = 0$$

$$\Rightarrow \qquad -kA\frac{d^2T}{dx^2} = 0$$

$$\frac{d^2T}{dx^2} = 0 \qquad \ldots(i)$$

This is the required differential equation for this case.

Boundary Conditions:

(*i*) Heat flux at left face = Rate of heat condition at the left face

$$\Rightarrow \qquad \dot{q} = -kA\left.\frac{dT}{dx}\right|_{x=0}$$

(*ii*) Temperature of left face is maintained at 80°C or 353 K°.

$$\therefore \qquad T|_{x=0} = 353\ \text{K}$$

Solving differential equation (i), we get

$$\frac{dT}{dx} = C_1$$

$\Rightarrow$ $$T = C_1 x + C_2$$

Applying 1st boundary condition,

$$\frac{\dot{q}}{A} = -k\frac{dT}{dx}\bigg|_{x=0} = -kC_1$$

$\Rightarrow$ $$C_1 = -\left(\frac{\dot{q}}{A}\right)\frac{1}{k} = \frac{-700 \text{ W/m}^2}{2.5 \text{ W/mK}} = -280 \text{ K/m}$$

Applying 2nd boundary condition,

$$T|_{x=0} = C_1 \cdot 0 + C_2 = 353 \text{ K}$$

$\Rightarrow$ $$C_2 = 353 \text{ K}$$

$\therefore$ Expression for the temperature distribution in the wall is

$$T = 353 - 280\,x$$

(*iii*) Temperature of right surface of the wall

$$T|_{x=0.3\text{ m}} = 269 \text{ K}$$

Q.6 The thermal conductivity of a hollow sphere of inside radius (R_i) and outside radius (R_0) is given by

$$K = K_i + (K_0 - K_i)\left(\frac{T - T_i}{T_0 - T_i}\right)$$

where, T_i = Inner surface temperature and T_0 = Outside surface temperature

Prove that:

(i) The heat flow rate is given by

$$Q = 4\pi R_i R_0\left(\frac{K_i + K_0}{2}\right)\left(\frac{T_i - T_0}{R_0 - R_i}\right)$$

(ii) Also determine the heat loss from a spherical shell whose D_i = 2.5 m and covered with 30 cm of insulation. The thermal conductivity of insulation is 0.3 W/mK and 0.2 W/mK at inner and outer surface temperatures of 150°C and –15°C respectively.

[CSE (Mains), 2009 : 20 Marks]

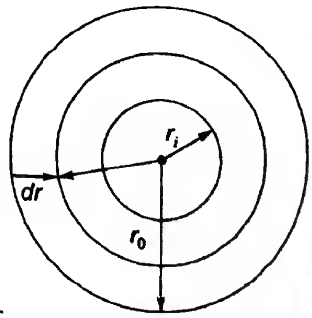
Solution:

(i) Consider an elemental thin shell of thickness *dr* at a distance *r* from the center.

Heat flow rate through this element will be

$$Q = -kA\frac{dT}{dr}$$

$\Rightarrow$ $$Q\cdot\frac{dr}{A} = -kdT$$



$$\frac{Q\cdot dr}{4\pi r^2} = \left[k_i + (k_0 - k_i)\left(\frac{T - T_i}{T_0 - T_i}\right)\right]dT$$

$$= -\left[\frac{k_i(T_0 - T_i) + T(k_0 - k_i) - T_i(k_0 - k_i)}{(T_0 - T_i)}\right]dT$$

Integrating both sides within suitable limits,

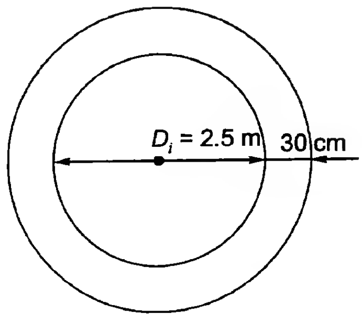
$$-\int_{r_i}^{r_0} \frac{Q}{4\pi r^2} dr = \int_{T_i}^{T_0} \left[\frac{k_i(T_0 - T_i) - T_i(k_0 - k_i)}{T_0 - T_i} + \frac{(k_0 - k_i)^T}{T_0 - T_i} \right] dT$$

$$\Rightarrow \quad -\frac{Q}{4\pi}\left[\frac{1}{r_0} - \frac{1}{r_i}\right] = k_i(T_0 - T_i) - T_i(k_0 - k_i) + \frac{(k_0 - k_i)}{2}(T_0 + T_i) = \frac{(T_0 - T_i)}{2} \cdot (k_i + k_0)$$

$$\Rightarrow \quad \frac{Q}{4\pi}\left[\frac{1}{r_i} - \frac{1}{r_0}\right] = \frac{(T_i - T_0)}{2(k_i - k_0)}$$

$$\Rightarrow \quad \text{Heat flow rate} = Q = 4\pi r_i r_0 \left(\frac{k_i + k_0}{2}\right) \cdot \left(\frac{T_i - T_0}{r_0 - r_i}\right)$$

(ii) Consider the thin spherical shell with an insulation of 30 cm. Assuming that thickness of shell as negligible.



$$\text{Internal radius of shell} = \frac{2.5}{2} = 1.25 \text{ m}$$

Outer radius of shell with insulation,

$$R_0 = 1.25 + 0.30 = 1.55 \text{ m}$$

$$\text{Thermal conductivity of shell} = 0.2 \text{ W/mK}$$

$$Q = 4\pi \times 1.25 \times 1.55 \times 0.25 \times \frac{165}{0.3} = 3347.76 \text{ W}$$

Q.7 **The volumetric heat generation rate in a cylindrical nuclear reactor fuel rod of 5 cm diameter is 5×10^7 W/m^3. Under steady state operating condition, the temperature distribution in the rod is $t = 800 - 4.167 \times 10^5 r^2$. Density, specific heat and thermal conductivity of rod material are 1100 kg/m^3, 800 J/kg-K and 30 W/m-K respectively.**

(i) What are the rates of heat transfer per unit length of the rod at the centre line (axis) and at the rod surface?

(ii) If the reactor power level is suddenly increased to 10^8 W/m^3, what are the initial time rate of change of temperature at the centre line and at the surface?

[CSE (Mains) 2012 : 20 Marks]

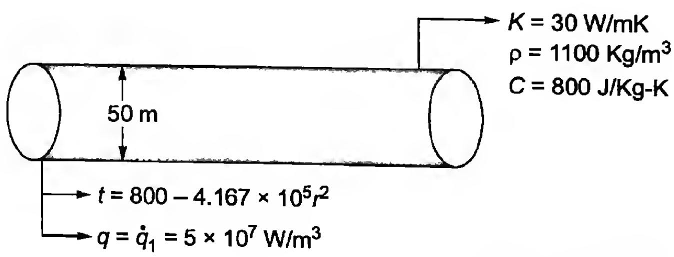
Solution:

Assumptions:

(i) One dimensional conduction in direction of r

(ii) Uniform generation

(iii) Steady state condition



$$q = -KA\frac{\partial T}{\partial r} \quad \therefore \quad q = -K(2\pi RL)\frac{\partial T}{\partial r}$$

$$\therefore \qquad q = -K2\pi R\frac{\partial T}{\partial r} \qquad \text{[per unit length]}$$

and also

$$\frac{\partial T}{\partial r} = -2 \times 4.167 \times 105\, r \text{ K/m}$$

(a) $\therefore$ $q_{r=0} = 0$ and $q_{r=0.025} = -30 \times 2 \times \pi \times 0.025 \times (-2) \times 4.167 \times 10^5 \times 0.025$ W/m

$$q_{r=0.025} = (\dot{q} \times \pi r^2 \times 1) = 0.9818 \times 10^5 \text{ W/m}$$

(b) Transient condition will exist when generation is increased

$$\frac{1}{r}\frac{\partial}{\partial r}\left(Kr\frac{\partial T}{\partial r}\right) + \dot{q}_2 = \rho C_p \frac{\partial T}{\partial t}$$

$$\therefore \qquad \frac{\partial T}{\partial t} = \frac{1}{\rho C_p}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(Kr\frac{\partial T}{\partial r}\right)+\dot{q}_2\right]$$

but at $t = 0$; $T(r) = 800 - 4.167 \times 10^5 r^2$

$$q = -5 \times 10^7 \text{ W/m}^3 \text{ (the original } q)$$

Now,
$$\frac{\partial T}{\partial t} = 56.82 \text{ k/s}$$

Alternatively: at $t = 0$

Net generation increased is responsible for transient condition and temperature gradient.

$$\Delta\dot{q} = (10^8 - 5\times10^7)$$

$$\therefore \qquad \Delta\dot{q} \text{ W/m}^3 = \rho C_p \frac{dT}{dt} W/m^3$$

$$\therefore \qquad \frac{dT}{dt} = 56.82 \text{ k/s}$$

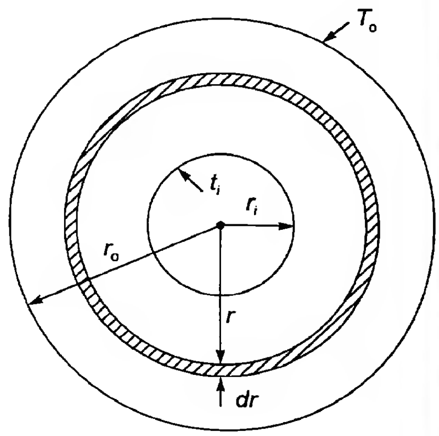
Q.8 **Inner and outer surfaces of a spherical shell are maintained at temperatures T_i and T_o respectively such that $T_i > T_o$. If inner and outer radii of the shell are r_i and r_o and its conductivity is k, derive an expression for the rate of heat conduction through the shell. Assume steady state and no heat generation within the shell.**

[CSE (Mains), 2013 : 10 Marks]

Solution:

Consider a spherical shell of inner and outer radius r_i and r_o and surfaces maintained at T_i and T_o.

Consider a thin spherical shell element of thickness dr at a distance r from the centre.



Rate of heat transfer through this element,

$$Q = -kA\frac{dT}{dr}$$

We know, here
$$A = 4\pi r^2$$

$$\therefore \qquad \frac{Q \cdot dr}{kA} = -dT$$

Integrating both sides within proper limits

$$\int_{r_i}^{r_o} \frac{Q \cdot dr}{k \cdot 4\pi r^2} = -\int_{T_i}^{T_o} dT$$

$$\Rightarrow \qquad \frac{Q}{4\pi k}\left[-\frac{1}{r}\right]_{r_i}^{r_o} = [-T]_{T_i}^{T_o}$$

$$\Rightarrow \qquad \frac{Q}{4\pi k}\cdot\left[\frac{1}{r_i} - \frac{1}{r_o}\right] = T_i - T_o$$

$\therefore$ Rate of heat transfer through the shell

$$\therefore \qquad Q = \frac{(T_i - T_o)4\pi k}{\left(\frac{1}{r_i} - \frac{1}{r_o}\right)} \Rightarrow Q = \frac{T_i - T_o}{R_{Th}}$$

where, R_{Th} = Thermal resistance for shell

$$R_{Th} = \frac{1}{4\pi k}\left(\frac{1}{r_i} - \frac{1}{r_o}\right)$$

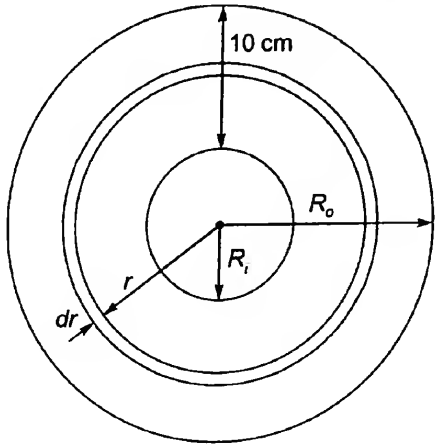
Q.9 **The outer and inner surfaces of a thick hollow cylinder have areas 1.25 m^2 and 0.25 m^2 respectively. The thickness of the cylinder is 10 cm and the thermal conductivity of the cylinder is 50 W/mK. Find the radial heat transfer through the cylinder for 100°C temperature difference at the surfaces. Derive the formula used.**

[CSE (Mains), 2014 : 10 Marks]

Solution:

Assume inner and outer radii as R_i and R_o and length of cylinder as L.

Given: $A_i = 0.25$ m², $A_o = 1.25$ m², Outer area = $2\pi R_o L = 1.25$ m², Inner area = $2\pi R_i L = 0.25$ m².



Also,

$$\text{Thickness} = R_o - R_i = 0.10$$

$$\Rightarrow \quad R_i = 0.025 \text{ m}$$

$$R_o = 0.125 \text{ m},$$

$$L = \frac{1.25}{2\pi \cdot 0.125} = \frac{5}{\pi} \text{ m}$$

Consider a thin cylindrical element at a distance r from centre of shell and thickness dr.

$$\text{Heat flow rate, } Q = -kA\frac{dT}{dr}$$

$$\Rightarrow \quad \frac{dr}{2\pi rL} = -\frac{k}{Q}dT$$

Integrating both sides,

$$\int_{r_i}^{r_o} \frac{dr}{2\pi rL} = -\frac{k}{Q}\int_{T_i}^{T_o} dT$$

$$\Rightarrow \quad \frac{1}{2\pi L}\ln\left(\frac{r_o}{r_i}\right) = \frac{k}{Q}(T_i - T_o)$$

$$\therefore \quad \text{Heat flow rate} = \boxed{Q = \frac{T_i - T_o}{\frac{1}{2\pi kL}\cdot ln\left(\frac{r_o}{r_i}\right)}}$$

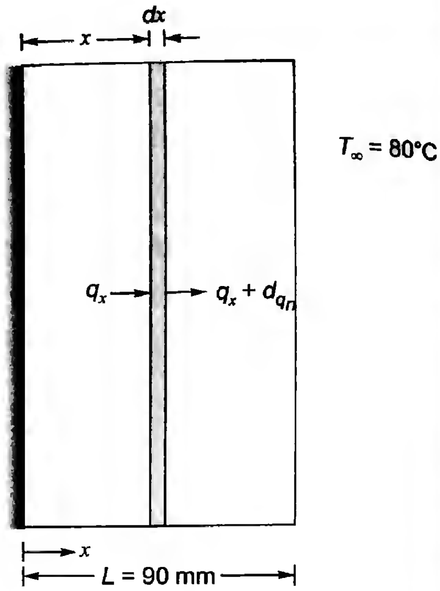
Radial heat transfer through cylinder for 100°C temperature difference between surfaces,

$$Q = \frac{100}{\frac{1}{2\pi \cdot 50 \cdot \frac{5}{\pi}}\cdot \ln(5)} = 31.067 \text{ kW}$$

Q.10 **A plane wall 90 mm thick ($K = 0.18$ W/m°C) is insulated on one side while the other side is exposed to environment at 80°C. The rate of heat generation within the wall is 1.3×10^5 W/m³. If the convective heat transfer coefficient between the wall and the environment (h) is 520 W/m²°C, determine the maximum temperature in the wall. Derive the expression used, starting from the steady state one-dimensional heat conduction with heat generating equation.**

[CSE (Mains), 2015 : 20 Marks]

Solution:



Given: $K = 0.18$ W/m°C, $h = 520$ W/m^2°C, $\dot{q} = 1.5 \times 10^5$ W/m^3

Consider a section of thickness dx at a distance 'x' from the insulated end as shown.

For steady state condition,

$$q_{x+} dq_x - q_x = \dot{q}_{gen}$$

$$\Rightarrow \quad \frac{dq_x}{dx} \cdot dx = \dot{q}(A \cdot dx)$$

$$\Rightarrow \quad \frac{\partial}{dx}\left(-kA\frac{dT}{dx}\right) \cdot dx = \dot{q} A dx$$

$$\Rightarrow \quad kA\frac{d^2T}{dx^2} + \dot{q}A = 0$$

$$\Rightarrow \quad \frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = 0$$

This is the required one dimensional steady state equation in this case.

Solving this equation, we get

$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = 0$$

$$\Rightarrow \quad \frac{dT}{dx} = -\frac{\dot{q}}{k}x + C_1 \quad \text{...(i)}$$

$$\Rightarrow \quad T = -\frac{\dot{q}}{k}\frac{x^2}{2} + C_1 x + C_2 \quad \text{...(ii)}$$

Boundary conditions:

(a) Insulated wall at $x = 0$

∴ No conduction at $x = 0$

$$\Rightarrow \quad kA\left.\frac{dT}{dx}\right|_{x=0} = 0$$

$$\Rightarrow \quad \left.\frac{dT}{dx}\right|_{x=0} = 0$$

Applying this in equation (i), we get

$$C_1 = 0$$

(b) Total heat generated inside wall = Heat transferred from right face by convention

$$\dot{q} \times A \cdot L = hA(T_W - T_\infty)$$ where, T_W = Temperature of right face

$$T_W = \frac{\dot{q}L}{h} + T_\infty = \frac{1.3 \times 10^5 \times 90 \times 10^{-3}}{520} + 80 = 102.5°\text{C}$$

∴ Temperature at right face,

$$T|_{x=L} = -\frac{\dot{q}}{k} \cdot \frac{L^2}{2} + C_2$$

$$\Rightarrow \quad C_2 = 102.5 + \frac{1.3 \times 10^5 \times (0.09)^2}{0.18 \times 2} = 3027.5$$

∴ Expression for variation of temperature in the wall is

$$T = -\frac{\dot{q}}{k}\frac{x^2}{2} + 3027.5$$

∴ Maximum temperature is attained on the insulated face of the wall at $x = 0$

∴ Maximum temperature in the wall at $x = 0$,

$$T = 3027.5°C$$

Q.11 A steam pipe (inner diameter = 150 mm and outer diameter = 160 mm) having thermal conductivity 58 W/m K is covered with two layers of insulation of thicknesses 30 mm and 50 mm respectively and thermal conductivities 0.18 W/m K and 0.09 W/m K respectively. The temperature of inner surface of the steam pipe is 320°C and that of the outer surface of the insulation layers is 40°C.

(i) Determine the quantity of heat lost per metre length of the steam pipe and layer contact temperature.

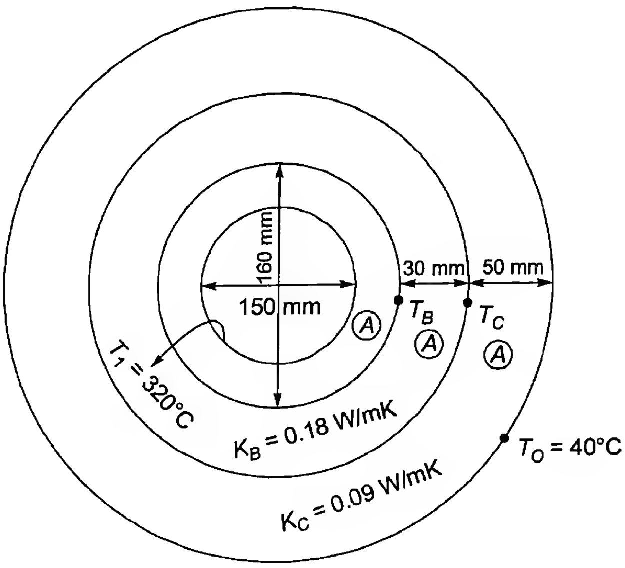
(ii) If the condition of the steam is dry and saturated, find the quality of the steam coming out of one metre pipe assuming that the quantity of steam flowing is 0.32 kg/min.

[Use the data: At 320°C saturation temperature $h_f = 1463$ kJ/kg, $h_{fg} = 1240$ kJ/kg, $h_g = 2703$ kJ/kg]

[CSE (Mains) 2016 : 20 Marks]

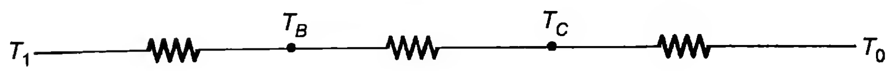
Solution:

Given, $r_{iA} = 75$ mm, $r_{OA} = 80$ mm, $r_{iB} = 80$ mm, $r_{OB} = 110$ mm, $r_{iC} = 110$ mm, $r_{OC} = 160$ mm



Resistance circuit

$$R_B = \frac{\ln\left(\frac{r_{OB}}{r_{iB}}\right)}{2\pi K_B L}$$



$$R_A = \frac{\ln\left(\frac{r_{OA}}{r_{iA}}\right)}{2\pi K_{pipe} L} \qquad R_C = \frac{\ln\left(\frac{r_{OC}}{r_{iC}}\right)}{2\pi K_C L}$$

$$R_{total} = R_A + R_B + R_C \text{ (Considering } L = 1 \text{ meter)}$$

$$= \frac{\ln(80/75)}{2\pi \times 58 \times 1} + \frac{\ln(10/80)}{2\pi \times 0.18 \times 1} + \frac{\ln(160/110)}{2\pi \times 0.09 \times 1} = 0.94435 \text{ K/Watt}$$

$$\dot{Q}_{\text{per meter}} = \frac{T_1 - T_2}{R_{total}} = \frac{320 - 40}{0.94435} = 296.49 \text{ Watt/m}$$

Now to calculate T_B and T_C

$$296.49 = \frac{320 - T_B}{R_A} \Rightarrow T_B = 319.94°C$$

Very small temperature gradient due to high conductivity of steel pipe.

Also,

$$Q = \frac{T_1 - T_C}{R_A + R_B}$$

$$296.49 = \frac{320 - T_C}{0.28175} \Rightarrow T_C = 236.46°C$$

(ii) Heat extracted by steam = 296.49 Watt

$$\dot{m}_{steam} = \frac{0.32}{60} = 5.333 \times 10^{-3} \text{ kg/sec}$$

Hence energy lost by 1 kg of steam $= \dfrac{296.49}{5.333 \times 10^{-3}} = 55.59$ kJ

So, the enthalpy of the steam coming out of one metre pipe is

$$h = 2703 - 55.59 = 2647.41 \text{ kJ/kg}$$

Now,

$$2647.71 = 1463 + x1240 \Rightarrow x = 0.955$$

So the value of dryness fraction $x = 0.955$

2. Fins

Q.12 With usual notations, develop an expression for the efficiency of a fin of uniform cross-section when the heat loss from the tip is considered negligible.

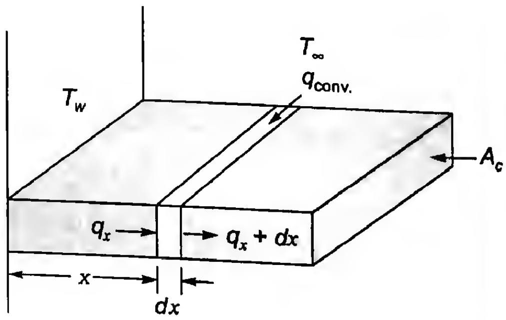
[CSE (Mains) 2005 : 30 Marks]

Solution:

Consider a finned surfaces as shown in figure with thermal conductivity (k), area of cross section (A_c), perimeter (p) and convection coefficient (h).

Consider a section at a distance x from base of fin of thickness dx.

For this section, Energy Balance:



$$q_{in} = q_{out}$$

$$\Rightarrow \quad q_x = q_{x+dx} + q_{conv}$$

$$\Rightarrow \quad q_{x+dx} - q_x + q_{conv} = 0$$

$$\Rightarrow \quad \frac{dq_x}{dx} \cdot dx + h(pdx)(T - T_\infty) = 0$$

$$\Rightarrow \quad KA_c \frac{d^2T}{dx^2} - hp(T - T_\infty) = 0$$

Since T_∞ = const, taking $\theta = T - T_\infty$, $d\theta = dT$

$$\therefore \quad \frac{d^2\theta}{dx^2} - \frac{hp}{KA_c}\theta = 0$$

$$\Rightarrow \quad \frac{d^2\theta}{dx^2} - m^2\theta = 0 \quad \text{where} \quad m = \sqrt{\frac{hp}{KA_c}}$$

General solution of this equation is,

$$\theta = C_1 e^{mx} + C_2 e^{-mx}$$

For heat loss from the fin tip to be negligible

$$\left.\frac{d\theta}{dx}\right|_{x=L} = 0 \quad \text{1st Boundary condition}$$

2nd Boundary condition $\quad \theta|_{x=0} = \theta_0 = T_w - T_\infty$

Solving using the above boundary conditions, we get

$$\text{Heat loss from fin} = Q = \sqrt{hpKA_c}(T_w - T_\infty)\tanh mL$$

Fin Efficiency: This is defined as the ratio of actual heat transfer from a fin surface and ideal heat transfer from fin.

Ideal heat transfer takes place when the temperature along the entire length of fin becomes equal to base or wall temperature.

$$\eta_{fin} = \frac{Q_{fin}}{Q_{fin,\,max}} = \frac{\sqrt{hpKA_c}(T_w - T_\infty)\tanh mL}{hA_{fin}(T_w - T_\infty)}$$

$$A_{fin} = pL$$

$$\eta_{fin} = \sqrt{\frac{kA_c}{hp}} \cdot \frac{\tanh mL}{L}$$

We know, $\quad m = \sqrt{\frac{hp}{kA_c}}$

$\therefore$ fin efficiency, $\quad \eta_{fin} = \frac{\tanh mL}{mL}$

Q.13 The temperature of a gas flowing through a pipe was measured by a mercury-in-glass thermometer, dipped in an oil-filled steel tube welded radially to the pipeline. The thermometer indicates a temperature lower than the gas temperature. How large is the error in the temperature measurement if the thermometer reads 85°C and the temperature of the pipe wall is 40°C? The steel tube is 125 mm long and has a 1.5 mm thick wall. The thermal conductivity of this tube material is 56 W/m-K and the local heat transfer coefficient between the gas and the tube is 23.5 W/m²-K.

In what way the thermometric error can be reduced?

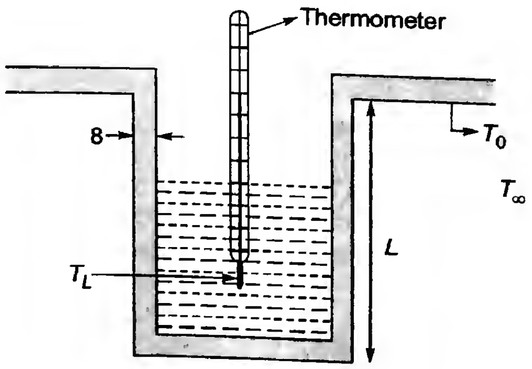
[CSE (Mains) 2006 : 30 Marks]

Solution:

Given: $h = 23.5$ W/m²K ; $K = 56$ w/mK; $L = 125$ mm

Temperature measured will not be T_∞, but it will be T_L that is the temperature at the end of the fin.

Considering the thermometer pocket as pin fin, asuming there is no heat flow from tip.



Using the general solution $\theta = C_1 e^{-mx} + C_2 e^{mx}$ and

Boundary condition $\theta = \theta_0$ at $x = 0$, $\frac{d\theta}{dx} = 0$ at $x = L$ we have

$$\theta = \frac{\theta_0 \cos h\, m(L-x)}{\cos h\; mL}$$

$$\text{Temperature at fin tip } x = L,\ \theta_L = \frac{\theta_0}{\cos h\; mL}$$

$$m = \sqrt{\frac{hP}{KA}} = \sqrt{\frac{h \times \pi d}{K \times \pi d S}} = \sqrt{\frac{23.5}{56 \times 1.5 \times 10^{-3}}} = 16.726$$

$$(T_L - T_\infty) = \frac{T_0 - T_\infty}{\cos h\, 16.726 \times 0.125} \Rightarrow 85 - T_\infty = \frac{45° - T_\infty}{4.10728}$$

$$T_\infty = \frac{85 \times 4.10728 - 45}{4.10728 - 1} = 99.48°C$$

$$\text{Error} = T_\infty - T_L = 99.48 - 85°C = 14.48°C$$

To reduce error, θ_L should be minimum, so ↑ $\cos h$ mL, ↑ mL, $\sqrt{\frac{h}{K\delta}} \cdot L$

(i) increase h

(ii) reduce K

(iii) long width in wall, product may be placed obliquely to produce longer insertion.

Q.14 A very long AISI 316 stainless steel (K = 14 W/m-K) rod 5 mm in diameter has one end maintained at 100°C. The surface of the rod is exposed to ambient air at 30°C with average convective heat transfer coefficient of 50 W/m²-K. Neglecting radiation heat transfer, estimate how long the rod must be to treat it as "infinitely long" to yield a reasonable accurate estimation of heat loss. If the rod is made of copper K = 350 W/m-K, will the length be different? How much will it be and why? Compare the heat transfer rates for both the rods. The analysis may be based on fin tip heat loss alone.

[CSE (Mains) 2012 : 12 Marks]

Solution:

For a very long rod temperature distribution given by

$q = \theta_0 e^{-mx}$; $\theta = T - T_\infty$

Thus heat transfer rate, $q = \int_0^L h \cdot P \cdot dx (T - T_\infty) = \int_0^L h \cdot P \cdot \theta \cdot dx$

$$= \int_0^L hP\theta_0 e^{-mx} dx = \frac{hP\theta_0}{m}(1 - e^{-mx})$$

For $L = \infty$,

$$q = \frac{hP\theta_0}{m} = \sqrt{hPKA}\,\theta_0$$

So, reasonably accurate estimate of heat loss, we need to have rod long enough so that

$$e^{-mL} = 0.01 = 1\%$$

$\therefore$ $q_L = q_\infty \times 0.99$ (reasonable estimation)

$$m = \sqrt{\frac{hP}{KA}} = \sqrt{\frac{50 \times 2\pi \times 2.5 \times 10^{-3}}{14 \times \pi (2.5 \times 10^{-3})}} = 53.45$$

Then, $e^{-53.45L} = 0.01$

$\therefore$ $L = 0.086$ m

$\therefore$
$$q_{ss} = \frac{50 \times 2\pi \times 2.5 \times 10^{-3} \times (100 - 30)}{53.45} \times 0.99 = 1.0183 \text{ Watt}$$

For copper rod,
$$m = \sqrt{\frac{50 \times 2\pi \times 2.5 \times 10^{-3}}{350 \times \pi \times (2.5 \times 10^{-3})^2}} = 10.69$$

Then, $e^{-mL} = 0.01$

$\therefore$
$$L = \frac{\ln 0.01}{-10.69} = 0.43 \text{ m}$$

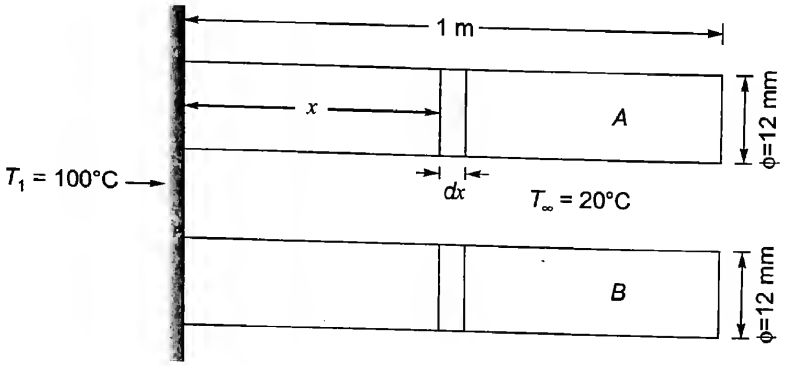
Heat transfer rate,
$$q = \frac{50 \times 2\pi \times 2.5 \times 10^{-3}}{10.69} \times (100 - 30) \times 0.99$$

$$q_{Cu} = 5.0915 \text{ Watt}$$

Q.15 Two long slender rods A and B, made of different materials having same diameter of 12 mm and length 1 m, are attached to a surface maintained at a temperature of 100°C. The surfaces of the rods are exposed to ambient still air at 20°C. By traversing along the length of the rods with a temperature sensor, it is found that the surface temperatures of rods A and B are equal at positions 15 cm and 7.5 cm respectively away from the base surface. If material of A is carbon steel with thermal conductivity 60 W/mK, what is the thermal conductivity of rod B? List the assumptions made. Assume that the average convection coefficient air is 5 W/m^{2}K. Find the ratio of the rate of heat transfer for rods A and B.

[CSE (Mains), 2014 : 20 Marks]

Solution:



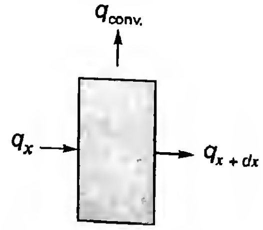
Base of both the rods is maintained at the same temperature as shown in figure.

Assumption:

(i) Since the length of the rods is very large 1 m as compared to their diameter, they can be assumed to be infinitely long with temperature at tip approaching surrounding temperature.

(ii) Heat transfer is purely one dimensional conduction in the rods and convection from rod surface.

Given: h = 5 W/m^{2}K, K_A = 60 W/mK, $A_c = \frac{\pi d^2}{4} = 1.131\times10^{-4}\text{m}^2$, $p = 2\pi r = 0.0377$ m



Consider an element of thickness dx at a distance x from the base on any rod.

Energy balance for this section

$$q_x - q_{x+dx} = q_{\text{conv}}$$

$$\Rightarrow \quad \frac{kA_c d^2T dx}{dx^2} - h(pdx)(T - T_\infty) = 0$$

$$\Rightarrow \quad \frac{d^2\theta}{dx^2} - m^2\theta = 0$$

where, $\theta = T - T_\infty$ and $m = \sqrt{\frac{hp}{kA_c}}$

Solution of this differential equation with following boundary conditions:

(i) $T|_{x=0} = T_1$

(ii) Infinitely long rod assumption due to which $T|_{x=L} = T_\infty$

Solution of this boundary condition

$$\theta = \theta_b e^{-mx}$$

For rod A:

$$m_A = \sqrt{\frac{hp}{kA_c}} = 5.271$$

Temperature at 15 cm from base

$$\frac{T - T_\infty}{T_b - T_\infty} = e^{-mx} = e^{-m \times 0.15}$$

$\Rightarrow$ $$T = 329.29 \text{ K}$$

This temperature is equal to temperature of rod B at a distance 7.5 cm from base.

For rod b:

$$\frac{T - T_\infty}{T_b - T_\infty} = e^{-m_B \times 0.075}$$

$\Rightarrow$ $$\frac{329.29 - 293}{373 - 293} = e^{-m_B \times 0.075}$$

We know, $$m_B = \sqrt{\frac{hp}{k_B A_C}} = 10.5398$$

$\Rightarrow$ $$m_B = 10.5398$$

$\Rightarrow$ $$k_B = 15 \text{ W/mK}$$

Heat transfer for a rod in this case $= \sqrt{hpkA_C} \cdot (T_b - T_\infty)$

$\therefore$ Ratio of heat transfer for rod A and B

$$\frac{Q_A}{Q_B} = \frac{\sqrt{hpk_A A_c} \cdot (T_b - T_\infty)}{\sqrt{hpk_B A_c} \cdot (T_b - T_\infty)}$$

$\Rightarrow$ $$\frac{Q_A}{Q_B} = \sqrt{\frac{k_A}{k_c}} = 2$$

3. Free and Forced Convection

Q.16 Show that: $\quad \text{Nu}\,(\text{Pr})^{-1/3} = \{\text{St}\,(\text{Pr})^{2/3}\}\,\text{Re}$

[CSE (Mains) 2001 : 20 Marks]

Solution:

For laminar flow of a fluid over a flat plate, we have

Local Nusselt Number, $\text{Nu}_x = 0.332\, P_r^{1/3}\, \text{Re}_x^{1/2}$ ... (i)

and Local skin friction coefficient, $C_{f,x} = 0.664 \cdot \text{Re}_x^{-1/2}$ ... (ii)

where, Pr = Prandtl number

Re_x = Local Reynolds number

We know that:

$$\text{Local Stanton number} = \frac{\text{Nu}_x}{\text{Re}_x \text{Pr}} = \text{St}_x$$

$\therefore$ $$St_x = \frac{\text{Nu}_x}{\text{Re}_x \text{Pr}} = \frac{0.332 \cdot P_r^{1/3} \cdot \text{Re}_x^{1/2}}{\text{Re}_x \cdot \text{Pr}} = \frac{0.332}{\text{Re}_x^{1/2} \cdot P_r^{2/3}}$$

$\Rightarrow$ $$(St_x) \cdot P_r^{2/3} \cdot \text{Re}_x = 0.332\, \text{Re}_x^{1/2} \quad \text{... (iii)}$$

From (ii) above, we have

$$0.332 \cdot \text{Re}_x^{1/2} \cdot \text{Re}_x = 0.332 \cdot \text{Re}_x^{1/2} = \text{Nu}_x \cdot P_r^{-1/3}$$

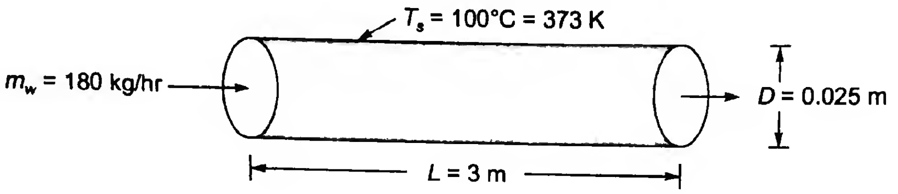
$\Rightarrow$ $$\text{Nu}_x\, P_r^{-1/3} = \{\text{St}_x \cdot P_r^{2/3}\} \cdot \text{Re}_x$$

Hence proved.

Q.17 **A 3 m long 25 mm dia tubes is held at 100°C by steam jacketing. Water flows through the tube at the rate 180 kg/hr at 20°C. Calculate the rate of heat transfer from the tube to water. For water (60°C): C_P = 4.178 kJ/kg K; k = 0.66 (W/mK); ρ = 983 kg/m³ and $\mu = 0.47 \times 10^{-3}$ kg/ms.**

[CSE (Mains) 2001 : 30 Marks]

Solution:



Consider the tube as shown through which water flows. Film temperature = $T_f = \dfrac{20+100}{2} = 60°C$

We have,
$$\dot{m}_w = 180 \text{ kg/hr} = \rho A\, v_{avg}$$

$$\Rightarrow \quad v_{avg} = \frac{180}{3600} \cdot \frac{1}{983 \cdot \frac{\pi}{4} \cdot 0.025^2} = 0.1036 \text{ m/s}$$

Reynolds number for the flow, $$Re_L = \rho \frac{V_{avg} D}{\mu} = \frac{983 \times 0.1036 \times 0.025}{0.47 \times 10^{-3}} = 5418.04$$

Prandtl number, $$Pr = \frac{\mu C_P}{k} = \frac{0.47 \times 10^{-3} \times 4.178 \times 10^3}{0.66} = 2.975$$

Since the flow is turbulent Re > 2300 and the fluid is heating, applying Dittus Boelter equation, we get

$$Nu_D = 0.023 \cdot Re^{0.8} \cdot Pr^n, \qquad \text{where, } n = 0.4 \text{ for heating of fluid}$$
$$= 0.023 \cdot (5418)^{0.8} \cdot (2.975)^{0.4} = 34.53$$

We know,
$$Nu_D = \frac{hD}{k} = 34.53$$

$\Rightarrow$ Convection coefficient, $$h = \frac{34.53 \times 0.66}{0.025} = 911.592 \text{ Watt/m}^2\text{K}$$

This will be equal to overall heat transfer coefficient of heat exchanger (U).

For steam and water heat exchanger,
$$C_{max} = \infty$$
$$C_{min} = \dot{m}_w \cdot C_{P,w} = 208.9 \text{ Watt/K}$$

Effectiveness
$$\epsilon = 1 - e^{-NTU}$$

$$NTU = \frac{UA}{C_{min}} = \frac{911.592 \times \pi \times 0.025 \times 3}{208.9} = 1.028$$

$\therefore$
$$\epsilon = 1 - e^{-NTU} = 0.64235$$

We know that,
$$\epsilon = \frac{T_{c,o} - T_{c,i}}{T_{h,i} - T_{c,i}} = \frac{T_{c,o} - 20}{100 - 20}$$

$\Rightarrow$
$$T_{c,o} = 71.38°C$$

$\therefore$ Rate of heat transfer from tube to water

$$Q = mC_P \Delta T_c = \frac{180}{3600} \times 4.178 \times (71.38 - 20) = 10.73 \text{ kW}$$

Q.18 Air at 30°C flows over a horizontal plate heated to 70°C at a speed of 3 m/s. Calculate the convective heat transfer coefficient and the rate of heat transfer between the plate and the air. The length of the plate is 2 m and width 1 m. Nusselt number is given by:

$$Nu = 0.664\, Re^{1/2}\, Pr^{1/3}$$

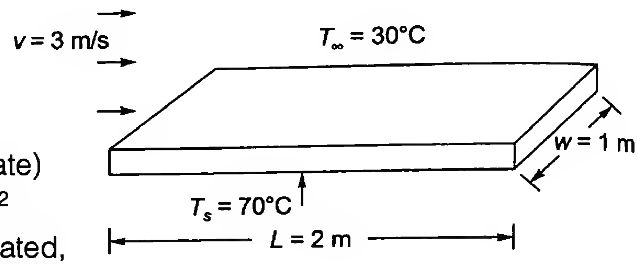
The properties of air at 50°C are:

$\nu = 17.95 \times 10^{-6}$ m²/s, $K = 0.0283$ W/mK, $\rho = 1.093$ kg/m³, $C_P = 1.005$ kJ/kgK

[CSE (Mains) 2002 : 30 Marks]

Solution:

Consider the case of forced convection between heated plate and air is shown in figure.



Characteristic length for a flat plate,

$$L_c = 2 \text{ m (length of plate)}$$

Area of heat transfer, $A_s = 2 \text{ m} \times 1 \text{ m} = 2 \text{ m}^2$

Film temperature at which properties of fluid are to be evaluated,

$$T_f = \frac{T_s + T_\infty}{2} = \frac{70+30}{2} = 50°\text{C}$$

$$\text{Reynolds number for flow} = Re_L = \frac{vL_c}{\nu} = \frac{3 \times 2}{17.95 \times 10^{-6}} = 334261.81 \simeq 33.4262 \times 10^4$$

$$\text{Prandtl number for fluid} = \frac{\nu}{\alpha} = \frac{\nu}{K} \cdot \rho C_p = \frac{17.95 \times 10^{-6} \times 1.093 \times 1005}{0.0283} = 0.6967$$

$$\text{Nusselt number} = Nu = 0.664\, Re^{1/2}\, Pr^{1/3} = 340.33$$

We know,

$$Nu = \frac{hL}{K} = 340.33$$

$$\Rightarrow \quad h = \frac{340.33 \times 0.0283}{2} = 4.816 \text{ W/m}^2\text{K}$$

∴ Rate of heat transfer between the plate and air is

$$\phi = h A_s (T_s - T_\infty) = 4.816 \times 2 \times 40 = 385.25 \text{ Watt}$$

Q.19 Define Reynolds number, Prandtl number, Stanton number and Nusselt number and give their physical significace. What is meant by fully developed flow in a pipe? Illustrate it with the help of a figure. State the parameters upon which the entrance length depends.

[CSE (Mains) 2003 : 20 Marks]

Solution:

Reynolds number (Re) can be defined as the ratio of inertia forces to viscous forces in a fluid flow.

By definition,

$$Re = \frac{\text{Inertia force}}{\text{Viscous force}} = \frac{vL}{\nu}$$

Reynolds number signifies which force-inertia or viscous is more dominant in a flow regime. This decides whether flow is turbulent (high inertia forces and high Re) or laminar (high viscous forces and low Re).

Prandtl Number can be defined as the ratio of molecular diffusivity of momentum and thermal diffusivity.

$$\therefore \quad Pr = \frac{\text{Momentum diffusivity}}{\text{Thermal diffusivity}} = \frac{\nu}{\alpha} = \frac{\mu C_P}{K}$$

Hence, Prandtl number signifies the relative thickness of thermal and velocity boundary layers. High and low values of Prandtl number signify a thicker velocity and thermal boundary layers, respectively. Prandtl number is strictly a property of fluid alone.

Nusselt Number can be defined as the ratio of heat transfer by convection to the heat transfer by conduction.

$$\therefore \quad \text{Nu} = \frac{\dot{q}_{\text{convection}}}{\dot{q}_{\text{conduction}}} = \frac{h\,\Delta T}{K\,\Delta T / L} = \frac{hL}{K}$$

Nusselt number signifies enhancement of heat transfer due to convection. High Nusselt number implies convection is more dominant than conduction at fluid-solid surface.

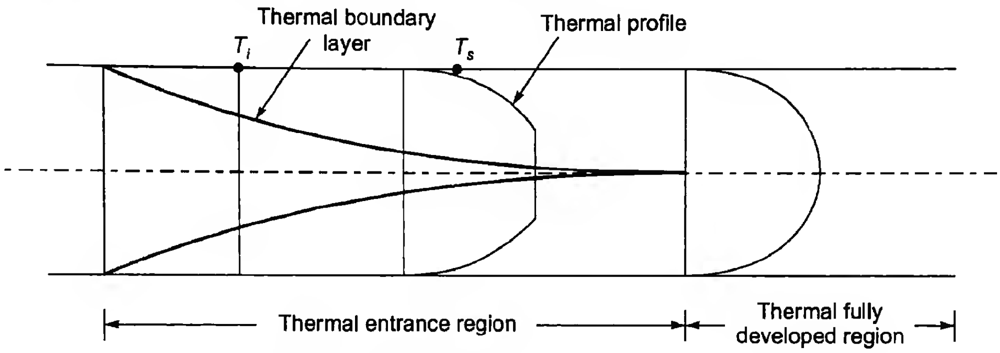
Stanton Number is defined as the ratio of heat transferred by convection to thermal capacity of fluid.

$$\therefore \quad \text{St} = \frac{h}{\rho v C_P} = \frac{\text{Nu}}{\text{Re Pr}}$$

Stanton number is used to express relationship between drag force and heat transfer in cases of geometric similarity between hydrodynamic and thermal boundary layers.

In case of internal forced convection, fully developed flow means that hydrodynamic (velocity) and thermal (temperature) profiles do not change with axial distance along the tube.

For example, as shown in figure below, post thermal entry length, temperature profile does not change in x-direction and depends on radial distance from axis alone.



Parameters upon which entrance length depends—

(a) Velocity and Reynolds number – nature of flow laminar or turbulent
(b) Smoothness or roughness of pipe/tube
(c) Dimensions and geometry of tube.

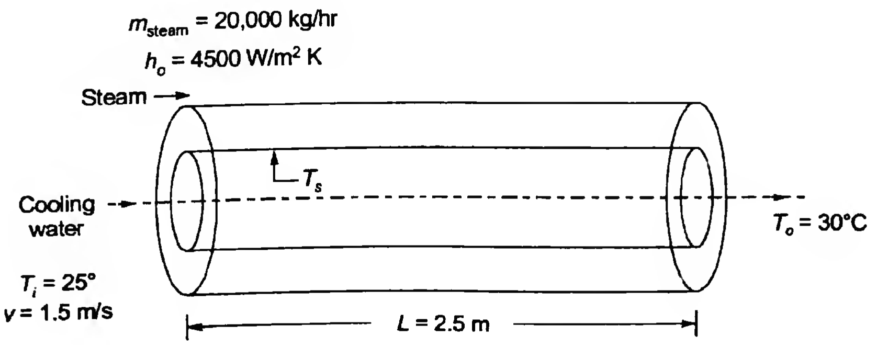
Q.20 Calculate the number of tubes required for a single pass steam condenser of 2.5 m length to handle 20000 kg/hr of dry saturated steam at 60°C. The cooling water enters the tubes at 20°C and leaves at 30°C. The tubes are of 25 mm outside diameter and 22.5 mm inside diameter. The thermal conductivity of tube material is 100 W/m-K. The water velocity is 1.5 m/s. Assume that the steam side film coefficient is 4500 W/m²-K. For water use, $\text{Nu} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.4}$

The properties of water at 25°C are:

$\rho = 996.95$ kg/m³, $C_P = 4178$ J/kg-K, $k = 60.78 \times 10^{-2}$ W/m-K, $\nu = 0.9055 \times 10^{-6}$ m²/s, Pr = 6.22, h_{fg} (at 60°C) = 2358.5 kJ/kg

[CSE (Mains) 2003 : 40 Marks]

Solution:



Given: Steam side convection coefficient, $h_0 = 4500\ \text{W/m}^2\text{K}$

Latent heat of steam at 60°C, $h_{fg} = 2358.5\ \text{kJ/kg}$

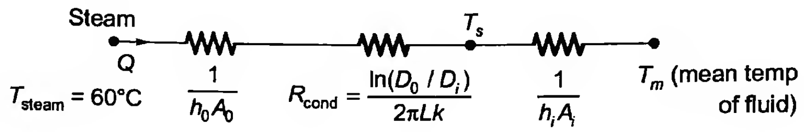
$\therefore$ Rate of heat transfer from steam to water $= \dot{m}_{\text{steam}} \cdot h_{fg} = \dfrac{20{,}000}{3600} \times 2358.5\ \text{kJ/kg}$

$$\dot{Q} = 13{,}102.78\ \text{kW}$$

Mean film temperature for water, $T_f = \dfrac{T_i + T_0}{2} = 25°\text{C}$

Reynolds number for water flow, $\text{Re} = \dfrac{vD_i}{\nu} = \dfrac{1.5 \times 0.0225}{0.9055 \times 10^{-6}} = 37272.225$

Consider the thermal resistance network for tube,



Thermal resistance of tube, $R_{\text{cond.}} = \dfrac{\ln(25/22.5)}{2\pi L k} = \dfrac{\ln(25/22.5)}{2\pi \times 2.5 \times 100} = 6.7075 \times 10^{-5}$ K/watt

For the water flowing in tube, we have

$$\text{Nu} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.4}$$
$$= 0.023 \times (37272.225)^{0.8}(6.22)^{0.4} = 216.74$$

We know $\text{Nu} = \dfrac{hD}{k} = 216.74$

$\Rightarrow$ $h = h_i = \dfrac{216.74 \times 60.78 \times 12^{-2}}{0.0225} = 5854.79\ \text{W/m}^2\text{K}$

Internal convection resistance, $\dfrac{1}{h_i A_i} = \dfrac{1}{5854.79 \times \pi \times 2.5 \times 0.0225} = 9.665 \times 10^{-4}$ K/watt

Overall thermal resistance, $R_{\text{net}} = \dfrac{1}{h_i A_i} + R_{\text{cond.}} + \dfrac{1}{h_0 A_0}$

$$= 9.665 \times 10^{-4} + 6.7075 \times 10^{-5} + \frac{1}{4500 \times \pi \times 0.025 \times 2.5}$$

$\Rightarrow$ $R_{\text{net}} = 2.1654 \times 10^{-3}$ K/watt

Overall heat transfer coefficient of heat exchanger, U and heat transfer surface $= A_s$

$\therefore$ $\dfrac{1}{UA_s} = R_{\text{net}}$

Log mean temperature difference for heat exchanger $= \dfrac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)}$

$\Rightarrow$ $\Delta T_{lm} = \dfrac{(60-20)-(60-30)}{\ln\left(\dfrac{60-20}{60-30}\right)} = 34.76\ \text{K}$

$\therefore$ Heat transfer rate through a single tube

$$\dot{q} = UA_s \cdot \Delta T_{lm} = \frac{\Delta T_{lm}}{R_{\text{net}}} = \frac{34.76}{2.1654 \times 10^{-3}} = 16.053\ \text{kW}$$

$\therefore$ No. of tubes required $= \dfrac{\text{Overall heat transfer rate}}{\dot{q}} = \dfrac{13102.78}{16.053} = 816.23 \simeq 817$ tubes

Q.21 Define the term Nusselt number and explain its significance in the case of forced convection heat transfer.

Atmospheric air at T_∞ = 275 K and a free-stream velocity u_∞ = 20 m/s flows over a plate L = 1.5 m long that is maintained at a uniform temperature T_w = 325 K.

(i) Calculate the average heat transfer coefficient h_m over the region where the boundary layer is laminar.

(ii) Find the average heat transfer coefficient over the entire length L = 1.5 m of the plate.

(iii) Calculate the total heat transfer rate Q from the plate to the air over the length L = 1.5 m and width W = 1 m.

Assume transition occurs at Re = 2×10^5.

[CSE (Mains) 2004 : 30 Marks]

Solution:

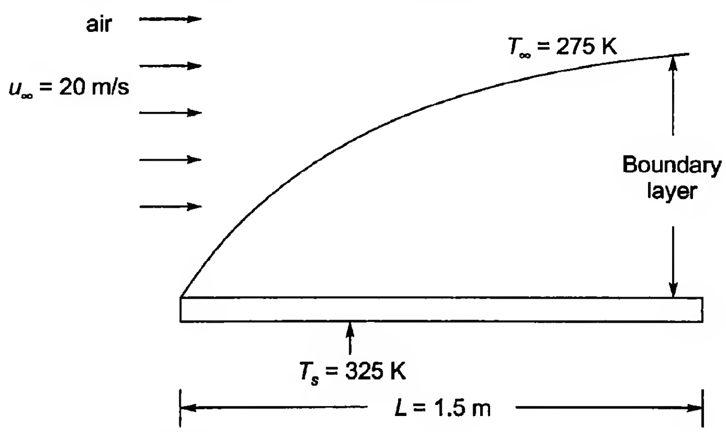
Nusselt Number can be defined as the ratio of rates of heat transfer by convection to heat transfer by conduction in a heat exchange between a fluid and a solid.

$$\text{Nu} = \frac{hL_c}{K}$$

Where,
h - convection coefficient
K - conduction coefficient
L_c - characteristic length

Nusselt number signifies the increase in heat transfer by convection mechanism in a forced convection case. If value of Nusselt number is high it signifies convection is the dominant form of heat transfer.

Consider the forced convection from plate as shown in figure



Film temperature for property calculation,

$$T_f = \frac{T_s + T_\infty}{2} = 300 \text{ K}$$

Properties of air at 300 K,

$$\mu = 1.846 \times 10^{-5} \text{ kg/m/s}$$
$$\rho = 1.177 \text{ kg/m}^3$$
$$K = 2.624 \times 10^{-2} \text{ W/mK}$$
$$\alpha = 22.07 \times 10^{-6} \text{ m}^2\text{/s}$$

Reynolds number at transition from laminar to turbulent boundary layer = 2×10^5.

Assume transition occurs at a distance 'x' from one end.

$$\therefore \quad \text{Re}_x = \frac{v\rho x_{cr}}{\mu} = 2 \times 10^5$$

$$\Rightarrow \quad x_{cr} = \frac{2 \times 10^5 \times 1.846 \times 10^{-5}}{20 \times 1.177} = 0.157 \text{ m}$$

Prandtl number for fluid flow,

$$Pr = \frac{\nu}{\alpha} = \frac{\mu}{\rho\alpha} = \frac{1.846\times10^{-5}}{1.177\times22.07\times10^{-6}} = 0.711$$

We know, for laminar region

$$Nu_x = 0.332\, Re_x^{0.5} \cdot Pr^{1/3}$$

$$\Rightarrow \quad \frac{h_x x}{k} = (0.332 \cdot Pr^{1/3}) \cdot \left[\left(\frac{v\rho}{\mu}\right)\cdot x\right]^{1/2}$$

$$\Rightarrow \quad h_x = 0.332 \cdot Pr^{1/3} \cdot K\left(\frac{v\rho}{\mu}\right)^{1/2} \cdot x^{-1/2}$$

∴ Average convection coefficient over laminar region,

$$h_{avg,\,Lam} = \frac{1}{x_{cr}}\int_0^{x_{cr}} h_x dx$$

$$= \frac{1}{0.157}\int 0.332 \cdot 0.711^{1/3} \cdot 2.624\times10^{-2}\left(\frac{20\times1.177}{1.846\times10^{-5}}\right)^{1/5} x^{-1/2}dx$$

$$= 55.926\int_0^{0.157} x^{-1/2}dx = 2\times55.926\times\sqrt{0.157} = 44.32 \text{ Watt/m}^2\text{ K}$$

For Turbulent region

$$Nu_x = 0.0296\times Re_x^{0.8}\cdot Pr^{1/3}$$

$$= 0.0296\times\left(\frac{20\times1.177}{1.846\times10^{-5}}\right)^{0.8}\times x^{0.8}\cdot 0.711^{1/3}$$

$$= 2024.765\times x^{0.8}$$

$$\therefore \quad Nu_x = \frac{h_x\cdot x}{K} = 2024.765\times x^{0.8} \Rightarrow h_x = 53.13\times x^{-0.2}$$

For the entire length of plate, average convection coefficient,

$$h = \frac{1}{1.5}\left\{\left[\int_0^{0.157}\frac{0.332\cdot Re_x^{0.5}\cdot Pr^{1/3}\cdot K\,dx}{x} + \int_{0.157}^{1.5}\frac{K(0.0296\cdot Re_x^{0.8}\cdot Pr^{1/3}}{x}\right]dx\right\}$$

$$= \frac{1}{1.5}\times\left[6.958 + \int_{0.157}^{1.5} 53.13\times x^{-0.2}dx\right]$$

$$h_{avg} = 55.81 \text{ Watt/m}^2\text{ K}$$

Total area for heat transfer, $A_s = 1.5\times1 = 1.5 \text{ m}^2$

∴ Total heat tranfser rate from the plate,

$$\dot{Q} = h_{avg}\cdot A_s\cdot(T_s - T_\infty)$$

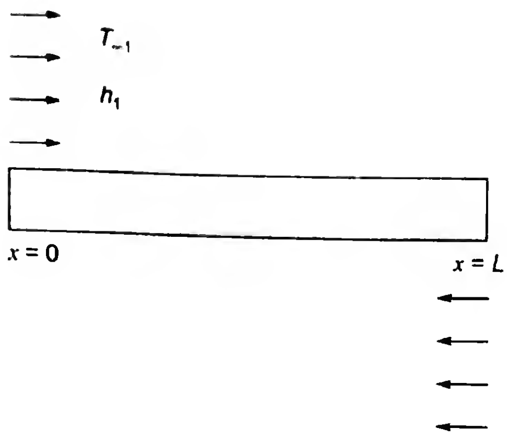
$$\dot{Q} = 55.81\times1.5\times(325-275) = 4.186 \text{ kW}$$

Q.22 A fluid at a temperature T_{∞_1}, with a heat transfer coefficient h_1 flows on one side of a slab at $x = 0$ and another fluid at a temperature T_{∞_2} with a heat transfer coefficient h_2 flows on the other side of the slab at $x = L$. Both the fluids flow in opposite directions. Develop an expression for the heat flow Q through an area 'A' of the slab.

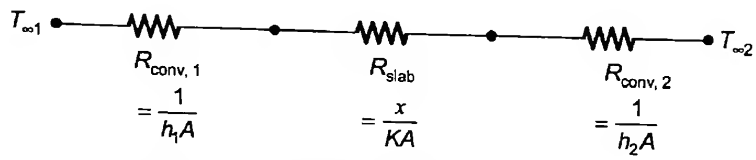
Calculate the heat transfer rate through an area of one square metre of the slab for the following data: $T_{\infty_1} = 150$°C, $h_1 = 300$ W/m²K, $T_{\infty_2} = 50$°C, $h_2 = 600$ W/m²K, thickness of slab = 5 cm and $K = 20$ W/mK.

[CSE (Mains) 2004 : 30 Marks]

Solution:



Consider heat transfer on two sides of the plate as shown in figure.
Taking thickness of slab to be 'x' m and its thermal conductivity to be 'K' W/mK.
Assumption: Temperature of air streams on both sides remains constant. Constructing thermal resistance network for this case,



∴ Net thermal resistance between the two sides of slab
⇒

$$R_{net} = R_{conv,\,1} + R_{slab} + R_{conv,\,2}$$

$$= \frac{1}{h_1 A} + \frac{x}{KA} + \frac{1}{h_2 A}$$

∴ Rate of heat transfer between the two sides, taking ($T\infty_2 > T\infty_1$),

$$Q = \frac{T\infty_2 - T\infty_1}{R_{net}} \Rightarrow Q = \frac{T\infty_2 - T\infty_1}{\frac{1}{h_1 A} + \frac{x}{KA} + \frac{1}{h_2 A}}$$

Given the values of temperature and heat transfer coefficients, we get

$$Q = \frac{T\infty_2 - T\infty_1}{\frac{1}{h_1 A} + \frac{x}{KA} + \frac{1}{h_2 A}}$$

⇒ heat transfer per square meter of area,

$$\frac{Q}{A} = \frac{T\infty_2 - T\infty_1}{\frac{1}{h_1 A} + \frac{x}{KA} + \frac{1}{h_2 A}} = \frac{150 - 50}{(300)^{-1} + \frac{0.05}{20} + (600)^{-1}} = 13.33 \text{ kW/m}^2$$

Q.23 A rectangular copper plate 10 cm × 50 cm, having a mass of 1 kg and at a temperature of 100°C, is suspended vertically in still air at 20°C such that 50 cm side is vertical. Neglecting radiation effect, find heat transfer coefficient due to natural convection and initial rate of cooling of the plate in °C/minute. Take C_p for copper = 383 J/kgK

The properties of air at mean temperature 60°C are:

ρ = 1.06 kg/m³, v = 18.97 × 10⁻⁶ m²/s, Pr = 0.696, C_p = 1.005 kJ/kg-K, k = 28.96 × 10⁻³ W/m-K

μ = 20.1 × 10⁻⁶ N-s/m²

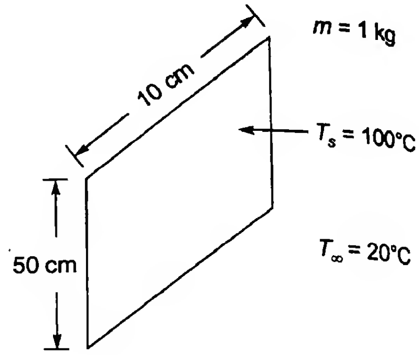
You may use the following correlation: Nu = 0.1 (Gr Pr)$^{1/3}$

Will the result change if 10 cm side is vertical? Why?

[CSE (Mains) 2006 : 20 Marks]

Solution:

Consider the plate as shown in figure undergoing natural convection in ambient air.



For vertical plate, we have,

$$\text{Characteristic length, } L_c = 50 \text{ cm}$$

Mean film temperature for evaluating properties of air

$$T_m = \frac{20+100}{2} = 60°C$$

$\therefore$ Prandtl number for air, $\text{Pr} = \dfrac{\mu C_p}{k} = \dfrac{20.1 \times 10^{-6} \times 1005}{28.96 \times 10^{-3}}$

$\Rightarrow$ $\text{Pr} = 0.6975$

Volume expansion coefficient, $\beta = \dfrac{1}{T_m} = \dfrac{1}{333} k^{-1}$

$\therefore$ Grashoff number, $\text{Gr} = \dfrac{g\beta(T_s - T\infty)L_c^3}{\nu^2} = \dfrac{9.81 \times 333^{-1} \times (80) \times 0.5^3}{(18.97 \times 10^{-6})^2} = 818634629.6$

$$\text{Nu} = 0.1\,(\text{Gr Pr})^{1/3} = 82.96$$

We know, $\text{Nu} = \dfrac{hL_c}{K} = 82.96$

$\Rightarrow$ $h = \dfrac{82.96 \times 28.96 \times 10^{-3}}{0.5}$

$\therefore$ Heat transfer coefficient, $h = 4.81$ Watt/m^{2}K

Initial rate of heat transfer from plate to air,

$$Q = h\,A_s\,(T_s - T_\infty) = 4.81 \times 0.5 \times 0.1 \times 80 = 19.22 \text{ Watt}$$

This is the rate of decrease of internal energy of the plate,

$\therefore$ $Q = \dfrac{m}{t} \cdot C_P \cdot \Delta T$

$\Rightarrow$ $\dfrac{\Delta T}{t} = \dfrac{Q}{mC_P} = \dfrac{19.22}{1 \times 383} = 0.05$ K/sec. $= 3.01$ K/min.

$\therefore$ Rate of cooling, $\dfrac{°C}{\min}\left(\dfrac{\Delta T}{t}\right) = 3.01$ °C/min.

Rate of heat transfer will remain unchanged even on keeping 10 cm side vertical as,

$$Nu = \frac{hL}{k} = (0.1)(\text{Gr}\,\text{Pr})^{\frac{1}{3}}$$

$$h = \text{independent of } L$$

Q.24 Steel ball bearings ($K = 50$ W/m-K, $\alpha = 1.3 \times 10^{-5}$ m^2/s) having a diameter of 40 mm are heated to a temperature of 650°C and then quenched in a tank of oil at 55°C. If the heat transfer coefficient between the ball bearings and the oil is 300 W/m^2-K, determine:

(i) the duration of time the bearings must remain in oil to reach a temperature of 200°C;

(ii) the total amount of heat removed from each bearing during this time;

(iii) the instantaneous heat transfer rate from the bearing when they are first immersed in oil and when they reach 200°C.

[CSE (Mains), 2007 : 20 Marks]

Solution:

Checking applicability of lumped analysis:

$$\text{Biot number} = Bi = \frac{hL_C}{k} = \frac{h\cdot\frac{4}{3}\pi R^3}{k4\pi R^2} = \frac{300\times 0.04}{3\times 50\times 2} = 0.04$$

Since $Bi < 0.1$, entire steel ball can be treated as a lumped body and temperature difference in the body of steel are negligible.

Assume temperature of ball drops by dT in a duration of time dt.

Heat lost by body by convection = Decrease in internal energy of the body

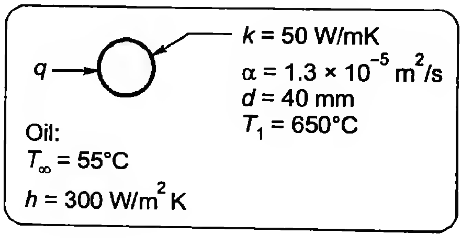
$$\therefore \quad dt\times hA\cdot(T-T_\infty) = -mC_p\,dT = -\rho V C_p\,dT$$

Since T_∞ = constant,

$$dT = d(T-T_\infty)$$

$$\frac{d(T-T_\infty)}{T-T_\infty} = -\frac{hA}{\rho C_p\cdot V}\cdot dt = -\frac{h}{(k/\alpha)\cdot L_C}\cdot dt$$

$$\text{Since, } \alpha = \frac{k}{\rho C_p} \quad \text{and} \quad \frac{V}{A} = L_C = \frac{R}{3}$$



$$\frac{d(T-T_\infty)}{T-T_\infty} = -\left(\frac{3h\alpha}{Rk}\right)dt$$

Integrating both sides,

$$\int_{T_1}^{T}\frac{d(T-T_\infty)}{(T-T_\infty)} = \int dt\left(-\frac{3h\alpha}{Rk}\right)$$

$$\Rightarrow \quad \ln\left[\frac{T-T_\infty}{T_i-T_\infty}\right] = t\cdot\left(-\frac{3h\alpha}{Rk}\right) \qquad \ldots(1)$$

This is the required relation between temperature attained of ball and time required.

Solving (1), we get

$$\ln\left[\frac{T-328}{595}\right] = -t0.0117$$

(*i*) Time required for bearing to reach 200°C

$$t = \frac{-1}{0.0117}\cdot\ln\left[\frac{473-328}{595}\right] = 120.67 \text{ sec.}$$

(*ii*) Total amount of heat removed during this time to reach 200°C

= decrease in internal energy of ball

$$= mC_p\Delta T = \rho VC_p\Delta T = \frac{k}{\alpha}\cdot\frac{4}{3}\pi R^3\cdot(T_i - T) = 57.978 \text{ kJ} \simeq 58 \text{ kJ}$$

(*iii*) Instantaneous heat transfer rates:

(a) When its first submerged, bearing temp = 650°C

$$\dot{Q} = h A_s(T-T_\infty) = 300.4\,\pi R^2\cdot(650-55) = 897.24 \text{ Watt}$$

(b) When it reaches 200°C,

$$Q' = h A_s(T-T_\infty) = 300.4\pi\cdot 0.02^2\cdot(200-55) = 218.65 \text{ Watt}$$

Q.25 **A 30 cm × 30 cm horizontal duct made of sheet metal carries warm air. The duct is maintained at 65°C and is 10 m long. Heat loss from duct to the ambient air takes place due to free convection only. If the ambient air is at 25°C, calculate the heat loss from the duct.**

Use the following free convection correlations. Heat transfer from the upper surface of a horizontal plate:

$\overline{Nu} = 0.54\,(Ra_L)^{1/4}$ Heat transfer from the lower surface of a horizontal plate:

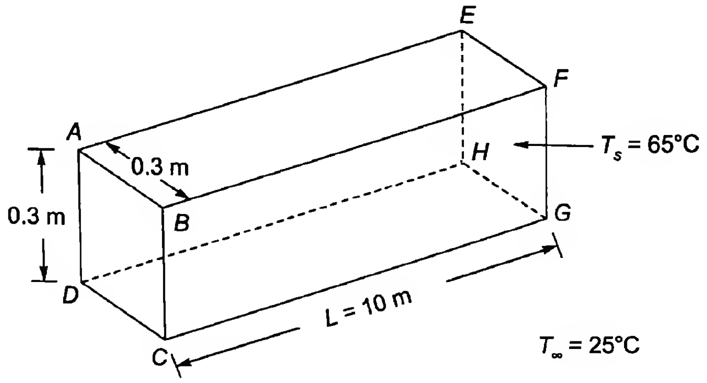
$\overline{Nu} = 0.27\,(Ra_L)^{1/4}$ Heat transfer from vertical plane $\overline{Nu} = 0.59\,(Ra_L)^{1/4}$

The properties of air at 45° are

$Pr = 0.695$, $\nu = 1.85 \times 10^{-5}$ m²/sec, $K = 0.028$ W/mK

[CSE (Mains) 2008 : 20 Marks]

Solution:



Heat transfer takes place from 4 surfaces - *ABFE*, *DCGH*, *BCGF* and *ADHE* by means of free convection.

$$\text{Film temperature, } T_f = \frac{T_s + T_\infty}{2} = \frac{65+25}{2} = 45°C$$

For horizontal surfaces - top and bottom

$$\text{Characteristic length, } L_c = \frac{A_s}{P} = \frac{0.3\times 10}{2(0.3+10)} = 0.146 \text{ m}$$

∴ Grashoff number for natural convection,

$$Gr = \frac{g\beta(T_s - T_\infty)\cdot L_c^3}{\nu^2} = \frac{9.81\times 318^{-1}\times 40\times 0.146^3}{(1.85\times 10^{-5})^2}$$

$$= 11220623.48$$

$$\text{Rayleigh number, } Ra_L = Gr_L \cdot Pr = 7798333.316$$

For upper plate:

$$\overline{Nu} = 0.54\times (Ra_L)^{1/4} = 28.536$$

We know

$$\overline{Nu} = \frac{hL_c}{k} = 28.54$$

$$\Rightarrow \quad h = \frac{28.54\times 0.028}{0.146} = 5.46 \text{ W/m}^2\text{K}$$

∴ Heat transfer from this surface, $Q_1 = hA_s(T_s - T_\infty) = 5.46 \times 0.3 \times 10 \times (40) = 655.48$ watt

For lower plate:

$$\overline{Nu} = 0.27\,(Ra_L)^{1/4} = 14.27 = \frac{hL_c}{k} = 14.27 \quad \Rightarrow h = 2.73 \text{ W/m}^2\text{K}$$

∴ Heat transfer from this surface, $Q_2 = h\,A_s(T_s - T_\infty) = 2.73 \times 0.3 \times 10 \times 40 = 327.74$ Watt

For vertical surfaces–

Characteristic length, $L_c = 0.3$ m

∴ Rayleigh Number, $Ra_L = Gr_L \cdot Pr$

$$= \frac{g\beta(T_s - T_\infty)L_c^3 \cdot Pr}{\nu^2} = \frac{9.81\times 318^{-1}\times 40\times 0.3^3}{(1.85\times 10^{-5})^2}\cdot 0.695$$

$$= 67656104.86$$

$$\overline{Nu} = 0.59\,(Ra_L)^{1/4} = 53.51$$

$$\Rightarrow \quad \overline{Nu} = \frac{h L_c}{k} = 53.51 \Rightarrow h = \frac{53.51 \times 0.028}{0.3} = 4.99 \text{ W/m}^2\text{K}$$

∴ Heat transfer from two such surfaces,

$$Q_3 = 2 \times h \cdot A_s \cdot (T_s - T_\infty) = 2 \times 4.99 \times 0.3 \times 10 \times 40 = 1198.61 \text{ Watt}$$

Total heat loss from duct $= Q_1 + Q_2 + Q_3 = 2.182$ kW

Q.26 A hot plate of 15 cm² area maintained at a temperature of 200°C is exposed to still air at 30°C temperature. When the smaller side of the plate is held vertical, convective heat transfer rate is 14% higher than when the bigger side of the plate is held vertical. Determine the dimensions of the plate. Neglect internal temperature gradient of the plate thickness. Also determine the heat transfer rate in both the cases.

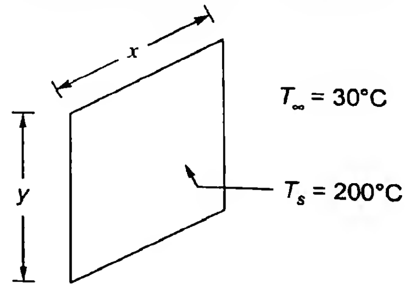
Use the following relation: $Nu = 0.59\,(Gr \cdot Pr)^{0.25}$

Take the following properties of air:

Temperature (°C)	ρ (kg/m³)	C_p (kJ/kg-K)	μ (N-s/m²)	K (W/m-K)
30	1.165	1.005	18.6×10^{-6}	0.0267
115	0.910	1.009	22.65×10^{-6}	0.0331
200	0.746	1.026	266×10^{-6}	0.0398

[CSE (Mains) 2009 : 30 Marks]

Solution:



Assume one of the sides of plate has length 'x' cm.

Since area of plate = 15 cm²

∴ Other side of plate $= y = \dfrac{15}{x}$ cm

Mean film temperature for evaluating properties of air,

$$T_m = \frac{T_s + T_\infty}{2} = \frac{30 + 200}{2} = 115°\text{C} = 388 \text{ K}$$

Heat transfer from plate, $Q = hA_s(T_s - T_\infty)$

$\Rightarrow \quad Q \propto h$

Also, Nusselt number, $Nu = \dfrac{hL_c}{k} \Rightarrow h = \dfrac{Nu\,K}{L_c}$

Case-I: Side of length 'x' m is kept vertical

∴ Characteristics length = 'x' m

$$\text{Prandtl number for fluid} = \frac{\mu C_P}{k} = \frac{22.65 \times 10^{-6} \times 1009}{0.0331} = 0.69$$

$$\text{Grashoff number} = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2} = \frac{9.8 \times 388^{-1} \times 170}{(22.65 \times 10^{-6} / 0.91)^2} \cdot x^3 \qquad (\text{since } \nu = \mu/\rho)$$

$$= 6937972516 \cdot x^3$$

∴ Nusselt number, $Nu = 0.59 \times (Gr \cdot Pr)^{0.25} = 155.193 \times (x^3)^{0.25}$

$Nu = 155.193 \times x^{0.75}$

$$\therefore \text{ Heat transfer coefficient, } h = \frac{Nu\,k}{L_c} = \frac{155.193 \times x^{0.75} \times 0.0331}{x} = 5.14\, x^{-0.25}$$

∴ Heat transfer coefficient when side x is vertical = $h_1 = 5.14\, x^{-0.25}$

heat transfer coefficient when side y is vertical = $h_2 = 5.14 \cdot (y)^{-0.25}$

Heat transfer in case of 'y' being vertical is greater than case I by 14%.

$$Q_2 = 1.14\, Q_1$$

$$\Rightarrow \quad h_2 = 1.14\, h_1 \quad (\text{since } Q \propto h)$$

$$\Rightarrow \quad 5.14 \cdot y^{-0.25} = 1.14 \cdot 5.14 \cdot x^{-0.25}$$

$$\Rightarrow \quad \left(\frac{x}{y}\right)^{0.25} = 1.14 \quad \Rightarrow \quad \frac{x}{y} = 1.689$$

$$\Rightarrow \quad \frac{x}{15/x} = 1.689$$

$$\Rightarrow \quad x = 5.03 \text{ cm}, \quad y = 2.98 \text{ cm}$$

∴ Heat transfer in Case-I, (bigger side is vertical)

$$Q_1 = h_1 A_s (T_s - T_\infty) = 5.14 \cdot (0.0503)^{-0.25} \times (15 \times 10^{-4}) \times 170$$
$$= 2.768 \text{ Watt}$$

Heat transfer in Case-II (smaller side is vertical)

$$Q_2 = 1.14\, Q_1 = 3.16 \text{ Watt}$$

Q.27 For a circular tube, explain with the help of neat sketches.

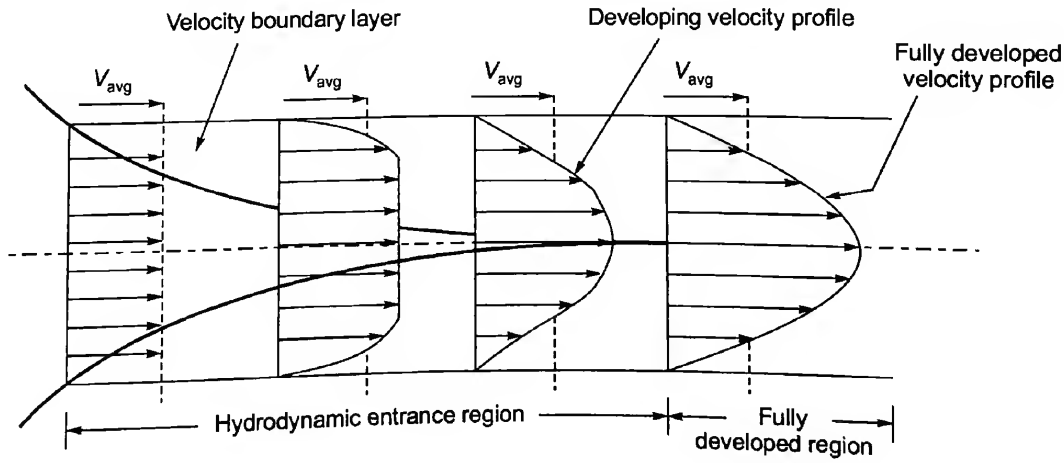
(i) Hydrodynamic entry region and hydrodynamically developed flows.

(ii) Thermal entry region and thermally developed flows.

[CSE (Mains) 2009 : 15 Marks]

Solution:

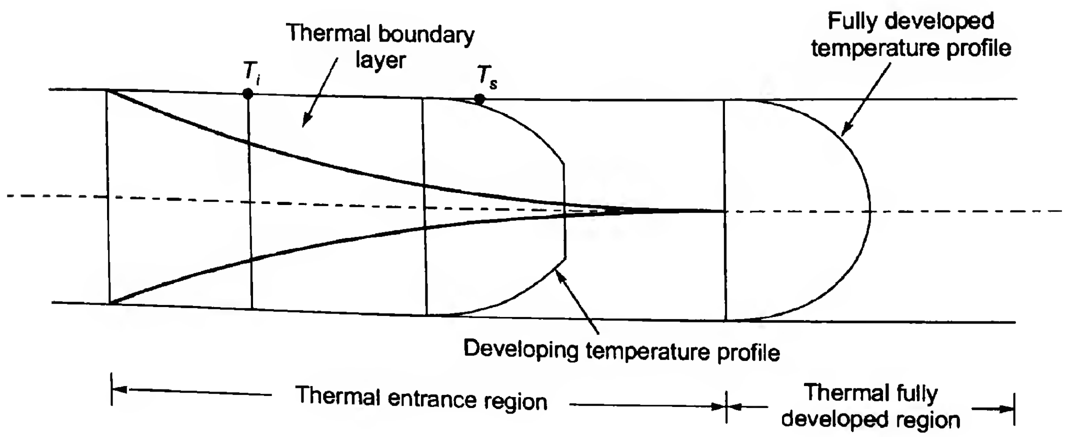
(i)



Hydrodynamic entry region as shown above is the region from pipe inlet to the point at which velocity profile is fully developed.

The region beyond entrance region where velocity profile is fully developed and remains unchanged is called hydrodynamically developed flow.

(ii)



Thermal entry region is defined as the region of flow from entry to the point where thermal boundary layer develops and reaches centre of tube. Length of this region is called thermal entry length.

The region beyond thermal entry region where thermal profile and dimensional temperature profile expressed as $(T_s - T)\backslash(T_s - T_m)$ remains unchanged is called thermally fully developed region.

Q.28 A decorative plastic film on a copper sphere having 10 mm diameter is cured in an oven at 75°C. Upon removal from the oven, the copper sphere is subjected to an air stream at a pressure, temperature and velocity of 1 bar, 23°C and 10 m/s respectively. How long it will take for the sphere to cool down to 35°C? State the assumptions made and justify the method of analysis used.

For, copper the density, specific heat and thermal conductivity are, respectively 8933 kg/m³, 388 J/kg-K and 350 W/m-K. The following correlation for forced convection may be used:

$Nu_d = 2 + [0.4\, Re_d^{1/2} + 0.06\, Re_d^{2/3}]\, Pr^{0.4}$

For air, kinematic viscosity, thermal conductivity and Prandtl number at the mean film temperature under consideration are 15.53×10^{-6} m²/s, 0.025 W/m-K and 0.708 respectively.

[CSE (Mains) 2010 : 20 Marks]

Solution:

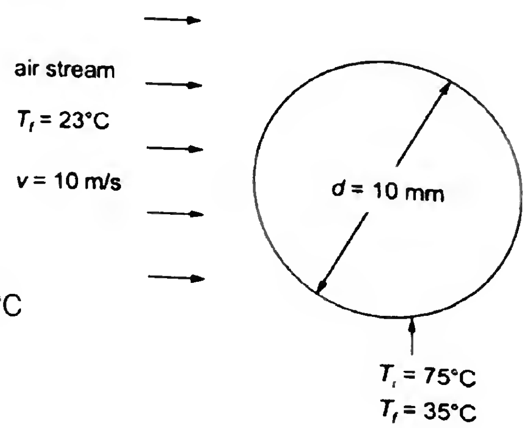
Consider the sphere as shown in figure being cooled in air steam at $T_\infty = 23°C$.

Assumptions:

(a) Properties of air remain constant and uniform throughout the process.

(b) Radiation effects are neglected.

(c) Thermal conductivity of copper and dimensions of sphere remain constant.



Characteristic length for sphere, $D = 0.01$ m

$$\text{Mean film temperature, } T_f = \frac{T_s + T_\infty}{2} = \frac{75 + 23}{2} = 49°C$$

$$\text{Reynolds number for flow} = \frac{vD}{\nu}$$

$$\Rightarrow \quad Re_D = \frac{10 \times 0.01}{15.53 \times 10^{-6}} = 6439.15$$

Prandtl Number, Pr = 0.708

We have, $Nu_d = 2 + [0.4\, Re_d^{1/2} + 0.06\, Re_d^{2/3}]\, Pr^{0.4} = 48.0445$

We know, $$Nu_d = \frac{hD}{k}$$

$$\Rightarrow \quad h = \frac{48.0445 \times 0.025}{0.01} = 120.11 \text{ W/m}^2\text{ K}$$

Biot number for transient heat transfer from sphere,

$$\Rightarrow \quad Bi = \frac{hD}{k} = \frac{120.11 \times 0.01}{350} = 3.43 \times 10^{-3}$$

Since Bi < 0.1, entire sphere can be considered as a lumped body for heat transfer.

Assume sphere is at a temperature T and for a small change in temperature (dT) of the sphere, we have with in dT, $Q = hA(T - T_\infty)\,dt$

This will be equal to reduction internal energy of sphere,

$$Q = -mC_P\,dT$$

$$\therefore \quad dt\,[hA(T - T_\infty) = -(\rho V)\,C_P\,dT$$

$$\Rightarrow \quad \frac{d(T - T_\infty)}{(T - T_\infty)} = -\frac{hA}{\rho V C_P} \cdot dt \qquad (dT = d(T - T_\infty) \text{ since } T_\infty \text{ is constant}$$

Integrating both sides within proper limits,

$$\int_{T_i}^{T_f} \frac{d(T - T_\infty)}{T - T_\infty} = -\frac{hA}{\rho V C_P} \int_0^t dt$$

$$\Rightarrow \quad \ln\left[\frac{T_f - T_\infty}{T_i - T_\infty}\right] = -\frac{hA}{\rho V C_P} \cdot (t)$$

$$\text{Time constant, } \tau = \frac{hA}{\rho V C_P} = \frac{120.11 \times 4\pi(0.005)^2}{8933 \times \frac{4}{3}\pi(0.005)^3 \times 388} = 0.02088^{-1}$$

∴ Time required for temperature at sphere to drop to 35° from 75°C,

$$t = -\ln\left[\frac{35 - 23}{75 - 23}\right] \times \frac{1}{0.0208} = 70.5 \text{ seconds}$$

Q.29 A steel strip emerges from hot roll section of a steel mill at a constant speed of 0.1 m/s and a temperature of 500°C. The length, width and thickness of the steel strip are 10 m, 1 m and 0.003 m respectively. Air at a mean velocity and free stream temperature of 20 m/s and 20°C respectively flows transversely over the strip. Density, specific heat and emissivity of steel strip, respectively, are 7850 kg/m³, 620 J/kg and 0.7. The thermal conductivity, kinematic viscosity and Prandtl number for air at 20°C are respectively 0.044 W/m-K, 4.5×10^{-5} m²/s and 0.68. Neglect the variation in strip temperature across its width and thickness.

(i) Write the governing equation for the temperature distribution along the length of the strip.

(ii) Neglecting radiation from the steel strip, derive an expression for the temperature of steel strip.

(iii) Neglecting radiation from the steel strip, calculate the steel strip temperature at the trailing edge.

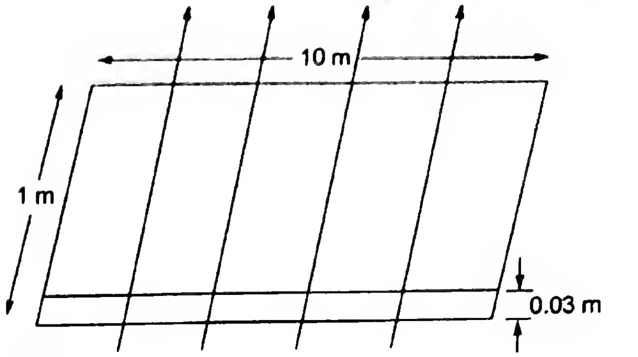
The following correlation for convection heat transfer may be used:

$\overline{Nu} = 0.037\ Re_{x-l}^{0.8}\ Pr^{1/3}$

Symbols have the usual meaning.

[CSE (Mains) 2012 : 20 Marks]

Solution:



$$Re = \frac{V_\infty W}{\nu}$$

$$= \frac{20 \times 1}{4.5 \times 10^{-5}} = 444444.444$$

(transverse flow)

$$N_{ux} = 0.037\,(444444.44)^{0.8}\,(0.68)^{1/3}$$

$$= 1073.063$$

$$\frac{hw}{R} = \bar{h} = \frac{1073.063 \times 0.044}{1} = 47.214 \text{ w/m}^2\text{K}$$

$$hA(T_s - T_\infty) = -\rho v C_p \left.\frac{dT_s}{dt}\right|_{ss}$$

V = volume

Assuming properties of air at 20°C are applicable for given range:

(i) $$\frac{-hA}{\rho V C_p}dt = \frac{dT_s}{T_s - T_\infty}$$

On integrating both sides,

$$\frac{hAt}{\rho V C_p} = \ln\left[\frac{T_s - T_\infty}{T_{500} - T_\infty}\right]$$

Time spent in stream = t, speed of strip = V_{ss}

Say x is the distance from trailing edge.

If time spent = t

Distance of trailing edge = x

$$\Rightarrow \quad x = vt$$

$$t = x/v$$

$$T_f = T_\infty + (T_i - T_\infty)e^{-\frac{hAt}{\rho V C_p}}$$

At $$t = \frac{10}{0.1} = 100 \text{ seconds}$$

(iii) $$T_f = 20° + (500 - 20)e^{-\frac{hAt}{\rho V C_p}}$$

For an element 'dx' $V = (1)(0.003)(dx)$

$$A = (dx)(1)$$

$$T_f = 20 + (480)e^{-\frac{(47.214)(dx) \times 100}{7850 \times 0.003 (dx) 620}}$$

$$T_f = 367.38°\text{C}$$

Q.30 **The average friction coefficient, when an incompressible fluid flows over a stationary flat surface with free stream velocity of 10 m/s, is 0.008. The average temperature of the plate and free stream fluid temperature are respectively at 120°C and 20°C respectively. Fluid properties known : density = 0.88 kg/m³ and specific heat = 1001 J/kg-K, Pr = 0.65.**

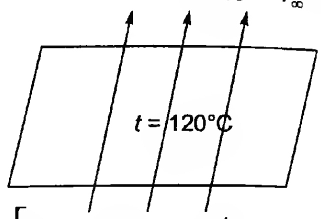
Estimate the average rate of heat transfer per unit area of the plate. Flow over the plate is laminar.

[CSE (Mains) 2012 : 12 Marks]

Solution:

$V_\infty = 10$ m/s, $t = 120°C$, $\rho = 0.88$ kg/m^3, $Pr = 0.65$, $\bar{C}_f = 0.008$, $t_\infty = 20°C$, $C_p = 1001$ J/Kg-K

According to Reynold-Coulomb analogy,



$$St.\ Pr^{2/3} = \frac{\bar{C}_f}{2}$$

$$\Rightarrow \quad St. = \frac{0.008}{2\times(0.65)^{2/3}}$$

$$\Rightarrow \quad \frac{h}{rV_\infty C_p} = \frac{0.008}{2\times(0.65)^{2/3}}$$

$$\Rightarrow \quad h = \frac{0.88\times10\times1001\times0.008}{2\times(0.65)^{2/3}}$$

$$h = 46.957 \text{ W/m}^2\text{/K}$$

$$St = \frac{Nu}{RePr} = \frac{h\cancel{L}}{K}\times\frac{m}{\rho V_\infty \cancel{L}\cdot Pr}$$

$$= \frac{h\times v\times a}{K\times V_\infty\times v} = \frac{h\times K}{C_p K\rho V_\infty}$$

$$St = \frac{h}{\rho V_\infty C_p}$$

$$\frac{Q}{A} = \bar{h}(T_s - T_\infty) = 46.957\times(120-20) = 4695.7 \text{ W/m}^2$$

Q.31 A hot plate of 100 cm height and 25 cm wide is exposed to atmospheric air at 25°C. The surface temperature of the plate is 95°C. Find the heat loss from both the surfaces of the plate. Also find the change in heat loss if the height of the plate is reduced to 50 cm and the width is increased to 40 cm.

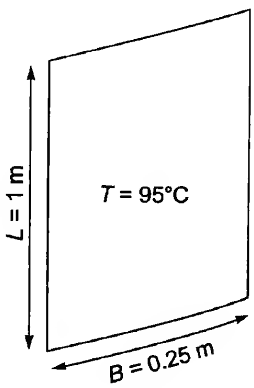
Use the following relations: Nu = 0.59 (Gr.Pr)$^{0.25}$ if Gr.Pr < 10^9, Nu= 0.10 (Gr.Pr)$^{0.33}$, if Gr.Pr > 10^9

The properties of air are: $\rho = 1.06$ kg/m^3, $C_P = 1004$ J/kg-K, $k = 0.029$ W/m-K, $\nu = 18.97\times10^{-6}$ m^2/sec

[CSE (Mains) 2013 : 15 Marks]

Solution:

Case I:



As we know that

$$Gr = \frac{g\beta\Delta TL^3}{\nu^2}, \quad \beta = \frac{2}{T_0+T_\infty} = \frac{1}{333}$$

$$GrPr = \frac{g\beta\Delta TL^3}{\nu^2}\times\frac{\nu}{\alpha} \qquad [\because \alpha = \frac{K}{\rho C_p}]$$

$$GrPr = \frac{\beta\Delta TL^3 g\rho C_p}{\nu K} = \frac{70\times1^3\times9.81\times1.06\times1004}{333\times18.97\times10^{-6}\times0.029}$$

$$= 3.9893\times10^9 > 10^9$$

∴ Flow is turbulent,

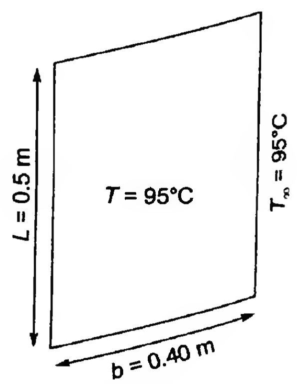
$$Nu = 0.1\times(GrPr)^{0.33} = 147.33$$

$$h = \frac{NuK}{L} = \frac{147.33\times0.029}{1} = 4.27 \text{ W/m}^2\text{K}$$

$$Q = U\cdot A\cdot\Delta T = 4.27\times(2\times1\times0.25)\times70$$

$$= 149.54 \text{ Watt}$$

Case II:



$L = 0.5$ m, $B = 0.4$ m,

$$GrPr = \frac{70\times0.5^3\times9.81\times1.06\times100^4}{333\times18.97\times10^{-6}\times0.029}$$

$$= 0.498\times10^9 < 10^9$$

Hence the flow is laminar

∴

$$Nu = 0.59\,(GrPr)^{0.25} = 88.166$$

$$h = \frac{NuK}{L} = 5.1136 \text{ W/m}^2\text{K}$$

$$Q = UA\Delta T = 5.1136\times(2\times0.5\times0.4)\times70 = 143.182 \text{ Watt}$$

Q.32 Do you think that velocity boundary layer and thermal boundary layer depend on Prandtl number? If yes, explain properly. Also explain, with the help of neat sketches, the significance of relative thickness of velocity boundary layer and thermal boundary layer for the following:

(i) Liquids (ii) Oils

[CSE (Mains) 2013 : 10 Marks]

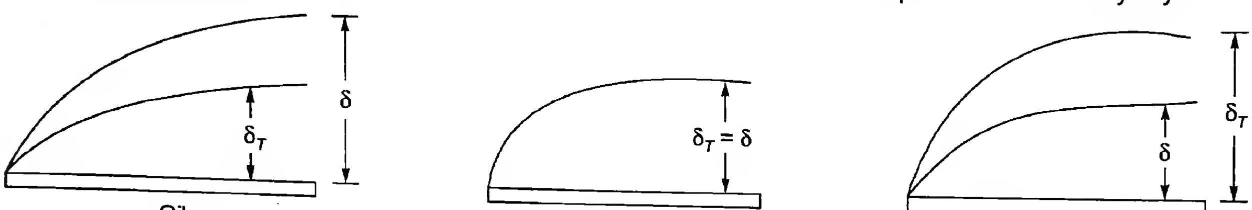
Solution:

Relative thickness of thermal and velocity boundary layers depends on the value of Prandtl number for the fluid.

By definition,

$$\text{Prandtl number} = \frac{\text{Molecular diffusivity of momentum}}{\text{Molecular diffusivity of heat}} = \frac{\nu}{\alpha} = \frac{\mu C_p}{K}$$

Higher the diffusivity of momentum or heat, higher is the thickness of respective boundary layer.



Oils Pr >> 1   Gases Pr ~ 1   Liquid metals Pr << 1

(i) For liquid metals Prandtl number is very low (Pr << 1). Hence due to high conductivity effect of heat transfer is felt through liquid much more and thermal boundary layer is much thicker than velocity boundary layer. This leads to application of liquid metals in heat exchangers and as coolants in nuclear sector.

(ii) For oil Pr >> 1. Hence velocity boundary layer is much thicker as compared to thermal boundary layer. Heat transfer in this case is slower than momentum transfer.

Q.33 A 2-stroke motor cycle petrol engine cylinder consists of 15 fins on its outer surface. If the outside and inside diameters of each fin are 200 mm and 100 mm respectively, the average fin surface temperature is 475°C and the atmospheric air temperature is 25°C, calculate the heat transfer rate from the fins for the following cases:

(i) when the motor cycle is stationary;

(ii) when the motor cycle is running at a speed of 60 kmph.

The fin may be idealized as a single horizontal plate of the same area, and the significant length may be taken as $L = 0.9\ d_0$, where d_0 is the outer diameter of the fin. Assume d_0 as 200 mm.

The properties of air may be taken as follows: $k = 4.266 \times 10^{-2}$ W/m°C; $\nu = 40.61 \times 10^{-6}$ m²/s; Pr = 0.677

For turbulent flow (forced convection) Nu = 0.036 (Re)^{08} (Pr)$^{0.33}$

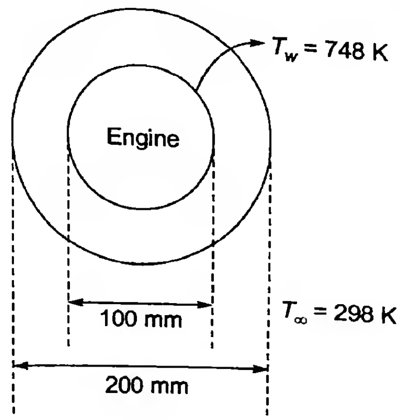
For natural convection: Nu = 0.54 (Gr. Pr)$^{1/4}$ if (Gr. Pr) < 10^9; Nu = 0.10 (Gr. Pr)$^{0.33}$ if (Gr. Pr) > 10^9

[CSE (Mains) 2015 : 20 Marks]

Solution:

Given: $D_o = 200$ mm, $D_i = 100$ mm, $T_\infty = 25°C$

(i) When motorcycle is stationary:



$$T_{base} = 475°C$$

$$Gr = \frac{g\beta\Delta T L_c^3}{\gamma^2}$$

$L_c = 0.9,\ d_o = 180 \text{ mm} = 0.18 \text{ m}.$

$$T_{avg} = \frac{475+25}{2} = 250$$

$$\beta = \frac{1}{T_{avg}} = \frac{1}{523}$$

$$Gr = \frac{(9.81)\left(\frac{1}{523}\right)(450)(0.18)^3}{(40.61\times10^{-6})^2} = 29849108.77$$

$$Pr = 0.677$$

$$Gr\,.\,Pr = 20.207 \times 10^6 < 10^9$$

$$\Rightarrow \quad Nu = 0.54\,(Gr\,.\,Pr)^{0.25} = 36.205$$

$$\frac{hL_c}{k} = 36.205$$

$$h = \frac{36.205\times4.266\times10^{-2}}{(0.18)} = 8.58068 \text{ W/m}^2\text{k}$$

$$q = hA\Delta T$$

$$= (8.58068)\ \frac{\pi}{4}(0.2^2 - 0.1^2)\times2\times450\times15$$

$$= 2.729 \text{ kW}$$

(ii) When motorcycle is running at 60 km/hr.

$$V = 60\times\frac{5}{18} = 16.66 \text{ m/s}$$

$$Nu = (0.036)(Re)0.8\,(Pr)^{0.333}$$

$$Pr = 0.677$$

$$Re = \frac{vL}{\gamma} - \frac{(16.66)(0.18)}{40.61\times10^{-6}}$$

$$Nu = (0.036)\,(73843.88)^{0.8}\,(0.677)^{0.33}$$

$$Nu = 248.34$$

$$h = \frac{248.34\times4.266\times10^{-2}}{0.18} = 58.85$$

$$Q = (58.85)2\times\frac{\pi}{4}(0.2^2 - 0.1^2)\times450\times15 = 18.721$$

Q.34 Air flows over a surface, 2 m in length, oriented in the direction of flow and of sufficient breadth, maintained at 150°C. The pressure is 1 atm and the bulk air temperature is 30°C. If the air velocity is 12 m/s, find the average heat transfer coefficient.

Use the following physical properties of air at film temperature, that is $\frac{(150+30)}{2} = 90°C$:

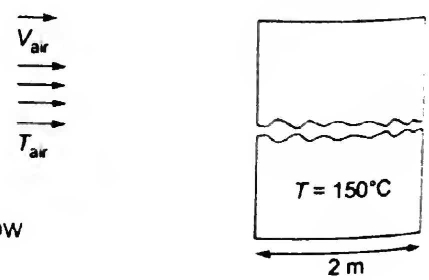
$\rho = 0.962$ kg/m^3, $\mu = 2.131 \times 10^{-5}$ kg/ms $k = 0.031$ W/mK, $C_p = 1.01$ kJ/kg K

$Nu = 0.332\, Re^{1/2} Pr^{1/3}$ for laminar flow $Nu = 0.0296\, Re^{0.8} Pr^{1/3}$ for turbulent flow

[CSE (Mains) 2016 : 20 Marks]

Solution:

Given the properties of air at $\frac{150+30}{2} = 90°C$ are as follows



$\Rightarrow \vec{V}_{air} = 12$ m/s, $\rho = 0.962$ kg/m^3, $m = 2.131 \times 10^{-5}$ kg/ms;

$R = 0.031$ W/mK, $C_p = 1.01$ kJ/KgK

$$Nu = 0.332\ Re^{1/2}\,Pr^{1/3} \text{ for laminar flow}$$

$$Nu = 0.0296\, Re^{0.8}\, Pr^{1/3} \text{ for turbulent flow]}$$

$$\text{Reynolds Number, } Re = \frac{\rho UL}{\mu} = \frac{0.962 \times 12 \times 2}{2.131 \times 10^{-5}} = 10.83435 \times 10^{5}$$

Since the value of Re > 5 × 10^5, Hence it is the case of turbulent flow.

Also,
$$\text{Prandtl Number} = \frac{\mu C_p}{K} = \frac{2.131 \times 10^{-5} \times 1.01 \times 10^{3}}{0.031} = 0.6943$$

Hence,
$$Nu = 0.0296 \times (10.83 \times 10^5)^{0.8} \times 0.6943^{1/3} = 1763.28$$

$$Nu = \frac{hL}{K} \Rightarrow h = \frac{Nu \times K}{L} = \frac{1763.28 \times 0.031}{2} = 27.33 \text{ W/m}^2\text{K}$$

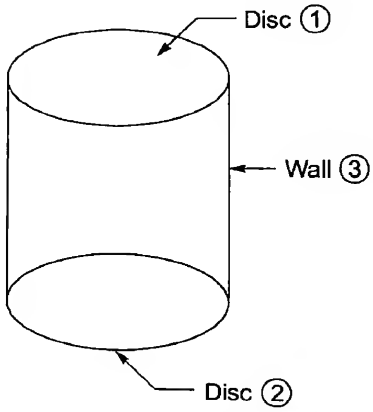
$$h = 27.33 \text{ W/m}^2\text{K}$$

4. Radiation

Q.35 The surface of two circular discs parallel to each other, each with 1 m² area, are exchanging thermal radiation with each other and with the surrounding walls. The temperatures of disc 1, disc 2 and the walls are 800 K, 500 K and 300 K respectively. The shape factor between discs is 0.5. Compute with help of network analysis the net radiative heat transfer between the two discs.

[CSE (Mains) 2001 : 30 Marks]

Solution:



For discs (1) and (2), we have

$$A_1 = A_2 = 1 \text{ m}^2$$
$$T_1 = 800 \text{ K}, T_2 = 500 \text{ K}, F_{12} = 0.5$$

For wall,

$$T_3 = 300 \text{ K}$$

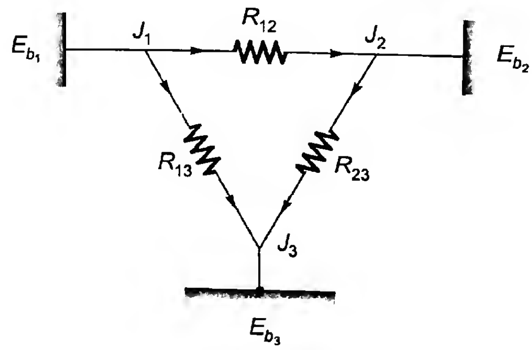
For disc (1),

$$F_{11} + F_{12} + F_{13} = 1 \qquad (\because F_{11} = 0)$$
$$F_{13} = 1 - F_{12} = 1 - 0.5 = 0.5$$

By symmetry, $F_{23} = 0.5$

Both the discs and the walls are assumed to be blackbodies.

Thermal network for this case can be drawn as:



Calculating thermal resistances,

$$R_{12} = \frac{1}{A_1F_{12}} = \frac{1}{1 \cdot 0.5} = 2 \text{ m}^{-2}$$

$$R_{13} = \frac{1}{A_1F_{13}} = \frac{1}{1 \cdot 0.5} = 2 \text{ m}^{-2}$$

$$R_{23} = \frac{1}{A_2F_{23}} = 2 \text{ m}^{-2}$$

Since the two bodies are black bodies,

$$J_1 = E_{b_1} = \sigma T_1^4$$

$$J_2 = E_{b_2} = \sigma T_2^4$$

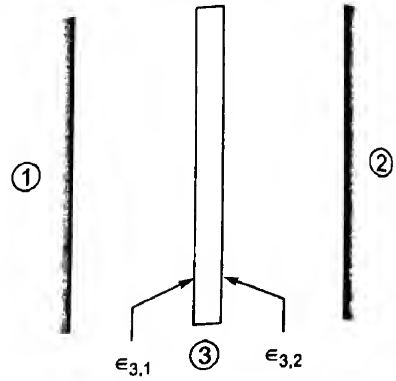
∴ Heat exchange between the two discs

$$q_{12} = \frac{J_1 - J_2}{R_{12}} = \frac{\sigma(T_1^4 - T_2^4)}{R_{12}} = \frac{5.67 \times 10^{-8} \times (800^4 - 500^4)}{2} = 9.84 \text{ kW}$$

Q.36 Two large parallel plates at $T_1 = 1000$ K and $T_2 = 750$ K has emissivities $\varepsilon_1 = 0.5$ and $\varepsilon_2 = 0.8$ respectively. A radiation shield having an emissivity of $\varepsilon_{3,1} = 0.12$ on one side and an emissivity of $\varepsilon_{3,2} = 0.08$ on the other side is placed between the plates. Calculate the heat transfer rate by radiation per square metre with and without radiation shield. Take $\sigma = 5.67 \times 10^{-8}$ W/m^2 K^4.

[CSE (Mains) 2005 : 30 Marks]

Solution:

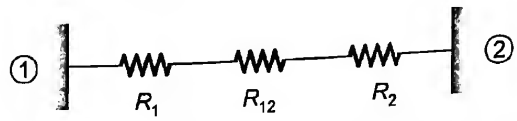


Given: For plates (1) and (2), we have $T_1 = 1000$ K; $T_2 = 750$ K

$\epsilon_1 = 0.5$ $\quad$ $\epsilon_2 = 0.8$

For radiation shield (3), we have $E_{3,1} = 0.12$, $\quad E_{3,2} = 0.08$

Thermal radiation network without heat shield,

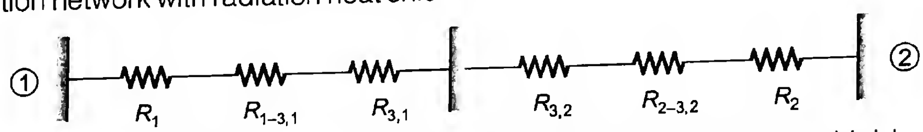


Heat transfer rate between the two plates without radiation shield per square meter.

$$\frac{q_{12}}{A} = \frac{E_{b_1} - E_{b_2}}{A(R_1 + R_{12} + R_2)} = \frac{\sigma(T_1^4 - T_2^4)}{\left[\left(\frac{1-\epsilon_1}{\epsilon_1 A}\right) + \frac{1}{AF_{12}} + \left(\frac{1-\epsilon_2}{\epsilon_2 A}\right)\right]} \quad [F_{12} = 1 \text{ for large parallel plates}]$$

$$= \frac{5.67 \times 10^{-8} \times (1000^4 - 750^4)}{\frac{1-0.5}{0.5} + \frac{1}{1} + \frac{1-0.8}{0.8}} = 17.23 \text{ kW}$$

Thermal radiation network with radiation heat shield:



Rate of heat exchange between the plates per square meter with radiation heat shield,

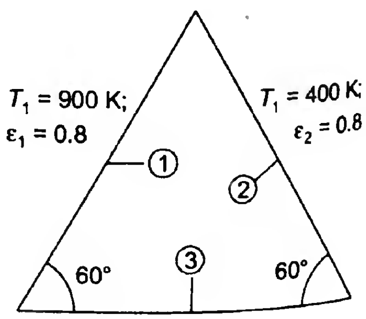
$$\frac{q'_{12}}{A} = \frac{E_{b_1} - E_{b_2}}{A \times \left[(R_1 + R_{1-3,1} + R_{3,1}) + (R_{3,2} + R_{2-3,2} + R_2)\right]}$$

$$= \frac{\sigma(T_1^4 - T_2^4)}{\left[\left(\frac{1-\epsilon_1}{\epsilon_1 A} + \frac{1}{AF_{1-3,1}} + \frac{1-\epsilon_{3,1}}{\epsilon_{3,1}}\right) + \left(\frac{1-\epsilon_{3,2}}{\epsilon_{3,2} A} + \frac{1}{AF_{2-3,2}} + \frac{1-\epsilon_2}{\epsilon_2}\right)\right]}$$

$$= \frac{5.67 \times 10^{-8} \times (1000^4 - 750^4)}{\left(\frac{1}{0.5} + \frac{1}{0.12} - 1\right) + \left(\frac{1}{0.8} + \frac{1}{0.08} - 1\right)} \quad \text{(Since large and parallel plates } F_{1-3,1} = F_{2-3,2} = 1\text{)}$$

$= 1.755$ kW

Q.37 The configuration of a furnace can be approximated as an equilateral triangular duct which is sufficiently long that the end effects are negligible. The hot wall is maintained at $T_1 = 900$ K and has an emissivity $\varepsilon_1 = 0.8$. The cold wall is at $T_2 = 400$ K and has an emissivity $\varepsilon_2 = 0.8$. The third wall is reradiating zone for which $Q_3 = 0$. The accompanying sketch illustrates the configuration. Calculate the net radiation heat flux leaving the hot wall.



[CSE (Mains) 2007 : 20 Marks]

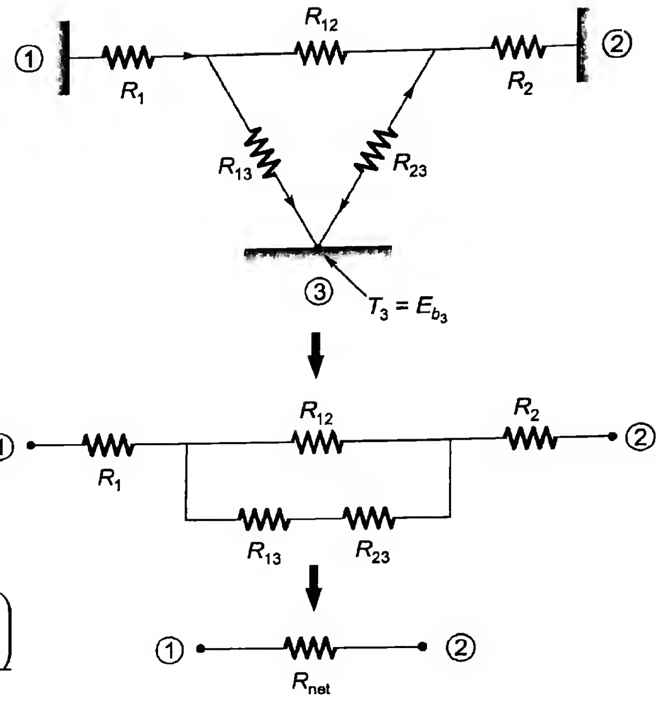
Solution:

Given: For the walls (1) and (2), we have

$$T_1 = 900\text{ K},\quad \epsilon_1 = 0.8,$$
$$T_2 = 400\text{ K},\quad \epsilon_2 = 0.8,$$

For reradiating surface, $Q_3 = 0$

Thermal network in this case can be drawn as below:



Assume length of each side of the equilateral triangular furnace are 'l' m.

We know,
$$F_{12} = \frac{l_1 + l_2 - l_3}{2l_1} = \frac{l + l - l}{2l} = 0.5$$

For triangular ducts

By symmetry $\quad F_{13} = F_{23} = 0.5$

Calculating overall thermal resistance,

$$R_{net} = R_1 + R_2 + \frac{1}{\dfrac{1}{R_{12}} + \dfrac{1}{R_{13} + R_{23}}}$$

$$= R_1 + R_2 + \frac{R_{12}\cdot(R_{13} + R_{23})}{R_{12} + R_{13} + R_{23}}$$

$$= \frac{1-\epsilon_1}{\epsilon_1 A} + \frac{1-\epsilon_2}{\epsilon_2 A} + \frac{\dfrac{1}{AF_{12}}\cdot\left(\dfrac{1}{AF_{13}} + \dfrac{1}{AF_{23}}\right)}{\dfrac{1}{AF_{12}} + \dfrac{1}{AF_{13}} + \dfrac{1}{AF_{23}}}$$

$$= \frac{1}{A}\left[\frac{1-0.8}{0.8} + \frac{1-0.8}{0.8} + \frac{\dfrac{1}{0.5}\left(\dfrac{1}{0.5} + \dfrac{1}{0.5}\right)}{\dfrac{1}{0.5} + \dfrac{1}{0.5} + \dfrac{1}{0.5}}\right]$$

$$\Rightarrow \quad R_{net} = \frac{1}{A}\cdot\frac{11}{6}\,\text{m}^{-2}$$

$\therefore$ Net radiation heat flux from hot wall,

$$\frac{q_{12}}{A} = \frac{E_{b_1} - E_{b_2}}{A\cdot R_{net}} = \frac{5.67\times10^{-8}\times(900^4 - 400^4)}{A\cdot\dfrac{11}{6A}} = 19.499\text{ kW} \simeq 19.5\text{ kW/m}^2$$

Q.38 Consider a diffuse circular disc of diameter D and area A_j and a plane diffuse surface of area $A_i << A_j$. The surfaces are parallel and A_i is located at a distance L from the centre of A_j. Obtain an expression for view factor.

[CSE (Mains) 2007 : 10 Marks]

Solution:

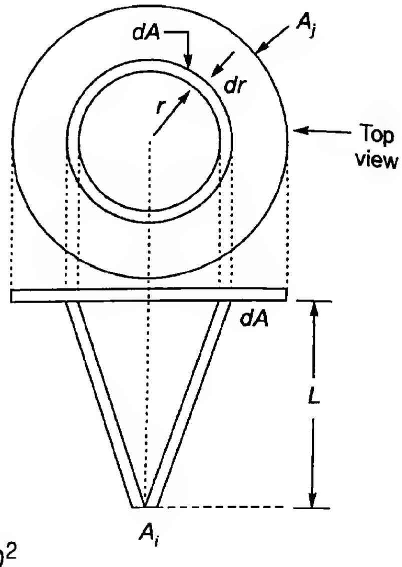
Consider the two diffuse circular disc A_j of diameter D and smaller diffuse disc A_i at a distance L from the centre of A_j.

Assume intensity of radiation from A_i in a direction perpendicular to it is I_n.
Consider an element on A_j of area dA.

Portion of heat emitted from A_i and incident on $dA = d_{q_{i-j}}$

Solid angle subtended by area dA on A_i = $d\psi = \dfrac{dA}{L^2}$

Since the body is diffuse, intensity of radiation emitted in all directions is same.



Hence, we have,
$$d_{q_{i-j}} = I_n \cdot A_i \cdot d\psi = I_n \cdot A_i \cdot \frac{dA}{L^2}$$

Also, we know,
$$dA = 2\pi r dr$$
$$d_{q_{i-j}} = I_n A_i \cdot \frac{2\pi r dr}{L^2}$$

Integrating both sides, we get,
$$\int d_{q_{i-j}} = q_{i-j} = \frac{I_n A_i}{L^2}\int_0^{D/2} 2\pi r\,dr = \frac{I_n A_i}{L^2}\cdot\pi\cdot\frac{D^2}{4}$$

We know, that for a diffuse body

Total radiation emitted by the body, $q_i = \pi I_n A_i$

∴ Fraction of radiation emitted by A_i falling on body A_j

$$\text{Shape/view factor} = F_{i-j} = \frac{q_{i-j}}{q_i} = \frac{I_n A_i}{L^2}\cdot\pi\frac{D^2}{4}\cdot\frac{1}{\pi I_n A_i}$$

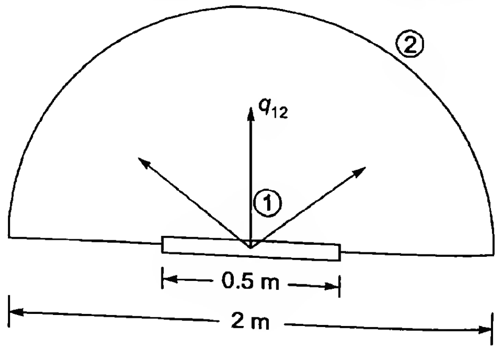
$$\Rightarrow \qquad F_{i-j} = \frac{D^2}{4L^2}$$

This is the required expression for view factor.

Q.39 **A 0.5 m diameter disc heater is horizontally placed and enclosed concentrically in a hemispherical shaped surface. The surface of the enclosure having an emissivity of 0.7 is maintained at 500 K. The disc heater, having emissivity of 0.8 is maintained at 1200 K. The diameter of the hemisphere is 2 m and the remaining base area enclosed is open to surroundings at 300 K and may be considered as black with reference to radiation network. Using thermal network method, calculate the heat exchange between heater and surroundings. Neglect convection heat transfer. Assume heater and hemispherical surface are opaque, diffuse and grey.** **[CSE (Mains) 2010 : 20 Marks]**

Solution:

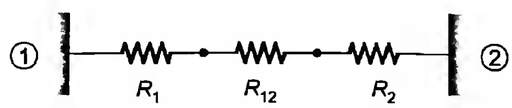
Consider the heater (1) and endosure (2) as shown.



For heater (1):
$$T_1 = 1200\text{ K}$$
$$\epsilon_1 = 0.8,\ d_1 = 0.5\text{ m}$$

For endosure (2):
$$T_2 = 500\text{ K}$$
$$\epsilon_2 = 0.7\text{ K}$$

Thermal network for this case between (1) and (2):



Heat transfer between heater and enclosure will be,
$$q_{12} = \frac{E_{b_1} - E_{b_2}}{R_{net}}$$

For heater

$$F_{11} + F_{12} = 1$$

$\Rightarrow$ $$F_{12} = 1 \qquad \text{(Since (1) is a plain surface) } (F_{11} = 0)$$

$\therefore$
$$q_{12} = \frac{E_{b_1} - E_{b_2}}{R_{net}}$$

$$= \frac{\sigma(T_1^4 - T_2^4)}{\frac{1-\epsilon_1}{\epsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{1-\epsilon_2}{\epsilon_2 A_2}}$$

$$= \frac{5.67\times10^{-8}\times(1200^4 - 500^4)}{\frac{1-0.8}{0.8\cdot\frac{\pi\cdot0.5^2}{4}} + \frac{1}{\frac{\pi\cdot0.5^2}{4}\cdot1} + \frac{1-0.7}{0.72\pi\cdot1^2}} = 17.72 \text{ kW}$$

The heater exchanges radiation with the surroundings from its other surface.
Assuming surroundings to be black w.r.t heat exchange, heat transfer between surroundings and heater.

$$q_{13} = \epsilon_1\cdot A_1\cdot\sigma(T_1^4 - T_3^4)$$

$$= 0.8\cdot\frac{\pi\cdot0.5^2}{4}\cdot5.67\times10^{-8}\times(1200^4 - 300^4)$$

$$= 18.396 \text{ kW} \simeq 18.4 \text{ kW}$$

Q.40 An electric furnace consisting of two flat surface heaters, top and bottom, is used to heat treat a coating that is applied to both surfaces of a thin metal plate inserted midway between the heaters. The heaters and the plate are 2 m × 2 m on a side, and each heater is separated from the plate by a distance of 0.5 m. Each heater is well insulated on its back side and has an emissivity of 0.9 at its exposed surface. The plate and side walls have emissivities of 0.6 and 0.3 respectively. Under steady operating conditions, both heaters are at 800 K while the side walls are at 400 K. View factor between the heaters and the plate is 0.62.

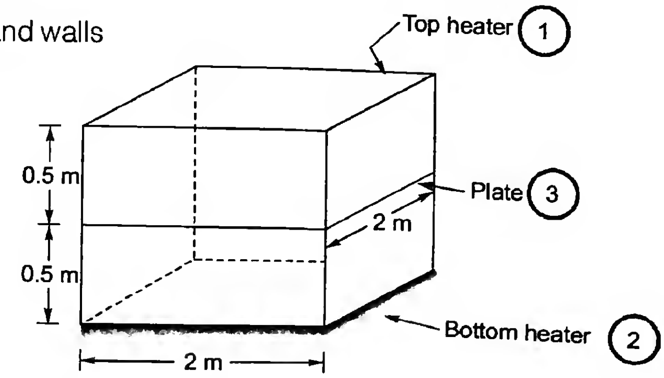
(i) Sketch the system and its equivalent thermal network and label all pertinent resistances and potentials.

(ii) Calculate the associated resistances and driving potentials.

(iii) Calculate the required electrical power. **[CSE (Mains) 2012 : 20 Marks]**

Solution:

Consider the schematic of heaters, plate with coating and walls



We have:

For heaters (1) and (2),

$$A_1 = A_2 = 2\times2 = 4 \text{ m}^2$$

$$\epsilon_1 = \epsilon_2 = 0.9,\ T_1 = T_2 = 800 \text{ K}$$

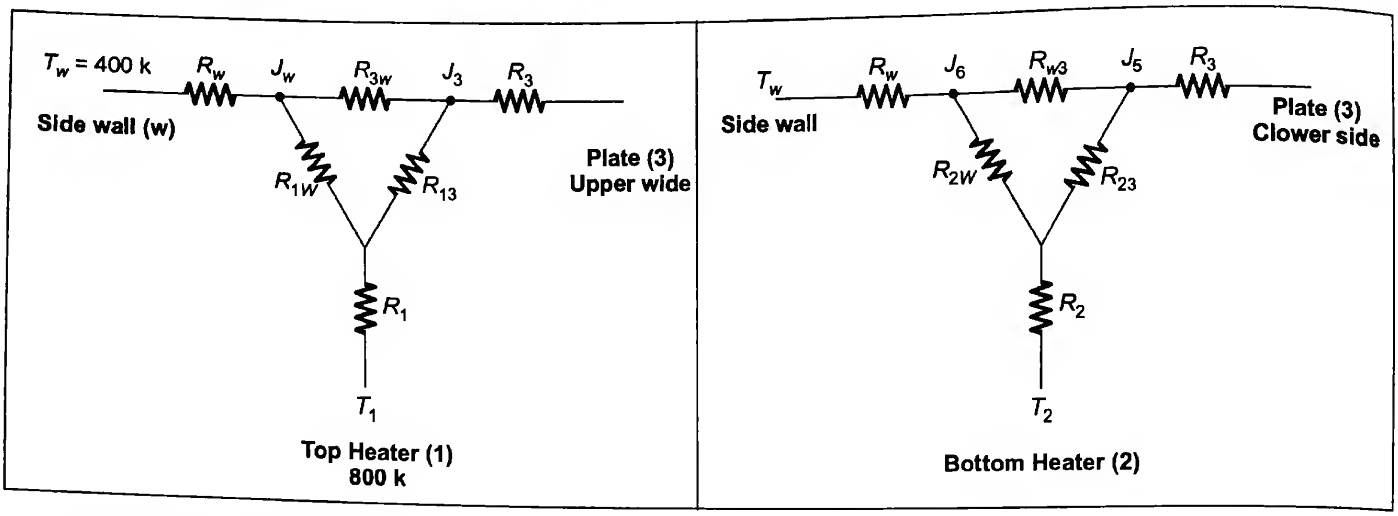
Also, $$F_{13} = F_{23} = 0.62$$

For walls (between heater and plate)

$$A_w = 4\times0.5 \text{ m}\times2 \text{ m} = 4 \text{ m}^2$$

$$\epsilon_w = 0.3,\ T_w = 400 \text{ K},\ A_3 = 4 \text{ m}^2,\ \epsilon_3 = 0.6$$

Thermal network along with resistances and driving potentials are as below:



Value of thermal resistance,

$$R_1 = R_2 = \frac{1-\epsilon_1}{\epsilon_1 A_1} = \frac{1-0.9}{0.9\cdot 4} = 0.028 \text{ m}^{-2}$$

$$R_3 = \frac{1-\epsilon_3}{\epsilon_3 A_3} = \frac{1-0.6}{0.6\cdot 4} = 0.167 \text{ m}^{-2}$$

$$R_w = \frac{1-\epsilon_w}{\epsilon_w A_w} = \frac{1-0.3}{0.3\cdot 4} = 0.5833 \text{ m}^{-2}$$

For heater (1):

$$F_{11} + F_{13} + F_{1w} = 1 \qquad (F_{11} = 0)$$

$\Rightarrow$ $$F_{1w} = 1 - F_{13} = 0.38$$

Similarly, $$F_{2w} = 0.38$$

For plate (3):

$$F_{33} + F_{3w} + F_{31} = 1$$

$\Rightarrow$ $$F_{3w} = 1 - F_{31} = 1 - 0.62 = 0.38$$

$$(\because\ A_1 = A_3,\ A_1 F_{13} = A_3 F_{31} \Rightarrow F_{13} = F_{31} = 0.62)$$

$$R_{13} = R_{23} = \frac{1}{A_1 F_{13}} = \frac{1}{4\times 0.62} = 0.403 \text{ m}^{-2}$$

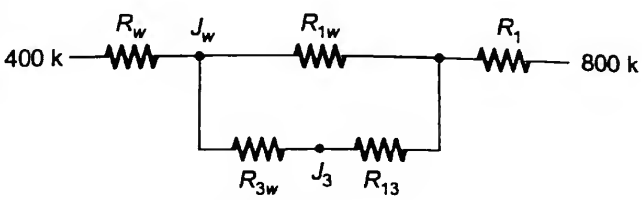
$$R_{1w} = R_{2w} = \frac{1}{A_1 F_{1w}} = \frac{1}{0.38\times 4} = 0.658 \text{ m}^{-2}$$

$$R_{3w} = \frac{1}{A_1 F_{3w}} = \frac{1}{4\times 0.38} = 0.658 \text{ m}^{-2}$$

By symmetry, potential on both sides of the plate remains same.

$\therefore$ During steady state no heat flows into the plate and it behaves like a re-radiating surface.

Thermal network reduces to :



Total thermal resistance between walls and heater,

$$R_{net} = R_1 + R_w + \frac{(R_{13} + R_{3w})R_{1w}}{R_{1w} + (R_{13} + R_{3w})} = 1.0172 \text{ m}^{-2}$$

∴ Heat transfer from heater $= q_{1w} = \dfrac{E_{b1} - E_{bw}}{R_{net}} = \dfrac{5.67 \times 10^{-8}(800^4 - 400^4)}{1.0172} = 21.404 \text{ kW}$

Same amount of heat is emitted by bottom heater as well.

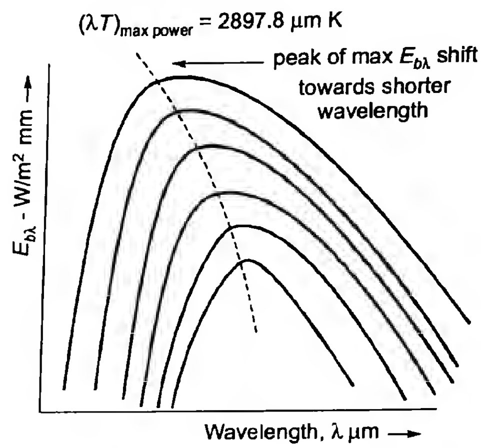
∴ Required electrical power $= 2 \times 36.5 = 42.8 \text{ kW}$

Q.41 Explain Wein's displacement law. Assuming sun to be a blackbody with a constant surface temperature of 8780 K, the wavelength at which it will have the maximum spectral emissive power.

[CSE (Mains) 2013 : 10 Marks]

Solution:

Wein's displacement law: As the temperature of a blackbody increase, peak of spectral emissive power shift towards shorter wavelengths. The wavelength at which the peak occurs for a specified temperature is given by Wein's displacement law as follows:



Assume wavelength at which sun has maximum spectral emissive power is λ_m.

From Wein's displaement law, we know that for blackbody,

$$\lambda_m \cdot T = 2898\ \mu m\ K$$

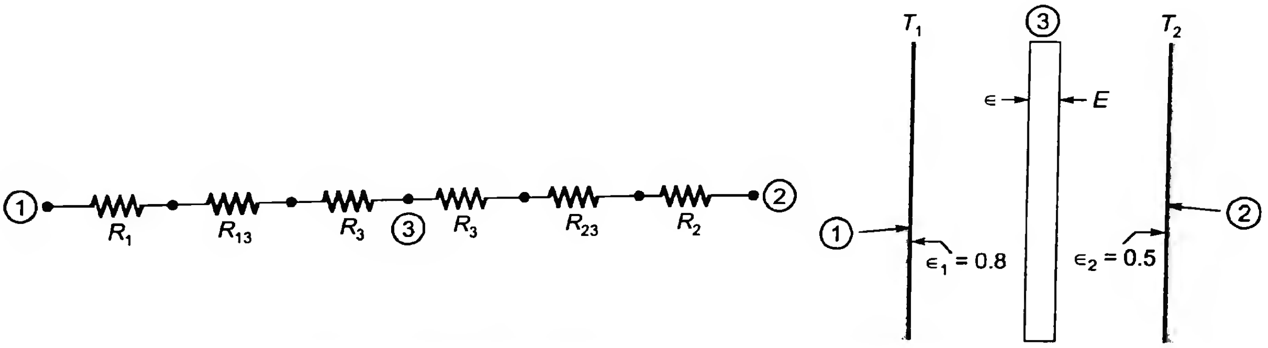
Surface temperature of sun $= 8780$ K

⇒ Wavelength of max. emission $= \lambda_m = 0.33\ \mu m$

Q.42 A thin radiation shield having equal emissivities on both sides is introduced parallel to and in between two large planes with emissivities 0.8 and 0.5 respectively. Determine the emissivity of the radiation shield to reduce the heat transfer rate by 92% of the original.

[CSE (Mains) 2014 : 10 Marks]

Solution:



Thermal radiation network corresponding to this problem

Assume that the two plates are at temperature T_1 and T_2.

Assume emissivity of radiation shield is ϵ.

Consider the figure above for radiation network corresponding to this problem.

Thermal resistance without heat shield placed between the two plates,

$$R = R_1 + R_{12} + R_2$$

$$= \frac{1 - \epsilon_1}{\epsilon_1 A} + \frac{1}{F_{12} A_1} + \frac{1 - \epsilon_2}{\epsilon_2 A_2}$$

Since the two plates are parallel and large,

$$A_1 = A_2 = A \quad \text{and} \quad F_{12} = 1$$

∴ Heat transfer between the plates without the heat shield,

$$Q_{1-2} = \frac{\sigma(T_1^4 - T_2^4)}{R} \Rightarrow \frac{Q_{12}}{A} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1-\epsilon_1}{\epsilon_1} + \frac{1}{F_{12}} + \frac{1-\epsilon_2}{\epsilon_2}}$$

$$= \frac{\sigma(T_1^4 - T_2^4)}{\frac{1-0.8}{0.8} + 1 + \frac{1-0.5}{0.5}} = 0.444 \cdot \sigma (T_1^4 - T_2^4) \text{ Watt/m}^2$$

For the case when radiation shield is introduced between the two plates from the radiation network:

Total thermal resistance, $R_{net} = (R_1 + R_{13} + R_3) + (R_3 + R_{23} + R_2)$

$$= \left(\frac{1-\epsilon_1}{\epsilon_1 A}\right) + \frac{1}{F_{13}A} + \left(\frac{1-\epsilon}{A\epsilon}\right) + \left(\frac{1-\epsilon}{A\epsilon}\right) + \frac{1}{F_{23}A} + \left(\frac{1-\epsilon_2}{\epsilon_2 A}\right)$$

$$= \frac{1}{A}\left[\left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon} - 1\right) + \left(\frac{1}{\epsilon_2} + \frac{1}{\epsilon} - 1\right)\right] \qquad \text{(since } F_{13} = F_{23} = 1\text{)}$$

$$\Rightarrow \quad R_{net} = \frac{1}{A}\left[\frac{1}{0.8} + \frac{1}{\epsilon} - 1 + \frac{1}{0.5} + \frac{1}{\epsilon} - 1\right] = \frac{1}{A}\left[1.25 + \frac{2}{\epsilon}\right]$$

We know that, new heat transfer (with radiation shield) is 92% less than original heat transfer (without radiation shield).

∴ Heat transfer with radiation shield

$$\frac{Q_{1-2}}{A} = \frac{\sigma(T_1^4 - T_2^4)}{A \cdot R_{net}} = (1 - 0.92) \times \frac{Q_{1-2}}{A}$$

$$\Rightarrow \quad \frac{\sigma(T_1^4 - T_2^4)}{1.25 + \frac{2}{\epsilon}} = 0.08 \times 0.444 \times \sigma (T_1^4 - T_2^4)$$

$$\Rightarrow \quad 1.25 + \frac{2}{\epsilon} = 28.125$$

$$\Rightarrow \quad \epsilon = 0.074$$

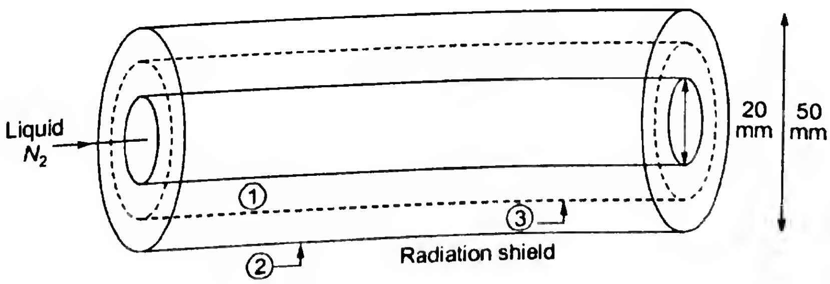
Hence required emissivity of radiation shield on both sides should be 0.074.

Q.43 Liquid N_2 enters a thin-walled 20 mm diameter tube at 77 K and flows steadily. The outer surface of the tube has an emissivity of 0.02. This tube is placed concentrically in another tube of 50 mm inner diameter, whose inner surface emissivity is 0.05. The inner surface of the outer tube is maintained at 300 K and the space in between the tubes is evacuated. Determine the heat gained by the liquid N_2 per unit length of the tube.

If a thin-walled radiation shield with emissivity 0.02 on both sides is inserted midway concentrically between inner and outer tubes, calculate the % change in heat gained in liquid N_2 per unit length of the tube.

[CSE (Mains) 2014 : 20 Marks]

Solution:



For inner tube (1):

$$d_1 = 20\,\text{mm}$$

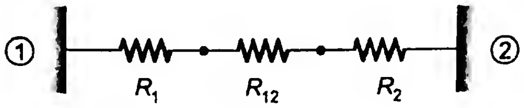
Since the tube is thin, its temperature will be equal to temperature of liquid N_2.

$$\therefore \qquad T_1 = 77\,\text{k}, \quad \epsilon_1 = 0.02$$

For outer tube (2):

$$d_2 = 50\,\text{mm},\ T_2 = 300\,\text{K},\ \epsilon_2 = 0.05$$

Thermal network in this case will be,



$$R_{net} = R_1 + R_{12} + R_2$$

$$= \frac{1-\epsilon_1}{\epsilon_1 A_1} + \frac{1}{A_1F_{12}} + \frac{1-\epsilon_2}{\epsilon_2 A_2} \qquad \text{[Since } F_{12} = 1\text{]}$$

$$= \frac{1}{A_1}\left[\frac{1-0.02}{0.02} + 1 + \left(\frac{1-0.05}{0.05}\right)\cdot\frac{A_1}{A_2}\right]$$

$$= \frac{1}{A_1}\left[\frac{1}{0.02} + \left(\frac{0.95}{0.05}\right)\cdot\frac{\pi\cdot 20\cdot L}{\pi\cdot 50\cdot L}\right] = \frac{1}{A_1}\cdot 57.6\,\text{m}^{-2}$$

Heat transfer to N_2 tube (1) from tube (2) per unit length,

$$= \frac{q_{21}}{L} = \frac{1}{L}\cdot\frac{E_{b_2} - E_{b_1}}{R_{net}} = \frac{\sigma\left(T_2^4 - T_1^4\right)}{L\cdot\dfrac{57.6}{A_1}}$$

$$= \frac{5.67\times 10^{-8}\times(300^4 - 77^4)}{L\cdot\dfrac{57.6}{\pi\cdot 20\times 10^{-3}\times L}} = 0.4988\ \text{watt} \simeq 0.5\ \text{watt}$$

If a radiation shield (3) is introduced midway between two tubes, for steady state condition,

$$q_{23} = q_{31}$$

$$\Rightarrow \quad \frac{E_{b_2} - E_{b_3}}{\dfrac{1-\epsilon_2}{\epsilon_2 A_2} + \dfrac{1}{A_2F_{23}} + \dfrac{1-\epsilon_3}{\epsilon_3 A_3}} = \frac{E_{b_3} - E_{b_1}}{\dfrac{1-\epsilon_3}{\epsilon_3 A_3} + \dfrac{1}{A_3F_{31}} + \dfrac{1-\epsilon_1}{\epsilon_1 A_1}}$$

$$\Rightarrow \quad \frac{\sigma\left(T_2^4 - T_3^4\right)}{\dfrac{1}{A_2}\left[\dfrac{1}{\epsilon_2} + \dfrac{A_2}{A_3}\left(\dfrac{1-\epsilon_3}{\epsilon_3}\right)\right]} = \frac{\sigma\left(T_3^4 - T_1^4\right)}{\dfrac{1}{A_3}\left[\dfrac{1}{\epsilon_3} + \dfrac{A_3}{A_1}\left(\dfrac{1-\epsilon_1}{\epsilon_1}\right)\right]}$$

$$\Rightarrow \quad \frac{300^4 - T_3^4}{\dfrac{1}{\pi\cdot 50\cdot L}\left[\dfrac{1}{0.05} + \dfrac{50}{35}\left(\dfrac{1-0.02}{0.02}\right)\right]} = \frac{T_3^4 - 77^4}{\dfrac{1}{\pi\cdot 35\cdot L}\left[\dfrac{1}{0.02} + \dfrac{35}{20}\left(\dfrac{1-0.02}{0.02}\right)\right]}$$

$$\Rightarrow \quad T_3 = 272.865\,\text{K}$$

$\therefore$ Heat transfer to liquid N_2 in this case per unit length,

$$\frac{q_{31}}{L} = \frac{1}{L}\cdot\frac{E_{b_3} - E_{b_1}}{\dfrac{1}{A_3}\left[\dfrac{1}{\epsilon_3} + \dfrac{A_3}{A_1}\left(\dfrac{1-\epsilon_1}{\epsilon_1}\right)\right]} = \frac{5.67\times 10^{-8}(272.86^4 - 77^4)}{\dfrac{1}{\pi\cdot 0.035}\left[\dfrac{1}{0.02} + \dfrac{35}{20}\left(\dfrac{1-0.02}{0.02}\right)\right]}$$

$$\Rightarrow \quad \frac{q_{31}}{L} = 0.25298 = 0.253\ \text{watt}$$

$\therefore$ % change in heat gained by liquid N_2 per unit length of tube

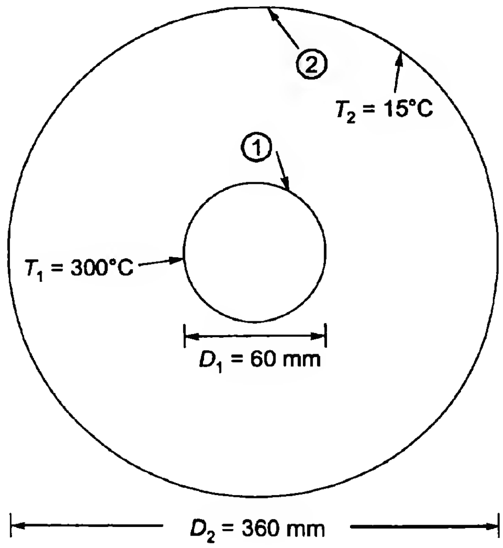
$$= \frac{0.5 - 0.253}{0.5} \times 100 = 49.4\%$$

$\therefore$ Heat transfer reduces by 49.4%.

Q.44 A small sphere (of outside diameter = 60 mm) with a surface temperature of 300°C is located at the geometric centre of a large sphere (of inside diameter = 360 mm) with an inner surface temperature of 15°C. Calculate how much of emission from the inner surface of large sphere is incident upon the outer surface of the small sphere. Assume that both bodies approach black body behaviour. What is the interchange of heat between the two spheres?

[CSE (Mains) 2015 : 10 Marks]

Solution:



Large sphere (body 2) has inner surface

Temperature T_2 = 15°C and body 1 - small sphere has surface

Temperature T_1 = 300°C.

Given: D_1 = 60 mm, D_2 = 360 mm

Since body 1 is a small sphere, view factor of body 1 with respect to itself = $F_{11} = 0$

Also, we know that, $F_{11} + F_{12} = 1$

$\Rightarrow$ $F_{12} = 1$

Fom reciprocity theorem,

$$A_1 F_{12} = A_2 F_{21}$$

$$\Rightarrow \quad F_{21} = \frac{A_1}{A_2} \cdot F_{12} = \frac{4\pi(D_1/2)^2}{4\pi(D_2/2)^2}$$

$$= \frac{60^2}{360^2} = \frac{1}{36} = 0.028$$

Also, we know that $F_{21} + F_{22} = 1 \Rightarrow F_{22} = 0.972$

$\therefore$ Fraction of heat emitted from inside surface of large sphere which falls an smaller sphere = $F_{21} = 0.028$

Since the two bodies approach black body behaviour, emissivity of both is unity.

$\therefore$ Net radiation exchange between the two spheres,

$$Q_{12} = A_1 \cdot F_{12} \cdot \sigma (T_1^4 - T_2^4) \qquad (F_{12} = 1)$$

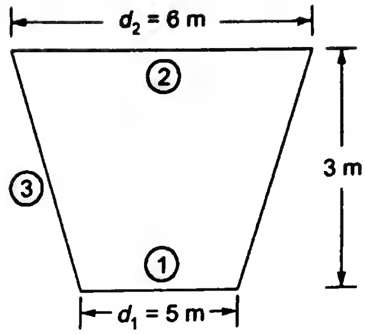
$$= 4\pi \cdot \left(\frac{60 \times 10^{-3}}{2}\right)^2 \text{m}^2 \cdot 5.67 \times 10^{-8} \cdot (573^4 - 288^4)$$

$$Q_{12} = 64.716 \text{ W}$$

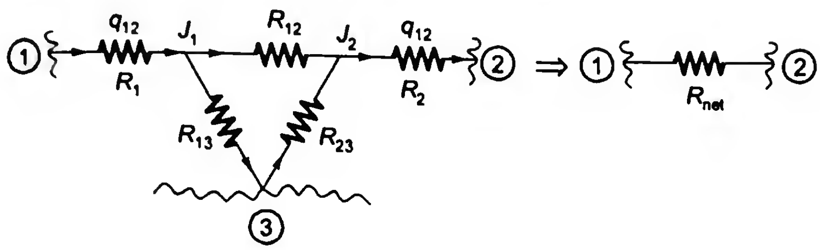
Q.45 A boiler furnace of 3 m height is made in the shape of a frustum of a cone with the bottom diameter 5 m (d_1) and top diameter 6 m (d_2). The emissivity of both the surface is 0.9. The bottom surface is at 1000 K and the top surface is at 500 K. Considering the inclined surface as refractory surface, find (i) the radiation heat transfer from the bottom to top surface and (ii) the inclined surface temperature. Radiation shape factor F_{1-2} = 0.4.

[CSE (Mains) 2015 : 10 Marks]

Solution:



Given: Top surface (2): $d_2 = 6$ m, $T_2 = 500$ K, $\epsilon_2 = 0.9$
Bottom surface (1): $T_1 = 1000$ K, $d_1 = 5$ m, $\epsilon_1 = 0.9$
Thermal network for this case can be drawn as —



From reciprocity theorem,

$$A_1 F_{12} = A_2 F_{21}$$

$$\Rightarrow \quad F_{21} = \frac{A_1}{A_2} F_{12} = \frac{d_1^2}{d_2^2} \cdot F_{12}$$

$$\Rightarrow \quad F_{21} = 0.28$$

For surface (1) $\quad F_{11} + F_{12} + F_{13} = 1$

$\Rightarrow \quad F_{13} = 1 - F_{12} = 1 - 0.4 = 0.6$ ($F_{11} = 0$, since (1) is a plain surface)

For surface (2) $\quad F_{22} + F_{23} + F_{21} = 1$

$\Rightarrow \quad F_{23} = 1 - F_{21} = 0.72$ ($F_{22} = 0$, since (2) is plain surface)

Total thermal resistance of the network,

$$R_{net} = R_1 + R_2 + \frac{1}{\dfrac{1}{R_{12}} + \dfrac{1}{R_{23} + R_{13}}} = R_1 + R_2 + R_t$$

$$R_t = \frac{1}{\dfrac{1}{R_{12}} + \dfrac{1}{R_{23} + R_{13}}} = \frac{1}{\dfrac{1}{\dfrac{1}{A_1F_{12}}} + \dfrac{1}{\dfrac{1}{A_2F_{23}} + \dfrac{1}{A_1F_{13}}}}$$

$$= \frac{1}{A_1 F_{12} + \dfrac{1}{\dfrac{1}{A_2 F_{23}} + \dfrac{1}{A_1F_{13}}}} = \frac{1}{\dfrac{\pi \cdot 5^2}{4} \cdot 0.4 + \dfrac{1}{\dfrac{1}{\dfrac{\pi \cdot 6^2}{4} \cdot 0.72} + \dfrac{1}{\dfrac{\pi 5^2}{4} \cdot 0.6}}}$$

$$= 0.06529 \text{ m}^{-2}$$

$$\therefore \quad R_{net} = R_1 + R_2 + R_t = \frac{1-\epsilon_1}{\epsilon_1 A_1} + \frac{1-\epsilon_2}{\epsilon_2 A_2} + 0.06529$$

$$= \frac{1-0.9}{0.9\dfrac{\pi \cdot 5^2}{4}} + \frac{1-0.9}{0.9\dfrac{\pi \cdot 6^2}{4}} + 0.06529 = 0.07488 \text{ m}^{-2}$$

$\therefore$ Net heat transfer from the bottom to top surface is

$$q_{12} = \frac{E_{b_1} - E_{b_2}}{R_{net}} = \frac{\sigma(T_1^4 - T_2^4)}{R_{net}} = \frac{5.67 \times 10^{-8} \times (1000^4 - 500^4)}{0.07488}$$

$$= 709.899 \text{ kW} \simeq 710 \text{ kW}$$

From thermal network, we can see that

$$q_{12} = \frac{E_{b_1} - J_1}{R_1} = \frac{\sigma T_1^4 - J_1}{\frac{1-\epsilon_1}{\epsilon_1 A_1}}$$

$\Rightarrow$
$$J_1 = \sigma T_1^4 - q_{12} \cdot \frac{1-\epsilon_1}{\epsilon_1 A_1} = 52.683$$

Also,
$$q_{12} = \frac{J_2 - E_{b_2}}{R_2}$$

$\Rightarrow$
$$J_2 = q_{12} \cdot R_2 + E_{b_2} = q_{12}\frac{1-\epsilon_2}{\epsilon_2 A_2} + \sigma T_2^4 = 6.333 \text{ kW}$$

Since surface (3) is reradiating surface, no heat is absorbed,

Hence,
$$\frac{J_1 - J_3}{R_{13}} = \frac{J_3 - J_2}{R_{23}}$$

$\Rightarrow$
$$J_1 R_{23} - J_3 R_{23} = J_3 R_{13} - J_2 R_{13}$$

$\Rightarrow$
$$J_3 = \frac{J_1 R_{23} + J_2 R_{13}}{R_{13} + R_{23}}$$

$$R_{13} = \frac{1}{A_1 F_{13}} = \frac{1}{\frac{\pi \cdot 5^2}{4} \cdot 0.6} = 0.0849 \text{ m}^{-2}$$

$$R_{23} = \frac{1}{A_2 F_{23}} = \frac{1}{\frac{\pi \cdot 6^2}{4} \cdot 0.72} = 0.0491 \text{ m}^{-2}$$

$\therefore$
$$J_3 = 23.316 \text{ kW}$$

If temperature of reradiating surface is T_3

$$J_3 = \sigma T_3^4 \Rightarrow T_3 = \left(\frac{J_3}{\sigma}\right)^{1/4} = 800.79 \text{ K}$$

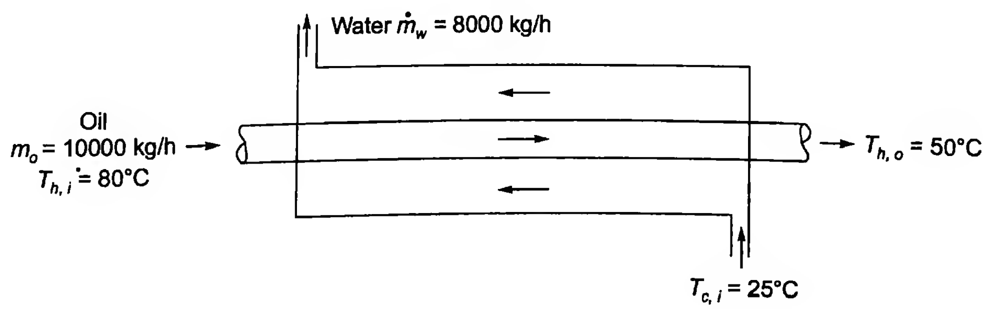
Hence, temperature of reradiating surface is 800.79 K

5. Heat Exchanger

Q.46 In a double pipe counterflow heat exchanger, 10,000 kg/h of an oil, having a specific heat of 2095 J/kg K, is cooled from 80° C to 50°C by 8000 kg/h of water entering at 25°C. Determine the heat exchanger area for an overall heat transfer coefficient of 300 W/m² K. Take C_p of water as 4180 J/kg K.

[CSE (Mains), 2004 : 20 Marks]

Solution:



Given:

Overall heat transfer coefficient, U_s = 300 W/m²K

Specific heat capacity of oil, $C_{p,o}$ = 2095 J/kgK

Specific heat capacity of water, $C_{p,w}$ = 4180 J/kgK

For steady state operation

$$\text{Heat lost by oil} = \text{Heat gained by water}$$

$$\Rightarrow \quad \dot{m}_o C_{p,o}(T_{h,i} - T_{h,o}) = \dot{m}_w C_{p,w}(T_{Go} - T_{c,i})$$

$$\Rightarrow \quad 10{,}000 \times 2095 \times (30) = 8000 \times 4180 \times (T_{Go} - 298)$$

$$\Rightarrow \quad T_{c,o} = 316.795 \simeq 316.8\text{ K} = 43.8°\text{C}$$

Log mean temperature difference,

$$\text{LMTD} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\dfrac{\Delta T_1}{\Delta T_1}\right)} = \frac{(80-43.8)-(50-25)}{\ln\left[\dfrac{80-43.8}{50-25}\right]} = 30.26\text{ K}$$

$$\text{Heat transfer rate, } Q = m_w C_{p,w}(T_{c,o} - T_{c,i}) = \frac{8000}{3600} \times 4180 \times 18.8 = 174.63\text{ kW}$$

Also

$$Q = UA_s(\text{LMTD})$$

$$\Rightarrow \quad A_s = \frac{Q}{U\cdot(LMTD)} = \frac{174.63\times10^3}{300\times30.26}$$

$$A_s = 19.236\text{ m}^2$$

Required area for heat exchange is $A_s = 19.236\text{ m}^2$

Q.47 Illustrate, with neat sketches, the temperature profile for hot and cold fluids as a function of the distance along the path for

(i) parallel flow heat exchangers,

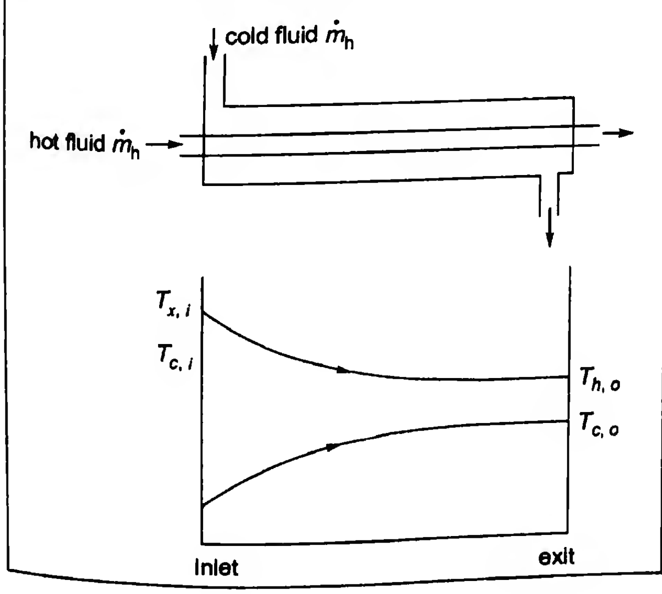
(ii) counter flow heat exchangers

(iii) condenser and gas heated boiler

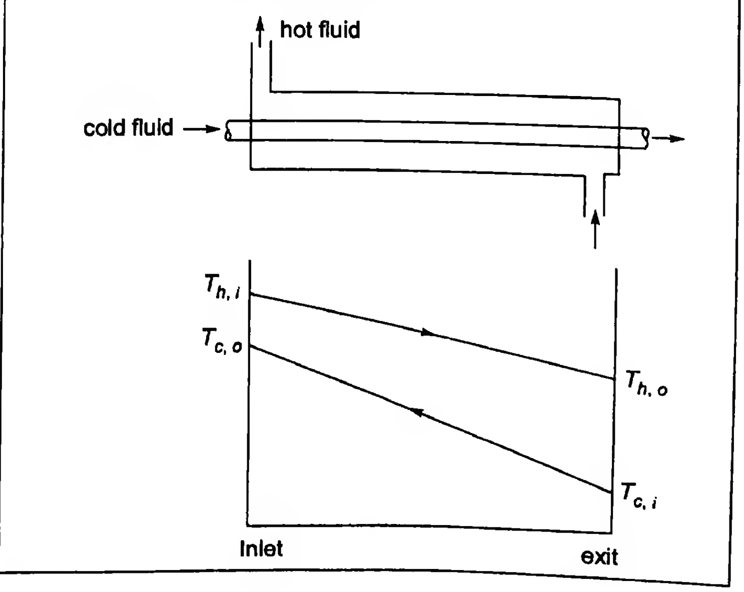
[CSE (Mains), 2005 : 20× 3 = 60 Marks]

Solution:

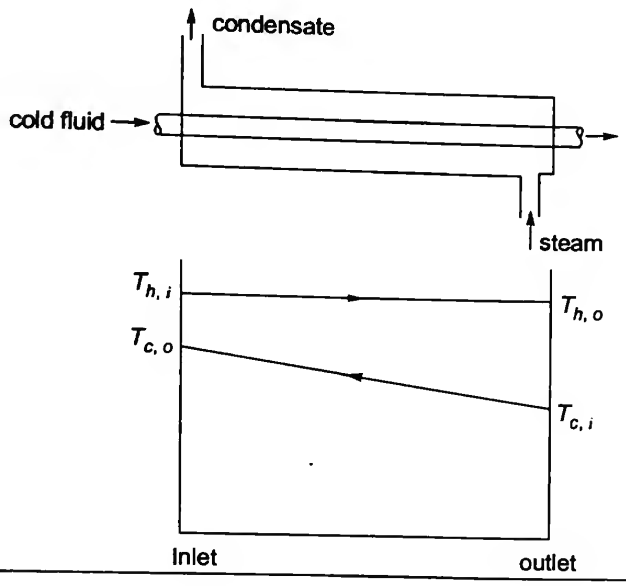
(i) Parallel flow heat exchangers



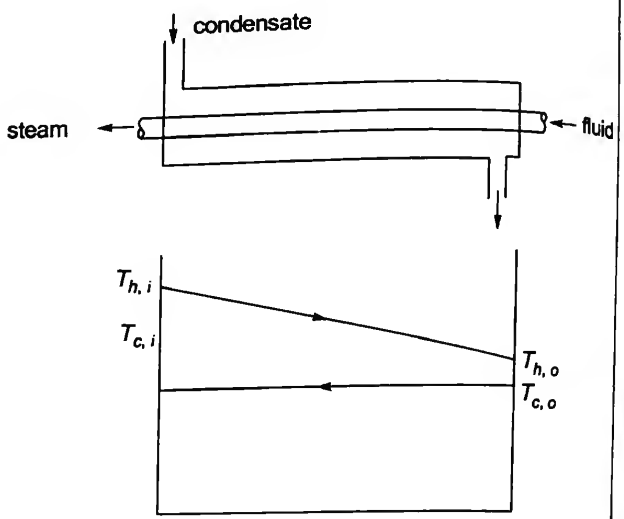
(ii) Counter flow heat exchangers



(iii) **Condenser:** For a condenser at constant pressure, temperature of hot fluid stays constant and a phase change from vapour to liquid phase occurs for the same.



(iv) **Gas heated boiler:** For a boiler, phase change from liquid to vapour takes place for cold fluid and its temperature stays constant.

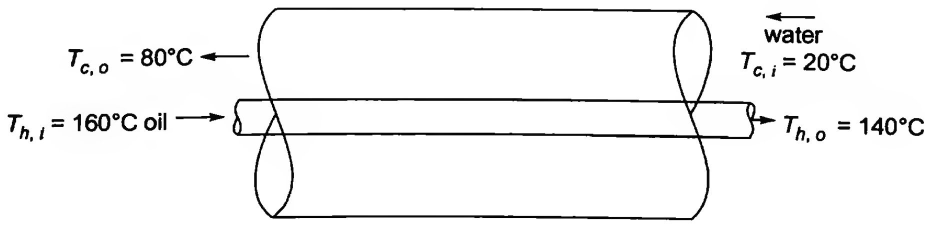


Q.48 **A counterflow, concentric tubes heat exchanger is designed to heat water from 20°C to 80°C using hot oil flowing through the annulus. The oil temperature gets reduced from 160°C to 140°C. The nominal diameter of the inner tube is 20 mm and the corresponding overall heat transfer coefficient is 500 W/m²–K. The heat transfer rate from the oil is 3000 watts. Determine the length of the exchanger. Because of fouling after some days the outlet temperature of water reduce to 65°C for the same flow rates and same inlet conditions. Determine the outlet temperature of oil, the fouling factor and the new heat transfer rate.**

Sketch the heat exchanger arrangement and the temperature profiles.

[CSE (Mains), 2006 : 30 Marks]

Solution:



Given: Diameter of inner tube = 20 mm,

Overall heat transfer coefficient, $U_i = 500\ \text{W/m}^2\text{K}$

Heat transfer from oil = 3000 watt

Log mean temperature difference, $\Delta T_{LM} = \dfrac{\Delta T_1 - \Delta T_2}{\ln\left(\dfrac{\Delta T_1}{\Delta T_2}\right)} = \dfrac{(160-80)-(140-20)}{\ln\left(\dfrac{160-80}{140-20}\right)}$

$\Rightarrow \quad \Delta T_{LM} = 98.65\ \text{K}$

Assume length of heat exchanger is 'l' m.

Heat transfer rate, $Q = UA_s \cdot \Delta T_{lm}$

$\Rightarrow \quad 3000 = 500 \times \pi \cdot 0.020 \times L \times 98.65$

$\Rightarrow \quad L = 0.9679\ \text{m} \simeq 0.97\ \text{m}$

Because of fouling outlet temperature of water reduces to 65°C

For initial condition:

$$\text{Heat transfer rate}, Q = m_w C_{pw}(80-20) = m_o C_{po}(160-140)$$

$$\Rightarrow \quad m_w C_{pw} \cdot 60 = m_o C_{po} \cdot 20 \qquad \text{...(i)}$$

For new condition:

$$Q = m_w C_{pw}(65-20) = m_o C_{po}(160 - T'_{c,o}) \qquad \text{...(ii)}$$

Dividing (ii) by (i)

$$\frac{160 - T'_{c,o}}{20} = \frac{45}{60}$$

$$\Rightarrow \quad T'_{c,o} = 145°\text{C}$$

$$\text{Old heat transfer rate}, Q = m_w C_{pw}(80-20) = 3000 \text{ Watt}$$

$$\Rightarrow \quad m_w C_{pw} = 50 \text{ Watt/K}$$

$$\therefore \quad \text{New heat transfer rate} = Q' = m_w C_{pw}(65-20) = 50 \times 45$$

$$Q' = 2250 \text{ Watt}$$

$$\text{New heat transfer rate} = Q' = UA_s \cdot \Delta T'_{\ln}$$

$$\Rightarrow \quad 2250 = U' \cdot (\pi \cdot 0.02 \times 0.97) . \frac{(160-65)-(145-20)}{\ln\left[\dfrac{160-65}{145-20}\right]}$$

$$\Rightarrow \quad U' = 337.72 \text{ Watt/m}^2\text{K}$$

Assume fouling factor is $R_{f,i}$

$$\frac{1}{U'A_i} = \frac{1}{UA_i} + \frac{R_{fi}}{A_i}$$

$$\Rightarrow \quad R_{fi} = \frac{1}{U'}(-)\frac{1}{U} = \frac{1}{337.72} - \frac{1}{500}$$

$\Rightarrow$ Fouling factor = 9.61×10^{-4} k-m²/Watt

Q.49 An oil is cooled to 100°C in a parallel flow heat exchanger by transferring its heat to cooling water, that leaves the exchanger at 30° C. However, it is now required that the oil must be cooled down to 75°C by increasing the length of heat exchanger, while oil and water flow rates, their inlet temperatures and other dimensions of the exchanger keeping constant. The inlet temperatures of water and oil are 15°C and 150°C respectively.

If the original cooler was 1 metre long, determine

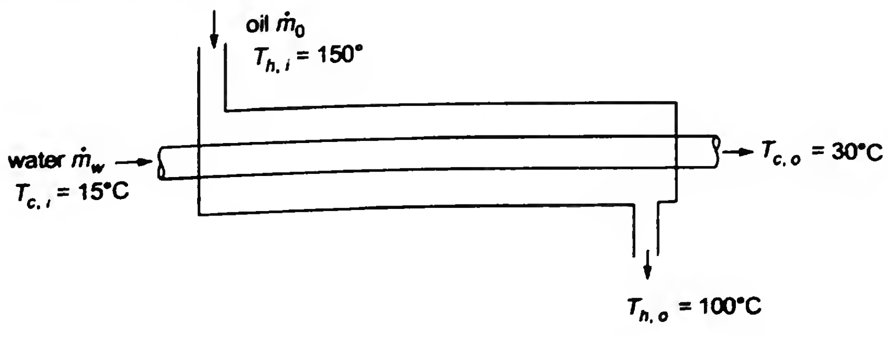
(i) outlet temperature of water in the new cooler and

(ii) length of the new cooler.

[CSE (Mains), 2009 : 30 Marks]

Solution:

Consider the parallel flow heat exchanger as showing in figure.



Let $\dot{m}_w$ and $\dot{m}_o$ are the mass flow rates of water and oil respectively.

Assume area of heat exchanger is A_s and overall heat transfer coefficient is U.

Total heat transferred tow after from oil

$$= Q = \dot{m}_w C_{p,w} \cdot (T_{Go} - T_{Gi})$$

$$= \dot{m}_w C_{p,w} \cdot 15 \quad ...(i)$$

$$= m_o C_{p,o} \cdot (150 - 100) = m_o C_{p,o} \cdot 50$$

Log mean temperature difference in this case

$$= \Delta T_{ln} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)} = \frac{(150-15)-(100-30)}{\ln\left[\frac{150-15}{100-30}\right]} = 98.968 \text{ K}$$

We know, heat transfer,

$$Q = UA_s \Delta T_{ln} \quad ...(ii)$$

From (i) and (ii)

$$= m_o C_{po} \times 50 = m_w C_{p,w} \cdot 15 = U \cdot \pi DL \cdot \Delta T_{ln}$$

$$\Rightarrow \quad \frac{m_o C_{po} \times 50}{U\pi \cdot D} = \frac{15 \times m_w C_{pw}}{U \cdot \pi \cdot D} = 98.968 \times 1 = 6.598 \times 15 \quad ...(iii)$$

Now length of cooler is to be increased to ensure

$$T_{h,o} = 75°$$

$$\therefore \quad \text{New } LMTD,\ \Delta T'_{LM} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)} = \frac{(150-15)-(75-T_{Go})}{\ln\left[\frac{150-15}{75-T_{Go}}\right]}$$

$$\Delta T'_{LM} = \frac{60 - T_{c,o}}{\ln\left(\frac{135}{75 - T_{c,o}}\right)}$$

Assume new length as l'

Heat transfer,

$$Q = UA'_s \cdot \Delta T'_{LM} = u \cdot \pi \Delta l' \cdot \frac{60 - T_{c,o}}{\ln\left(\frac{135}{75 - T_{c,o}}\right)} \quad ...(iv)$$

Heat transfer,

$$Q = m_w C_{p,w}(T_{cp} - 15) = m_o C_{p,o}(150 - 75)$$

$\Rightarrow$

$$Q = m_w C_{pw}(T_{c,o} - 15) = m_o C_{p,o} \cdot 75 \quad ...(v)$$

From (iv) and (v)

$$Q = m_o C_{po} \cdot 75 = u\pi D l' \cdot \frac{60 - T_{c,o}}{\ln\left(\frac{135}{75 - T_{c,o}}\right)}$$

$$\Rightarrow \quad \frac{m_o C_{po}}{U\pi D} = \frac{l'}{75} \cdot \frac{60 - T_{c,o}}{\ln\left(\frac{135}{75 - T_{c,o}}\right)} \quad ...(vi)$$

Dividing (v) by (i), we get

$$\Rightarrow \quad T_{c,o} = 37.5°\text{C}$$

From (iv) and (vi) we get

$$\frac{15 \times 6.598}{50} = \frac{l'}{75} \cdot \frac{60 - 37.5}{\ln\left(\frac{135}{75 - 37.5}\right)}$$

$$\Rightarrow \quad l' = 1.95 \text{ m}$$

$\therefore$ Length of new cooler is $l' = 1.95$ m

Q.50 Water enters a tube of dia D and length L. Inlet and outlet temperature are T_1 and T_2 respectively and wall temperature is T_0. Convective heat transfer is h.

(i) Derive the following expression for heat transfer:

$$Q = \frac{h\pi DL(T_1 - T_2)}{\ln\left(\frac{T_1 - T_0}{T_2 - T_0}\right)}$$

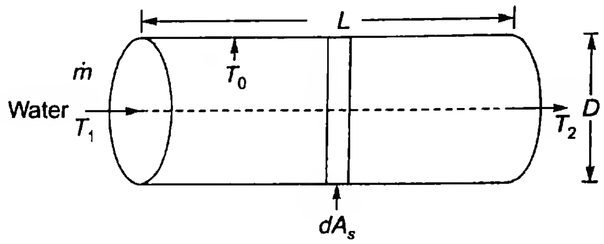
(ii) What is the significance of the logarithmic term in part (*i*) above?

(iii) State assumptions used in part (*i*).

[CSE (Mains) 2011 : 20 Marks]

Solution:

Consider the tube through which water is flowing, walls of the tube are maintained at temperature T_0. Heat transfer between water flowing and wall takes place by convection. Consider a differential element of area dA_s where heat transfer to tube wall takes place.



$$\text{Heat transferred to tube} = d_q = hdA_s(T - T_0) \quad \text{...(i)}$$

where, T is the temperature of that section of fluid. Assume this heat transfer changes the temperature of fluid by dT.

$\therefore$ Heat transfer $= \dot{m}C_p dT$ (where C_p specific heat capacity of fluid) ...(ii)

$\therefore$ From (i) and (ii), we get

$$\dot{m}C_p dT = hdA_s(T - T_0)$$

Since, T_0 is constant : $dT = d(T - T_0)$

$$\therefore \quad \frac{d(T-T_0)}{(T-T_0)} = \frac{hdA_s}{mC_p}$$

Integrating both sides with proper limits,

$$\int_{T_1}^{T_2} \frac{d(T-T_0)}{T-T_0} = \frac{h}{mC_p}\int dA_s$$

$$\Rightarrow \quad \ln\left(\frac{T_2-T_0}{(T_1-T_0)}\right) = \frac{h}{mC_p}\cdot(\pi DL) \quad \text{...(iii)}$$

Total heat transfer from fluid to wall is equal to change in internal energy of fluid,

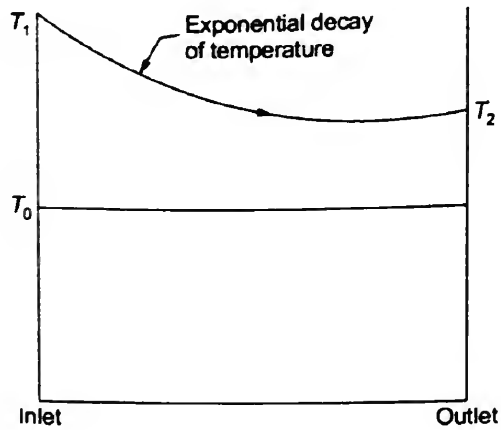
$$\therefore \quad Q = mC_p(T_2 - T_1)$$

Replacing (mC_p) in the expression (3) above and rearranging, we get

$$Q = h(\pi DL)\cdot\frac{(T_1-T_2)}{\ln\left(\frac{T_1-T_0}{T_2-T_0}\right)}$$

(*ii*) Temperature difference between the wall and fluid changes from $(T_1 - T_0)$ to $(T_2 - T_0)$. Average difference will be $\frac{\Delta T_1 + \Delta T_2}{2}$.

However if we use logarithmic term as obtained above it truly reflects the exponential decrease in temperature of local temperature of the fluid.



(*iii*) Assumption used in part above are:

(a) Steady state has been achieved.

(b) Convective heat transfer coefficient and specific heat capacity of fluid remain constant.

(c) Radiation effects are neglected.

(d) Heat transfer from outside of tube can be neglected.

Q.51 A tubular gas heater heats air flowing at the rate of 5.5 kg/s from 20°C to 75°C using saturated steam condensing at 1.3 bar (saturation temperature, t_{sat} = 107°C). It is proposed to double the flow rate of air to heat the same for the same rise in temperature in the same gas heater. One way of doing this is to increase the condensing pressure of saturated steam. What should be the pressure needed if the overall heat transfer coefficient remains the same for both the operating conditions? Specific heat of air = 1.005 kJ/kg-K.

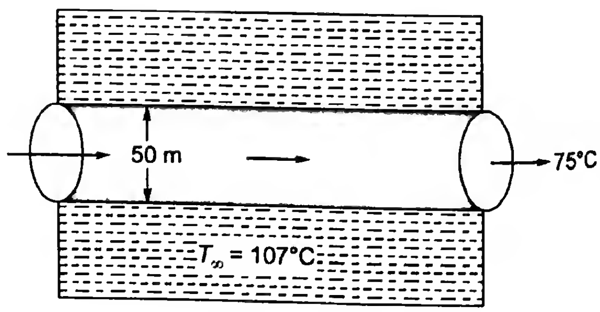
For steam, the following pressure and the corresponding saturation temperature are known:

p (bar)	2	3	4	5	6	7	8	9
t_{sat} (°C)	120.2	133.5	143.6	151.8	158.8	165.0	170.4	175.4

[CSE (Mains) 2012 : 20 Marks]

Solution:

Given: $\dot{m} = 5.5$ kg/sec, $t_i = 20°C$, $t_e = 75°C$, $T_{sat} = 107°C$



Case I: when $\dot{m}$ = 5.5 kg/sec,

$$Q_1 = \dot{m}C_p\Delta T = h\,A\,\Delta T_m$$

$$\Delta T_m = \frac{(107-20)-(107-75)}{\ln\left(\frac{107-20}{107-75}\right)}$$

$$= 54.9905°C$$

Case II: when $\dot{m}$ = 5.5 × 2,

$$Q_2 = \dot{m}_2 C_p \Delta T = h\,(A\,\Delta T_m)_2 = 5.5 \times 2\, C_p\,\Delta T$$

Since C_p & ΔT remains same and Since h & A remains same.

$$\therefore \quad Q_2 = 2 \times Q_1$$

$$(\Delta T_m)_2 = (\Delta T_m)_1 \times 2$$

$$109.98105 = (\Delta T_m)_2$$

Assuming condensing temperature of steam as T

$$109.98105 = \frac{(T-20)-(T-75)}{\ln\left(\frac{T-20}{T-75}\right)} = \frac{55}{\ln\left(\frac{T-20}{T-75}\right)}$$

$$\ln\left(\frac{T-20}{T-75}\right) = \frac{55}{109.981}$$

$$(T-20) = 1.6488\,(T-75)$$

$$\therefore \quad T = 159.763°C$$

Now applying interpolation to find out the pressure = P

$$\frac{159.763-158.8}{165-158.8} = \frac{P-6}{7-6}$$

$$\therefore \quad P = 6.17 \text{ bar}$$

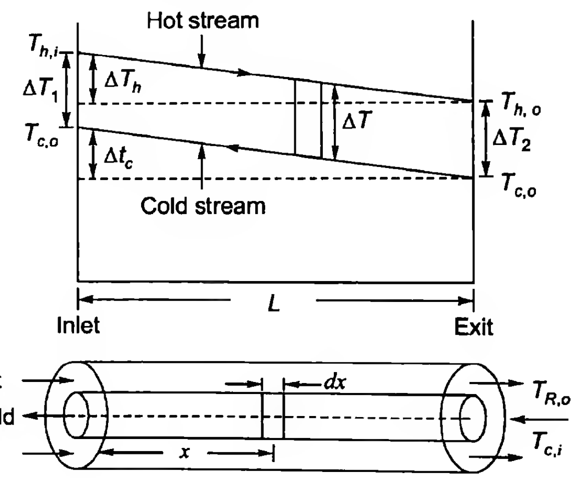
Q.52 In a balanced counterflow heat exchanger, where $m_c C_{p,c} = m_n C_{p,h}$, show that:

(i) $\Delta T_1 = \Delta T_2 = \Delta T$ at any section;

(ii) the temperature profiles of two fluids are parallel and linear.

[CSE (Mains) 2013 : 20 Marks]

Solution:



Consider the heat exchanger (counter flow) and associated temperature profile as shown in Figure.

$$m_c C_{p,c} = m_h C_{p,h}$$

Consider a section at a distance x from inlet of thickness dx.

Rate of heat given out by hot fluid in this section

$$= d_{\dot{q}h} = \dot{m}_h C_{p,h} d_{Th} = u dA_x (T_h - T_c)$$

Rate of heat transferred to cold fluid

$$= d_{qc} = \dot{m}_c C_{p,c} dT_c$$

At steady state,

Heat transferred from heat fluid = Heat transferred to cold fluid

$$\Rightarrow \quad d_{\dot{q}h} = d_{\dot{q}c}$$

$$\Rightarrow \quad \dot{m}_h C_{p,h} \cdot dT_h = \dot{m}_c C_{p,c}\, dT_c$$

$$\Rightarrow \quad dT_h = dT_c$$

$$\Rightarrow \quad \Delta T_h = \Delta T_c$$

$$\Rightarrow \quad T_{h,i} - T_{h,o} = T_{c,o} - T_{c,i}$$

$$\Rightarrow \quad T_{h,i} - T_{c,o} = T_{h,o} - T_{c,i}$$

$$\Rightarrow \quad \Delta T_1 = \Delta T_2$$

Comparing variation of temp along the length of tube we have $d_{\dot{q}h} = d_{\dot{q}c}$.

$$\Rightarrow \quad \dot{m}_h C_{p,h} \cdot dT_h = \dot{m}_c C_{p,c}\, dT_c$$

$$\Rightarrow \quad dT_h = dT_c$$

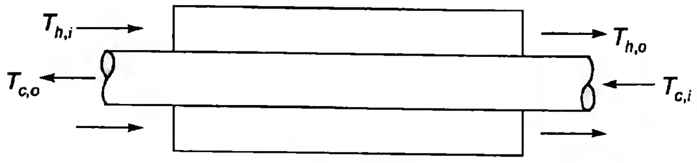
$$\Rightarrow \quad \frac{T_{h,i} - T_{h,o}}{L} = \frac{T_{c,o} - T_{c,i}}{L}$$

$\therefore$ Slope of temp-length graph is same for both hot and cold streams along the length of heat exchanger.

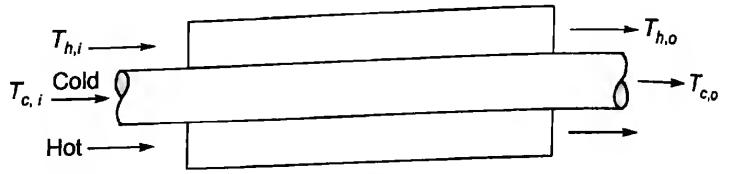
Q.53 In a double-pipe heat exchanger, $\dot{m}_h C_{p,h} = 0.5\,\dot{m}_c C_{p,c}$. The inlet temperature of hot and cold are $T_{h,i}$ and $T_{c,i}$. Determine an expression, in terms of $T_{h,i}$, $T_{c,i}$ and $T_{h,o}$, for the ratio of area of counterflow heat exchanger to that of parallel-flow heat exchanger, which will given same hot fluid outlet temperature $T_{h,o}$. Also find out the ratio, if $T_{h,i} = 150$ °C, $T_{c,i} = 30$ °C and $T_{h,o} = 90$ °C.

[CSE (Mains) 2013 : 25 Marks]

Solution:



Counter-Flow



Parallel

Given: $T_{h,i} = 150$ °C, $T_{c,i} = 30$ °C and $T_{h,o} = 90$ °C.

Also $\dot{m}_h C_{p,h} = 0.5\dot{m}_c C_{p,c}$ for both heat exchangers.

Heat transferred between the two fluids, $q = \dot{m}_h\, C_{p,h}(T_{h,i} - T_{h,o}) = m\,\dot{m}_c\; C_{p,c}(T_{c,o} - T_{c,i})$

$$\Rightarrow \qquad T_{c,o} = T_{c,i} + 0.5\,(T_{h,i} + 2T_{h,o}) \qquad \text{...(i)}$$

Heat transfer for both heat exchanger is same.

Hence,
$$Q = UA_{\text{count}} \cdot \Delta T_{Lm,\text{counter}} = UA_{\text{Parallel}}\, \Delta T_{Lm,\text{parallel}}$$

Assuming same heat transfer overall coefficient for both heat exchangers, we get

$$\frac{A_{\text{counter}}}{A_{\text{parallel}}} = \frac{\Delta T_{Lm,\text{parallel}}}{\Delta T_{Lm,\text{counter}}} = \frac{\left(\dfrac{\Delta T_1 - \Delta T_2}{\ln \Delta T_1 / \Delta T_2}\right)_{\text{parallel}}}{\left(\dfrac{\Delta T_1 - \Delta T_2}{\ln \Delta T_1 / \Delta T_2}\right)_{\text{counter}}}$$

$$\frac{A_{\text{counter}}}{A_{\text{parallel}}} = \frac{(T_{h,i} - T_{c,i}) - (T_{h,o} - T_{c,o})}{\ln\left(\dfrac{T_{h,i} - T_{c,i}}{T_{h,o} - T_{c,o}}\right)} \times \frac{\ln\left(\dfrac{T_{h,i} - T_{c,o}}{T_{h,o} - T_{c,i}}\right)}{(T_{h,i} - T_{c,o}) - (T_{h,o} - T_{c,i})}$$

Since,
$$T_{c,o} = T_{c,i} + 0.5T_{h,i} - T_{h,o}$$

$(T_{h,i} - T_{c,i}) - (T_{h,o} - (T_{c,i}\,(0.5)(T_{h,i} - T_{h,o})$

$$= T_{h,i} - T_{h,o} + 0.5\,(T_{h,i} - T_{h,o})$$
$$= 1.5\,(T_{h,i} - T_{h,o})$$

Also, $(T_{h,i} - T_{c,o}) - (T_{h,o} - T_{c,i})$

$[T_{h,i} - T_{c,i} - 0.5\,(T_{h,i} - T_{h,o})] - (T_{h,o} - T_{c,i})$

$0.5(T_{h,i} - T_{h,o})$

$$\Rightarrow \qquad \frac{A_{\text{counter}}}{A_{\text{parallel}}} = \frac{3\ln\left(\dfrac{T_{h,i} - T_{c,o}}{T_{h,o} - T_{c,i}}\right)}{\ln\left(\dfrac{T_{h,i} - T_{c,i}}{T_{h,o} - T_{c,o}}\right)} = \frac{3\ln\left[\dfrac{T_{h,i} - [T_{c,i} + 0.5(T_{h,i} - T_{h,o})]}{T_{h,o} - T_{c,i}}\right]}{\ln\left[\dfrac{T_{h,i} - T_{c,i}}{T_{h,o} - (T_{c,i} + 0.5(T_{h,i} - T_{h,o}))}\right]}$$

This is the required expression for ratio of area of heat exchanger.

Substituting values of temperature as given,

$$\Rightarrow \quad \frac{A_{counter}}{A_{parallel}} = \frac{3\ln\left[\dfrac{0.5(T_{h,i} - T_{h,o}) - T_{c,i}}{T_{h,o} - T_{c,i}}\right]}{\ln\left[\dfrac{T_{h,i} - T_{c,i}}{1.5T_{h,o} - T_{c,i} - 0.5T_{h,i})}\right]}$$

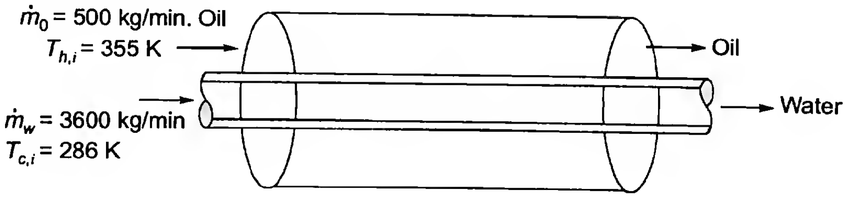
$$\Rightarrow \quad \frac{A_{counter}}{A_{parallel}} = \frac{3\ln\left(\dfrac{(0.5)(150+90)-30}{90-30}\right)}{\ln\left(\dfrac{150-30}{1.5\times 90 - 30 - 0.5\times 150}\right)} = 3\times\frac{\ln(1.5)}{\ln 4}$$

$$\Rightarrow \quad \frac{A_{counter}}{A_{parallel}} = 0.8775$$

Q.54 The vapour, at the saturation temperature of an oil flowing at the rate of 500 kg/min, enters a heat exchanger tube, at 355 K and condenses while it is cooled by water flowing at the rate of 3600 kg/min entering the concentric tube of a parallel flow heat exchanger at 286 K. Assuming overall heat transfer coefficient of 475 W/m² K, latent heat of oil as 600 kJ/kg K, calculate the number of tubes required of 25 mm outer diameter and 2 mm thick with a length of 4.87 m. What will be the number of tube passes, if cooling water velocity should not exceed 2 m/s? Take C_p for water as 4.18 kJ/kg K and density of water as 1000 kg/m³.

[CSE (Mains) 2014 : 10 Marks]

Solution:



Given: Length of each tube = 4.87 m, d_o = 25 mm, t = 2 mm, $d_i = 25 - 2 \times 2 = 21$ mm,
h_{fg} for oil = 600 kJ/kg K, U = 475 W/m² K

Maximum possible heat transfer takes place when water achieve temperature of oil.

$$\therefore \quad Q_{max} = \dot{m}_w c_p (T_{h,i} - T_{c,i}) = \frac{3600}{60}\times 4.18\times(355-286) = 17305.2 \text{ kW}$$

$$\text{Actual heat transfer rate, } \dot{Q} = \dot{m}_0 h_{f_g} = \frac{500}{60}\times 600 = 5000 \text{ kW}$$

$$\therefore \quad \text{Effectiveness of heat exchanger, } \epsilon = \frac{Q}{Q_{max}} = 0.2889$$

We know, for a parallel flow heat exchanger, for a condenser,

$$\epsilon = 1 - \exp(-\text{NTU})$$

$$\Rightarrow \quad \text{NTU} = 0.341$$

Also

$$\text{NTU} = \frac{UA_s}{C_{min}} = \frac{U\cdot\pi D_i L\times n}{\dot{m}_w c_{pw}}$$

$$\Rightarrow \qquad n = 0.341 \times \frac{3600 \times 4.18 \times 10^3}{60 \times 475 \times \pi \times 0.021 \times 4.87} = 560.39 \simeq 561$$

∴ 561 tubes of given dimensions will be required for heat transfer.

$$\text{Internal area of the tubes, } A_i = \frac{\pi d_i^2}{4} = 3.464 \times 10^{-4} m^2$$

$$\text{Maximum allowable velocity} = 2 \text{ m/s}$$

$$\text{Max. flow rate per tube, } \rho A v = 0.6927 \text{ kg/s}$$

$$\therefore \qquad \text{Number of tubes required} = \frac{3600}{60 \times 0.6927} = 86.61 \simeq 87 \text{ tubes}$$

Assume each tube requires 'n' passes.

Total length of tubes required for required heat transfer = 561 × 4.87 = 2732.07 m

∴ For 87 tubes with n passes each, total length of heat exchange surface required will be

$$= 87 \times n \times 4.87 = 2732.07 \text{ m}$$

$$\Rightarrow \qquad n = 6.45 \text{ passes} \simeq 7 \text{ passes}$$

∴ 87 tubes of 7 passes each or 609 tube passes are required to limit maximum velocity to 2 m/s.

Q.55 Steam at atmospheric pressure enters the shell of a surface condenser, in which water flows through a tube bundle, at the rate of 0.05 kg/s. The inner diameter of the tube is 25 mm. The overall heat transfer coefficient (U_i) based on the inner diameter is 230 W/m²C. The inlet and outlet temperatures of water are 15 °C and 70 °C, respectively. The condensation of steam takes place on the outside surface of the tubes.

Calculate the following:

(i) The effectiveness of the heat exchanger and the NTU.

(ii) Length of each tube.

(iii) The rate of steam condensation.

Assume c_p of water = 4.18 kJ/kg °C; h_{fg} = 2257 kJ/kg °C (latent heat of condensation)

[CSE (Mains) 2015 : 10 Marks]

Solution:

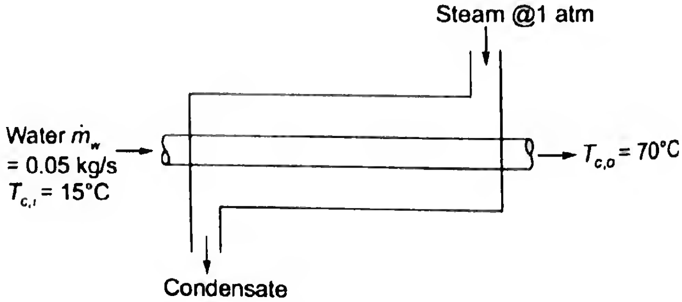
Given: U_i = 230 W/m² °C ; d = 25 mm

For condensation of steam, change of temperature is zero.

$$\Delta T_h = 0$$

∴ Heat capacity for steam is infinite or $C_h \to \infty$

$$\therefore \qquad \frac{C_{min}}{C_{max}} = C \to 0$$



Maximum heat transfer in this case will be,

$$Q_{max} = \dot{m}_w c_p (T_{h_i} - T_{c_i})$$

$$= 0.05 \text{ kJ/s} \times 4.18 \text{ kJ/kg °C} (100 - 15)$$

$$\Rightarrow \qquad Q_{max} = 17.765 \text{ kJ/s} = 17.765 \text{ kW}$$

$$\text{Actual heat transfer rate, } Q = \dot{m}_w c_p (T_{c,o} - T_{c,i}) = 0.05 \times 4.18 \times (70 - 15) = 11.495 \text{ kW}$$

(i) ∴ Effectiveness of heat exchanger,

$$\epsilon = \frac{Q}{Q_{max}} = \frac{11.495}{17.765} = 0.647$$

For a condenser heat exchanger, we know,

$$\epsilon = 1 - \exp(-NTU)$$

$$\Rightarrow \quad -NTU = \ln(1-\epsilon) \Rightarrow NTU = -\ln(1-\epsilon)$$

$$= -\ln(1-0.647)$$

$$\therefore \quad NTU = 1.041$$

(ii) We know,

$$NTU = \frac{UA_s}{C_{min}} = \frac{U\cdot \pi dL}{\dot{m}_w c_p}$$

$$\Rightarrow \quad L = \frac{1.041\times 0.05\times 4.18\times 10^3}{\pi\times 0.025\times 230}$$

$$\therefore \quad \text{Length of tube, } L = 12.044 \text{ m}$$

(iii) Rate of steam condensation $= \dfrac{Q_{actual}}{h_{f_g}} = \dfrac{13.495}{2257} = 5.09\times 10^{-3}$ kg/s

Q.56 Define effectiveness, NTU and heat capacity ratio in case of heat exchanger analysis, and also establish a relationship among them for counterflow heat exchanger.

[CSE (Mains) 2016 : 10 Marks]

Solution:

Effectiveness: It is defined as the ratio between actual heat transfer rate taking place between hot and cooled fluids and the maximum possible heat transfer rate that can occur between them.

$$\epsilon = \frac{q_{actual}}{q_{maximum\ possible}}$$

where, q_{act} = The rate of enthalpy charge of either fluids

$$q_{maximum\ possible} = (\dot{m}C_p)_{small}(T_{hi} - T_{ci})$$

Case I: If $\dot{m}_h C_{ph} < \dot{m}_c C_{pc}$,

$$\epsilon_{x\cdot\epsilon} = \frac{\dot{m}_h C_{ph}(T_{hi} - T_{he})}{\dot{m}_h C_{ph}(T_{hi} - T_{ci})} = \frac{T_{hi} - T_{he}}{T_{hi} - T_{ci}}$$

Case II: If $\dot{m}_c C_{p_c} < \dot{m}_h C_{p_h}$

$$\epsilon_{x\cdot\epsilon} = \frac{\dot{m}_c C_{pc}(T_{ce} - T_{ci})}{\dot{m}_c C_{pc}(T_{hi} - T_{ci})} = \frac{T_{ce} - T_{ci}}{T_{hi} - T_{ci}}$$

Number of Transfer Units (NTU): It is defined as the ratio between product of UA and the smaller capacity rate between hot and cold fluids.

or

$$NTU = \frac{UA}{(\dot{m}C_p)_{small}}$$

NTU being directly proportional to the area of heat exchanger it indicaes the overall size of heat exchanger.

Heat capacity ratio : It is defined as the ratio between the heat capcity of the two fluid flowing in heat exchanger. The numerator value is occupied by fluid having lower heat capacity whereas the denominator is by the fluid having higher value of heat capacity.

$$C = \frac{(\dot{m}C_p)_{small}}{(\dot{m}C_p)_{Big}} \quad 0 \le c \le 1$$

For the counterflow heat exchanger,

$$\epsilon_{x\cdot\epsilon} = \frac{1 - e^{-(1-c)NTU}}{1 - ce^{-(1-c)NTU}}$$

Q.57 In a large steam power plant, a shell and tube type condenser is used which has the following data:

Heat exchange data = 2100 MW Number of shell passes = 1
Number of tubes = 31500 Number of tube passes = 2
Diameter of each tube = 25 mm
Condensation temperature = 50°C
Mass flow rate of cooling water = 3.4×10^4 kg/s
Heat transfer coefficient on the steam side = 11400 W/m²K
Inlet water temperature = 20°C
Heat transfer coefficient on the water side = 8018 W/m²K
Using only ε-NTU method, calculate:
(i) the outlet temperature of cooling water;
(ii) the length of tube pass.
[Properties of water at 27°C are:
C_p = 4.18 kJ/kg K, $\mu = 855 \times 10^{-6}$ N s/m², k = 0.613 W/mK and Pr = 5.83]
Neglect the thermal resistance due to tube wall.

[CSE (Mains) 2016 : 20 Marks]

Solution:

Given: $\dot{Q}$ = 2100 mω, No. of shell passes = 1, No. of tubes (n) = 31500, No. of tube passes, $K = 2$, $\dot{m}_\omega = 3.4 \times 10^4$ Kg/s, Diameter of each tube = 25 mm, Condensation temperature = 50°C, h_{steam} = 11400 w/m²K
$T_{inlet,\, water}$ = 20°C, h_{water} = 8018 w/m²K

Now,
$$\frac{1}{U} = \frac{1}{h_{steam}} + \frac{1}{h_{water}} = \frac{1}{11400} + \frac{1}{8018} \Rightarrow U = 4707.24 \text{ W/m}^2\text{K}$$

(i) By the use of the energy balance, the heat lost by the steam is the heat gained by water..

So,
$$2100 \times 10^6 = 3.4 \times 10^4 \times 4180 \times (T_{ce} - 20)$$
$$T_{ce} = 34.77°\text{C}$$

(ii) The effectiveness of heat exchanger
$$= \frac{T_{ce} - T_{ci}}{T_{hi} - T_{ci}} = \frac{34.77 - 20}{50 - 20} = 0.4923$$

Since the heat capacity of the condensing steam is infinite.
Also we know that effectiveness in the case of steam condensor,
$$\epsilon = 1 - e^{-NTU}$$
$$0.4923 = 1 - e^{-NTU} \Rightarrow NTU = 0.6778$$

Also
$$NTU = \frac{UA}{(mC_p)_{small}}$$

$$(mC_p)_{small} = 3.4 \times 10^4 \times 4180 = 14212 \times 10^4$$
$$\text{Area} = \text{No. of tubes} \times \text{tube passes} \times \pi \times D \times \text{length}$$
$$= 31500 \times 2 \times \pi \times 0.025\, L = 4948.008\, L \text{ m}$$

So,
$$0.6778 = \frac{4707.24 \times 4948.008 L}{14212 \times 10^4}$$
$$L = 4.135 \text{ m}$$

■■■■

4 Internal Combustion Engine

1: Basics of I.C. Engines and Air Standard Cycles

Q.1 A three litre V 6 S.I. engine operates on a four stroke cycle at 3600 rpm. The compression ratio is 9.5, the length of the connecting rod is 16.6 cm and the engine is square (bore = stroke). At this speed, the combustion ends at 20° TDC. Calculate:

(i) cylinder bore and stroke length (ii) average piston speed
(iii) clearance volume of one cylinder (iv) piston speed at the end of combustion
(v) distance the piston has travelled from TDC at the end of combustion.
(vi) volume in the combustion chamber at the end of combustion.

[CSE (Mains) 2001 : 30 Marks]

Solution:

Given: Swept volume $V_s = 3 \times 10^{-3}$ m^3 , No. of cylinders, $k = 6$, Speed, $N = 3600$ rpm, compression ratio, $r = 9.5$,
4-stroke (S.I Engine), Length of connecting rod = L_{cr} = 16.6 cm = 0.166 m, Stroke, (L) = Bore (D)

$$S = r_{cr}\cos\theta + \sqrt{L_{cr}^2 - r_{cr}^2\sin^2\theta}$$

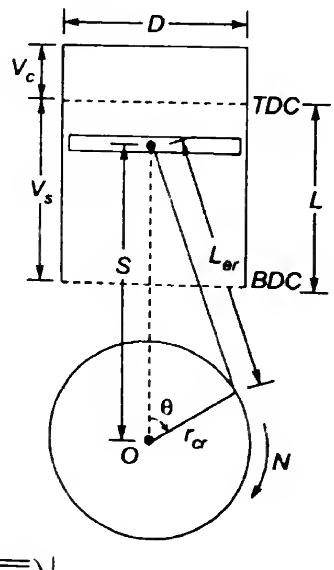
Cylinder vol. at crank angle, θ $\quad V(\theta) = V_C + \frac{\pi}{4}D^2 \times (L_{cr} + r_{cr} - s)$

In non-dimensionless term, $\quad \frac{V}{V_c} = 1 + \frac{\frac{\pi}{4}D^2 \times (L_{cr} + r_{cr} - s)}{V_c}$

where, $\quad V_c = \frac{V_s}{(r-1)} = \frac{\frac{\pi}{4}D^2 \times L}{(r-1)}$

then, $\quad \frac{V}{V_c} = 1 + \frac{(L_{cr} + r_{cr} - s)}{L} \times (r-1)$

where, $\quad L = 2\,r_{cr}$



$$\Rightarrow \quad \left|\frac{V}{V_c} = 1 + \frac{(r-1)}{2} \times \left(\frac{L_{cr}}{r_{cr}} + 1 - \cos\theta - \sqrt{\left(\frac{L_{cr}}{r_{cr}}\right)^2 - \sin^2\theta}\right)\right|$$

(i) $\quad$ swept volume per cycle $= \frac{\text{total swept volume } (V_s)}{\text{No. of cylinders } (k)}$

(ii) $\quad V_{s_{\text{per cyl}}} = \frac{V_s}{6} = \frac{3 \times 10^{-3}}{6} m^3 = \frac{\pi}{4}D^2 \times L$

$$\frac{3 \times 10^{-3}}{6} = \frac{\pi}{4}D^3 \qquad (\because D = L)$$

⇒ $D = L = 0.086 = 8.6$ cm; crank radius $= r_{cr}$

$$r_{cr} = \frac{L}{2} = 4.3 \text{ cm}$$

(ii) Average piston speed, $\overline{V_p} = 2 \times L \times N = 10.32$ m/sec

(iii)
$$r_c = 9.5 = \frac{V_s + V_c}{V_c}$$

⇒
$$V_{c,\text{ per cylinder}} = 59 \text{ cm}^3$$
$$V_{c,\text{ total}} = V_{c,\text{ per cylinder}} \times 6 = 354 \text{ cm}^3$$

(iv)
$$V_p = \frac{ds}{dt}$$
$$\frac{V_p}{\overline{V_p}} = \frac{\pi}{4}\sin\theta\left[1+\left(\frac{\cos\theta}{\sqrt{\frac{L_{cr}^2}{r_{cr}^2}-\sin^2\theta}}\right)\right]$$
$$V_p = 0.668\,\overline{V_p} = 6.89 \text{ m/sec}$$

(v)
$$s = r_{cr}\cos\theta + \sqrt{L_{cr}^2 - r_{cr}^2\sin^2\theta} = 0.206 \text{ m}$$

Distance from TDC is
$$x = L_{cr} + r_{cr} - s = 0.003 \text{ m} = 0.3 \text{ cm}$$

(vi)
$$\frac{V}{V_c} = 1 + \frac{1}{2}(r-1)\times\left[\frac{L_{cr}}{r_{cr}} + 1 - \cos\theta - \sqrt{\frac{L_{cr}^2}{r_{cr}^2} - \sin^2\theta}\right]$$

⇒
$$V = 77.9 \text{ cm}^3 = 0.0000799 \text{ m}^3$$

Q.2 The brake thermal efficiency of a diesel engine is 30%. If the air to fuel ratio by weight is 20 and the calorific value of fuel is 41800 kJ/kg, find brake mean effective pressure at S.T.P. (15°C and 760 mm of Hg).

[CSE (Mains) 2003 : 20 Marks]

Solution:

Given: Brake thermal efficiency, $\eta_{BT} = 0.3$, Air-fuel ratio, AFR $= \frac{\dot{m}_a}{\dot{m}_f} = 20$,

Calorific value of fuel, $(CV)_f = 41800$ kJ/kg, STP: temperature, $T_1 = 15°\text{C} = 288$ K,

Pressure,
$$P_1 = 760 \text{ mm Hg} = 13600 \times 9.81 \times \frac{760}{1000} \text{ Pa}$$
$$P_1 = 101.396 \text{ kPa}$$

Ideal gas equation;
$$P_1 = \rho_a R T_1$$

⇒
$$\rho_a = \frac{P_1}{RT_1} = \frac{\dot{m}_a}{\dot{V}_a} = 1.226 = \frac{\dot{m}_a}{\dot{V}_a} \Rightarrow \dot{m}_a = 1.226\,\dot{V}_a$$

$$\eta_{BT} = \frac{\text{Brake Power (BP)}}{\text{Rate of Heat Added } (Q_s)}$$

$$Q_s = \dot{m}_f (CV)_f = \frac{\dot{m}_a}{AFR} \times (CV)_f = \left(\frac{1.226\,\dot{V}_a}{20}\right) \times 41800 = 2562.34\,\dot{V}_a$$

⇒
$$BP = Q_s \times \eta_{BT} = 768.702\,\dot{V}_a$$

(ii) Let brake mean effective pressure be bmep

$$BP = \text{bmep} \times \dot{V}_s$$

Assume 100% volumetric efficiency,

$$\Rightarrow \quad \dot{V}_s = \dot{V}_a$$

$$\Rightarrow \quad 768.702\,\dot{V}_a = \text{bmep} \times \dot{V}_a$$

$$\Rightarrow \quad \text{bmep} = 768.702\text{ kPa} = 7.69\text{ bar}$$

Thus, brake mean effective pressure = 7.69 bar

Q.3 The air flow to a four cylinder 4-stroke oil engine is measured by means of a 4.5 cm diameter orifice, having $C_d = 0.65$. During a test the following data was recorded:
Bore = 10 cm, Stroke = 15 cm, Engine speed = 1000 RPM, Brake torque = 135 Nm, Fuel consumption = 5.0 kg/hour, CV_{fuel} = 42600 kJ/kg, Head across orifice = 6 cm of water, Ambient temperature and pressure are 300 K and 1.0 bar respectively. Calculate:
(i) Brake thermal efficiency
(ii) The brake mean effective pressure
(iii) The volumetric efficiency
Take R = 287 J/kg K for air.

[CSE (Mains) 2004 : 30 Marks]

Solution:

Given: 4 cylinder, 4 stoke engine, No. of cylinders, $k = 4$, Orifice Diameter, d = 4.5 cm = 0.045 m,

Area of orifice, $A = \frac{\pi}{4}d^2 = 1.59\times10^{-3}\text{m}^2$, Coefficient of discharge = C_d = 0.65, Cylinder bore, D = 10 cm = 0.1 m,

Stroke, L = 15 cm = 0.15 m, Engine speed, N = 1000 rpm

$$\text{Swept volume, } V_s = \frac{\pi}{4}D^2\,L\times\frac{N\times K}{2\times60} = 0.03927\text{ m}^3/\text{s}$$

Brake Torque, τ_b = 135 Nm $\quad \omega = \frac{2\pi N}{60} = 104.72$ rad/s

$$\text{Mass flow rate of fuel, } \dot{m}_f = 5\text{ kg/hr} = 0.001388\text{ kg/s}$$

$$\text{Calorific value of fuel, } (CV)_f = 42600\text{ kJ/kg}$$

(i) Brake thermal efficiency (η_{BT})

$$\eta_{BT} = \frac{\text{Brake Power}}{\text{Rate of Heat Addition}}\left(\frac{BP}{\text{HA/sec}}\right)$$

$$\text{Brake Power} = BP = T_b \times \omega = 14.137\text{ kW}$$

$$\text{Rate of Heat Addition} = \text{HA/sec} = \dot{m}_f \times CV_f = 59.167\text{ kW}$$

$$\Rightarrow \quad \eta_{BT} = 0.23893 = 23.893\%$$

(ii) Brake Mean Effective Pressure (bmep)

$$\dot{V}_s \times \text{bmep} = BP$$

$$\Rightarrow \quad \text{bmep} = \frac{BP}{\dot{V}_s} = 359.99\text{ kPa} = 3.599\text{ bar}$$

(iii) $$\text{Volumetric Efficiency, } \eta_{vol} = \frac{\text{Actual Volume }(\dot{V}_a)}{\text{Swept Volume }(\dot{V}_s)}$$

Applying Bernoulli's equation between inlet and outlet of orifice (Assuming Air-flow to be incompressible)

$$\frac{P_1}{\rho_a g}+\frac{C_1^2}{2g}+Z_1 = \frac{P_2}{\rho_a g}+\frac{C_2^2}{2g}+Z_2$$

Let air velocity at inlet of orifice, $C_1 = 0$

Assume constant datum heat $\Rightarrow Z_1 = Z_2$

$$\Rightarrow \qquad C_2 = \sqrt{2\left(\frac{P_1-P_2}{\rho_a}\right)}$$

Initial conditions: Inlet pressure, P_1 = 1 bar = 100 kPa

Inlet temperature, T_1 = 300 K

$$\Rightarrow \qquad P_1 = \rho_a R T_1 \Rightarrow \rho_a = \frac{P_1}{RT_1} = 1.16 \text{ kg/m}^3$$

Head across orifice,

Head in terms of water column = h_w = 6 cm

$$\text{Head in terms of air column} = h_a = \frac{h_w \times \rho_w}{\rho_a} = \frac{6\times 1000}{1.16} = 5172.413 \text{ cm}$$

$$h_a = 51.724 \text{ m}$$

$$\Rightarrow \qquad \frac{P_1-P_2}{\rho_a g} = h_a = 51.724$$

$$C_2 = \sqrt{2g\,h_a} = 31.856 \text{ m/s}$$

$$\text{Air flow rate through orifice} = \dot{m}_a = c_d \times \rho_a \times A \times C_2 = 0.0382 \text{ kg/s}$$

$$\text{Actual volume } \dot{V}_a = \frac{\dot{m}_a}{\rho_a} = 0.0329 \text{ m}^3\text{/s}$$

$$\eta_{vol} = \frac{\dot{V}_a}{\dot{V}_s} = 0.8386 = 83.86\%$$

Q.4 A six-cylinder, 4-stroke petrol engine has a swept volume of 3 litres with a compression ratio of 9.5. Brake output torque is 205 N-m at 3600 r.p.m. Air enters at 85 N/m² and 60°C. The mechanical efficiency of the engine is 85% and air-fuel ratio is 15 : 1. The heating value of fuel is 44,000 kJ/kg and the combustion efficiency is 97%. Calculate:

(i) Rate of fuel flow

(ii) Brake thermal efficiency

(iii) Indicated thermal efficiency

(iv) Volumetric efficiency

(v) Brake specific fuel consumption

[CSE (Mains) 2005 : 30 Marks]

Solution:

Given: No. of cylinders, $k = 6$, 4-stroke, petrol engine, Swept volume, $V_s = 3 \times 10^{-3}$ m³, Rate of volume swept, $\dot{V}_s = V_s \dfrac{N \times k}{2 \times 60} = 0.09 \text{ m}^3\text{/s}$, Compression ratio, $r = 9.5$, Brake output torque, T_b = 205 N-m.

Speed, N = 3600 rpm, Ambient pressure, P_1 = 85 kN/m²

Ambient temperature, T_1 = 60 + 273 = = 343 K

Mechanical efficiency, η_m = 85%

Air fuel ratio, AFR = 15 : 1

Calorific value of fuel, CV = 44,000 kJ/kg

Combustion efficiency, η_{comb} = 97% = 0.97

Ideal Gas Equation, $P_1 v_1 = RT_1$

$$\Rightarrow \quad v_1 = \frac{RT_1}{P_1} = 1.124 \text{ m}^3/\text{kg}$$

$$v_2 = \frac{V_1}{r} = 0.1183 \text{ m}^3/\text{kg}$$

Swept volume/kg mass flow rate, $v_s = v_1 - v_2 = 1.006 \text{ m}^3/\text{kg}$

For air flow, $$\dot{m}_a = \frac{\dot{V}_s}{v_s} = 0.089 \text{ kg/s} = 322.064 \text{ kg/hr}$$

(i) AFR = 15 : 1

$$\Rightarrow \quad \frac{\dot{m}_a}{\dot{m}_f} = 15 \Rightarrow \dot{m}_f = \frac{\dot{m}_a}{15} = 21.471 \text{ kg/hr}$$

(ii) Brake thermal efficiency = η_{bth}

$$\Rightarrow \quad \eta_{bth} = \frac{B.P.}{(\dot{m}_f \times CV) \times \eta_{comb}}$$

$$BP = T_b \times \omega = T_b \times \frac{2\pi N}{60} \Rightarrow B.P. = 77.283 \text{ kW} \quad = 30.36\%$$

(iii) Indicated thermal efficiency = η_{ith}

$$\eta_{ith} = \frac{I.P.}{(\dot{m}_f \times CV) \times \eta_{comb}}$$

$$\eta_{mech} = \frac{BP}{IP} \Rightarrow IP = \frac{BP}{\eta_{mech}} = 90.921 \text{ kW}$$

$$\Rightarrow \quad \eta_{ith} = 35.718\%$$

(iv) volumetric efficiency = $\eta_{vol} = \dfrac{\dot{V}_a}{\dot{V}_s}$

$$\dot{V}_a = \dot{m}_a \frac{RT_1}{P_1} = 0.103 \text{ m}^3/\text{s}$$

$$\dot{V}_s = 0.09 \text{ m}^3/\text{s}$$

$$\Rightarrow \quad \eta_{vol} = 114.53\%$$

($\eta_{vol} > 100\%$ implies that it is a supercharged engine)

(v) Brake specific fuel consumption (bsfc)

$$= \frac{\dot{m}_f}{BP} = 0.278 \text{ kg/KWhr}$$

Q.5 **Two identical petrol engines having the following specifications are used in vehicles:**
Engine 1: Swept volume = 3300 cc, Normally aspirated, bmep = 9.3 bar, rpm = 4500, Compression ratio = 8.2, Efficiency ratio = 0.5, Mechanical efficiency = 0.9, Mass of the engine = 200 kg.
Engine 2: Super charged, Swept volume = 3300 cc, bmep = 12.0 bar, rpm = 4500, Compression ratio = 5.5, Efficiency ratio = 0.5, Mechanical efficiency = 0.92, Engine mass = 220 kg.
If both the engines are supplied with just adequate quantity of petrol for the test run, determine the duration of test run so that the specific mass per kW of brake power is same for both the engines.
Calorific value of petrol = 44000 kJ/kg.
Assume both the engines operate on four stroke cycle.
Also compare two engines and suggest their applications with reasoning.

[CSE (Mains) 2010 : 20 Marks]

Solution:

Engine-I:

$$\text{Swept Volume, } V_S = 3300 \times 10^{-6}\ \text{m}^3$$

$\Rightarrow$
$$\dot{V}_s = \frac{V_s \times N}{2\times 60} = \frac{3300\times 10^{-6}\times 4500}{2\times 60} = 0.12375\ \text{m}^3/\text{sec}$$

As,
$$\text{BMEP} = 9.3\ \text{bar}$$

$\Rightarrow$
$$(BP)_I = (\text{BMEP})\times \dot{V}_s = 9.3\times 10^2 \times 0.12375\ \text{kW} = 0.12375\ \text{m}^3/\text{sec}$$

$$\eta_{\text{air - std}} = 1-\frac{1}{r^{\gamma-1}} = 1-\frac{1}{(8.2)^{0.4}} = 0.569$$

$$\text{Efficiency ratio, } \eta_{\text{ratio}} = \frac{\eta_{\text{ith}}}{\eta_{\text{air-std}}}$$

$$(\eta_{\text{ith}})_I = 0.5\times 0.569 = 0.284$$

$$(\eta_{\text{ith}})_I = \frac{I\cdot P}{\dot{m}_F \times C\cdot V} = \frac{B\cdot P/\eta_m}{\dot{m}_F\times C\cdot V}$$

$$(\dot{m}_F)_I = \frac{115.0875/0.9}{0.284\times 44000} = 36.84\ \text{kg/hr}$$

If the test duration is t hours, then specific mass per kW of brake power is

$$\text{Specific mass} = (\dot{m}_F)_I + (m_{\text{egine}})_I = (36.84\text{ t} + 200)\ \text{kg}$$

$$\frac{\text{specific mass}}{\text{kW of (BP)}_I} = \frac{36.84t+200}{115.0875} \quad \text{...(i)}$$

Engine II:

$$\text{Swept Volume, } V_s = 3300\ \text{cc}$$
$$N = 4500\ \text{rpm}$$
$$\dot{V}_s = 0.12375\ \text{m}^3/\text{sec}$$
$$\text{BMEP} = 12\ \text{bar}$$
$$(\text{B}\cdot\text{P})_{II} = 12\times 102\times 0.12375 = 148.5\ \text{kW}$$
$$(I\cdot \text{P})_{II} = \frac{148.5}{0.92} = 161.41\ \text{kW}$$
$$\eta_{\text{air-std}} = 1-\frac{1}{(5.5)^{0.4}} = 49.43\%$$
$$(\eta_{\text{ith}})_{II} = 0.493\times 0.5 = 0.2471$$
$$(\eta_{\text{ith}})_{II} = \frac{(I\cdot P)_{II}}{\dot{m}_F\times CV} = \frac{161.41}{0.2471\times 44000} = 53.44\ \text{kg/hr}$$

$$\left[\frac{\text{specific mass}}{\text{kW of (B}\cdot\text{P})_{II}}\right] = \frac{53.44t+220}{148.5} \quad \text{...(ii)}$$

From (i) and (ii)
$$\frac{36.84t+200}{115.0875} = \frac{53.44t+220}{148.5}$$
$$47.5236t + 258 = 53.44\text{ t} + 220$$
$$t = 6.42\ \text{h}$$

Application Criteria:

Engine-I: Naturally aspirated engine with more η_{bth} with less power should be used in passenger automobile for economical purposes, as less power is demanded.

Engine-II: Supercharged engine with less η_{bth} but with high power (B.P.) is recommended for high: speed application automobile for higher speed and torque.

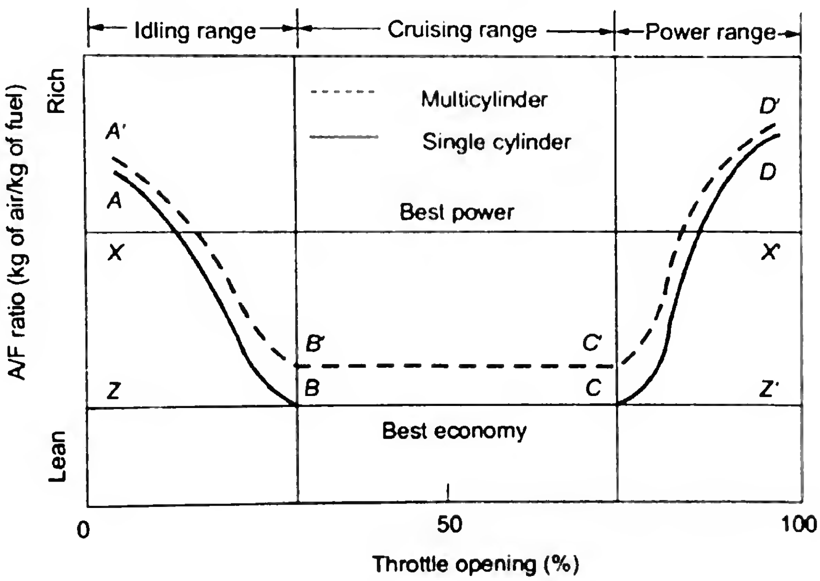
2. Combustion in S.I. and C.I. Engines

Q.6 Explain the following:
(i) Rich mixture is used during idling.
(ii) As the engine speed increases, the ignition timing should be advanced.
(iii) S.I. engines are generally not supercharged.
(iv) Two stroke C.I. engines find wide applications in marine propulsion.

[CSE (Mains) 2001 : 30 Marks]

Solution:

(i) **Rich mixture used during idling:** During idling, engine runs without load and produces power only to overcome friction between parts. Rich mixture is required in this case to sustain combustion. During idling, pressure in the intake manifold is about 20 to 25 percent of atmospheric pressure. At suction stroke, inlet valve opens and the products of combustion trapped in the clearance volume, expands in the inlet manifold. Later when piston moves downwards, the gases along with the fresh charges go into the cylinder. A very rich mixture must be supplied during idling to counteract the tendency of dilution and get an combustible mixture.



(ii) **Ignition timing should be advanced with increase in engine speed:** Ignition timing needs to be increasingly advanced (relative to top dead centre, TDC) as the engine speed increases because of the following reasons:

1. To ensure that the air fuel mixture has the correct amount of time to burn completely. As the engine speed increases, the time available to burn the mixture decreases but the burning itself proceeds at the same speed, so it needs to be started increasingly earlier to complete in time.
2. The correct timing advance for a given engine speed will allow for maximum cylinder pressure to be achieved at correct crankshaft angular position.
3. Poor volumetric efficiency at high engine speed also requires increased advancement of ignition timing.

(iii) **SI engines are generally not supercharged:** Supercharging is forced induction in which the density of air charge is raised before it enters the cylinder, thus inducting the increased mass of air and then compressing it in each cylinder. The increased mass of the inducted charge into the cylinder raises both the temperature and density of the charge at the time of ignition. Moreover the supercharging increases the temperature and pressure of the inducted air .As a result, the temperature of the unburnt mixture in the

cylinder might exceed the self-ignition temperature of the fuel and might remain at or above this temperature during the period of pre-flame reactions resulting in auto-ignition ,collision of flame fronts and knocking. Knocking may cause loss of engine power, damage to engine components and structure and might lead to complete engine failure.

(iv) **Two stroke engines find wide applications in marine propulsion because of the following reasons:**

1. A two stroke engine provides a working stroke in every revolution. Hence a more uniform turning moment is obtained on the crankshaft. Therefore the mass of the flywheel used is lesser.
2. For the same power output, less space is occupied by a two stroke engine.

Q.7 A 4-stroke petrol engine of 2 litres capacity is to develop maximum power at 4000 rpm. The volumetric efficiency at this speed is 0.75 and the air-fuel ratio is 14 : 1. The venturi throat diameter is 28 mm. The coefficient of discharge of venturi is 0.85 and that for fuel jet is 0.65. Calculate:

(i) the air velocity at the throat and (ii) the diameter of the fuel jet.

The specific gravity of petrol is 0.76. Atmospheric pressure and temperature are 1 bar and 17°C respectively.

[CSE (Mains) 2002 : 40 Marks]

Solution:

Given: 4 stroke, petrol engine, Swept volume = $V_s = 2\ l = 0.002\ m^3$, Speed, $N = 4000$ rpm

Rate of volume swept, $\dot{V}_s = V_s \times \frac{N}{2\times 60} = 0.0667\ m^3/s$, Volumetric efficiency, $\eta_{vol} = 0.75$

$$\frac{\text{Actual volume}}{\text{Swept volume}} = 0.75$$

$$\Rightarrow \quad \frac{\dot{V}_a}{\dot{V}_s} = 0.75 \Rightarrow \dot{V}_a = 0.75 \times \dot{V}_s$$

$$\dot{V}_a = 0.05\ m^3/s$$

$$\text{Air fuel ratio} = AFR = \frac{\dot{m}_a}{\dot{m}_f} = 14$$

Venturi (Air-flow):

Venturi throat diameter, d_v = 28 mm = 0.028 m

Area of throat, $A = (\pi/4)\ d_v^2 = 6.157 \times 10^{-4}\ m^2$

Coefficient of discharge of venturi, $C_{dv} = 0.85$

Coefficient of discharge of fuel jet, $C_{df} = 0.65$

Density of petrol, $\rho_f = 760\ kg/m^3$

Inlet pressure, P_1 = 1 bar = 100 kPa

Inlet temperature, T_1 = 290 K

(i) Ideal equation, $P_1\dot{V}_a = \dot{m}_a R T_1$

$\Rightarrow$ Actual mass flow rate = $\dot{m}_a = 0.06$ kg/s = 216 kg/hr

$\Rightarrow$ AFR = 14

$$\dot{m}_f = \frac{\dot{m}_a}{AFR} = 15.447\ \text{kg/hr}$$

Assuming flow of air to be incompressible,

$$\dot{m}_a = C_{dv}(\rho_a \times A_v \times C_2) \Rightarrow C_2 = \frac{\dot{m}_a}{C_{dv}\rho_a A_v}$$

ρ_a = density of inlet air

$P_1 = \rho_a R T_1$

$$\Rightarrow \quad \rho_a = \frac{P_1}{RT_1} = 1.2 \text{ kg/m}^3$$

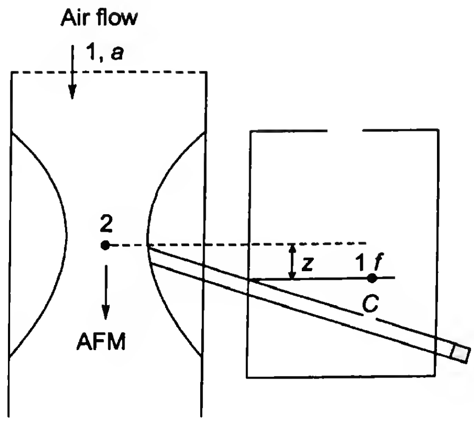
$$\Rightarrow \quad C_2 = 95.413 \text{ m/s}$$

(ii) Using Bernoulli's equation for air-flow

$$\frac{P_1}{\rho_a g} + \frac{C_1^2}{2g} + Z_1 = \frac{P_2}{\rho_a g} + \frac{C_2^2}{2g} + Z_2$$

$$Z_1 = Z_2$$

$$C_1 \simeq 0$$



$$\Rightarrow \quad C_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho_a}}$$

$$\Rightarrow \quad P_1 - P_2 = 5462.18 \text{ Pa}$$

$$\text{Velocity of fuel jet} = C_f = \sqrt{\frac{2(P_1 - P_2)}{\rho_f}} = 3.79 \text{ m/s}$$

$$\dot{m}_f = C_{df} \times \rho_f \times A_f \times C_f$$

$$\Rightarrow \quad A_f = 2.286 \times 10^{-6} \text{ m}^2$$

$$\Rightarrow \quad \frac{\pi}{4} d_f^2 = 2.286 \times 10^{-6} \text{ m}^2$$

$$\Rightarrow \quad d_f = 1.706 \times 10^{-3} \text{ m}$$

$$\text{diameter of fuel jet, } d_f = 1.706 \text{ mm}$$

Q.8 Discuss why a carburettor is being replaced by an injection system in S.I. engine these days.

[CSE (Mains) 2002 : 20 Marks]

or

Discuss the advantages of using multi-point fuel injection system in place of conventional carburettor.

[CSE (Mains) 2005 : 10 Marks]

Solution:

Fuel-injection system is the most vital component in the working of SI engines as engine performance such as power output, economy etc. is dependent on the effectiveness of the fuel-injection system.

Though, both carburettor and fuel-injection system serve the common purpose of preparation of combustible charge, but their working is different. Carburettor is replaced by fuel injection systems because of the following reasons:

1. Carburettor utilizes the relative air speed with respect to fuel speed at the fuel nozzle to atomize the fuel, thus the amount of fuel delivered being dependent on air velocity in the venturi.

   In case of fuel-injection system, the fuel speed at the point of delivery is greater than the air speed to atomize the fuel. The amount of fuel delivered into engine is controlled by pump which forces the fuel under pressure.
2. A fuel injection system enables accurate metering of the fuel injected per cycle and can handle efficiently small quantities of fuel.
3. The fuel injection system offers smoother and more consistent transient throttle response such as during quick throttle transitions, easier cold starting, more accurate adjustment to account for extremes of ambient temperatures and changes in air pressures.
4. It provides more stable idling, decreased maintenance needs and better fuel efficiency.
5. Fuel injection also dispenses with the need for a separate mechanical choke, which on carburettor-equipped vehicles must be adjusted as the engine warms up to the normal temperature.

6. In SI engines, fuel injection has the advantage of being able to facilitate stratified combustion which is not possible with carburettors.
7. Multi-point fuel injection system allows certain engine configurations such as inline five cylinder gasoline engines have become more feasible for mass production. The traditional carburettor arrangement with single or turn carburettors could not provide even fule distribution between cylinders, unless a more complicated, individual carburettor per cylinder is used.
8. Fuel injection systems are also able to operate normally regardless of orientation, whereas carburettors with floats are not able to operate upside down or in zero gravity, such as encountered on airplanes.

Q.9 Describe various stages of combustion in C.I. engine by illustrating on pressure-crank angle diagram. Discuss the effect of following operating variables on the first stage of combustion:
(i) ignition advance (ii) compression ratio (iii) speed

[CSE (Mains) 2003 : 20 Marks]

or

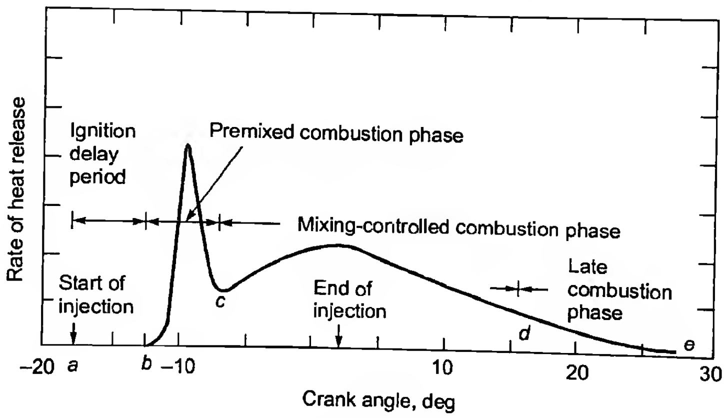
Explain the various stages of combustion in a diesel engine.

[CSE (Mains) 2005 : 15 Marks]

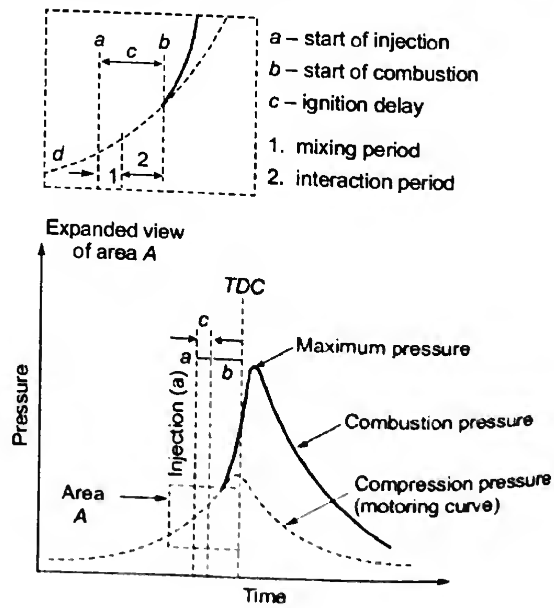
Solution:

Stages of combustion in C.I. engine

- Ignition delay period
- Premixed combustion phase (period of rapid combustion)
- Mixing-controlled combustion phase
- Late combustion phase



Ignition delay period : The period between the start of fuel injection into the combustion chamber and the start of combustion is termed as ignition delay period.



The definite period of inactivity between the time when the first droplet of fuel hits the hot air in the combustion chamber and the time it starts through the actual burning phase. The start of combustion is determined from the change in slope on the p ~ α diagram, or from heat release analysis of the p(α) data, or from luminosity detector in experimental conditions.

Both physical and chemical processes must take place before a significant fraction of the chemical energy of the injected liquid fuel is released.

Physical delay: It is the time between the beginning of injection and attainment of chemical reaction conditions. During this period, the fuel is atomised, vaporised, mixed with air and raised to its self-ignition temperature.

The physical delay is greatly reduced by using high injection pressures, high combustion chamber temperatures and high turbulence to facilitate breakup of jet and improving evaporation.

Chemical delay: The chemical processes are the precombustion reactions of the fuel, air and residual gas mixture which lead to auto ignition. In this phase, the reactions start slowly and then accelerate until inflammation or ignition take place.
Chemical delay is more accurate measure for the duration of the ignition delay period.
Ignition delay period is in the range of:
0.6 to 3 ms for low-compression ratio CI diesel engines
0.4 to 1 ms for high-compression ratio, turbocharged CI diesel engines

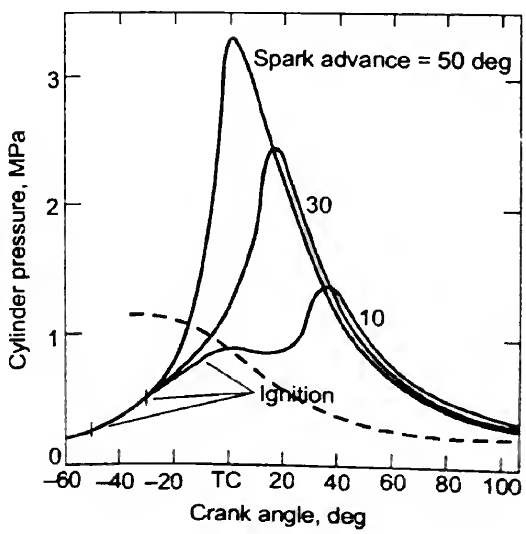
Premixed combustion phase or period of rapid combustion: Combustion of the fuel, which has mixed with air within flammability limits during ignition delay period, occurs rapidly in a few crank angle degrees. This phase is characterised by high heat release. If the amount of fuel collected in the combustion chamber during the ignition delay is large - high heat release rate results in a rapid pressure rise, which might also cause diesel knock. For fuels with low cetane number, with long ignition delay, ignition occurs late in the expansion stroke accompanied by incomplete combustion, reduced power output, poor fuel conversion efficiency.

Mixing-controlled combustion phase: Once the fuel and air which is pre-mixed during the ignition delay is consumed, the burning rate (heat release rate) is controlled by the rate at which mixture becomes available for burning. The rate of burning in this phase is mainly controlled by the mixing process of fuel vapour and air. Liquid fuel atomization, vaporization, pre-flame chemical reactions also effect the rate of heat release. Heat release rate sometimes reaches a second peak (which is lower in magnitude) and then decreases as the phase progresses. Generally it is desirable to have the combustion process near the TDC for low particulate (soot) emissions and high performance (and efficiency). This period is assumed to end at maximum cycle temperature.

Late combustion phase: Heat release rate continues at a lower rate into the expansion stroke - there are several reasons for this: a small fraction of the fuel may not yet be burnt, a fraction of the energy is present in soot and fuel-rich combustion products are not released. The cylinder charge is non-uniform and mixing during this phase promotes more complete combustion and less dissociated product gases. Kinetics is slower.

Effect of operating variables on first stage of combustion:

1. **Ignition Advance:** Maximum compression from the piston occurs at TDC. Increasing the spark advance makes the end of combustion crank angle approach TDC and thus gets higher pressure and temperature in the unburned gases just before burnout.



2. **Compression ratio:** The higher compression ratio increases the pressure and temperature ratio of the mixture and decreases the concentration of residual gases. All these factors reduce the ignition lag and help to speed up the second phase of combustion. The maximum pressure of the cycle as well as mean effective pressure of the cycle increases with increase in compression ratio of the cycle. Higher compression ratio increases the surface to volume ratio and thereby increases the part of the mixture which after-burns in the third phase.
3. **Engine Speed:** The turbulence of the mixture increases with an increase in engine speed. For this reason, the flame speed almost increases linearly with engine speed. If the engine speed is doubled, flame to traverse the combustion chamber is halved. Double the original speed and half the original time, time

required for flame give the same number of crank degrees for flame propagation. The crank angle required for the flame propagation, which is main phase of combustion, will remain almost constant at all speeds. This is an important characteristic of all petrol engines. However at high engine speeds there is less heat loss so the unburned gas temperature is higher which promotes auto-ignition.

Q.10 The venturi of a simple carburettor has a throat diameter of 20 mm and the fuel orifice has a diameter of 1.12 mm. The level of petrol surface in the float chamber is 6.0 mm below the throat venturi. Coefficient of discharge for venturi and fuel orifice are 0.85 and 0.78 respectively. Specific gravity of petrol is 0.75. Calculate

(i) the air-fuel ratio for a pressure drop of 0.08 bar,

(ii) petrol consumption in kg/hr and

(iii) the critical air velocity.

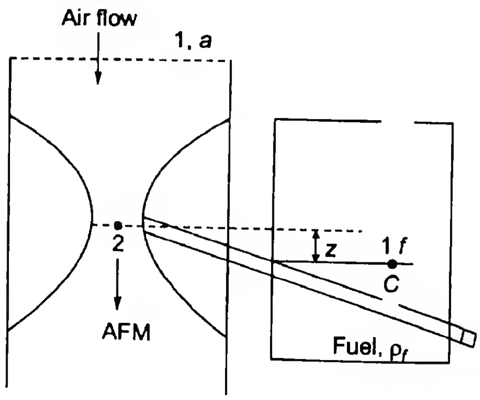
The intake conditions are 1.0 bar and 17°C. For air C_p = 1.005 and C_v = 0.718 kJ/kg-k.

[CSE (Mains) 2003 : 30 Marks]

Solution:

Given: Throat diameter, D_t = 20 mm, Orifice diameter, d_0 = 1.12 mm, z = 6 mm, Coefficient of discharge for venturi, $C_{d,t}$ = 0.85, Coefficient of discharge for fuel orifice = $C_{d,o}$ = 0.78, Pressure drop = ΔP = 0.08 bar, Density of fuel, ρ_f = 750 kg/m³

Density of air = ρ_a $\qquad \rho_a = \dfrac{P_1}{RT_1} = 1.201 \text{ kg/m}^3$



(i)
$$\text{AFR} = \frac{\dot{m}_a}{\dot{m}_f} = \frac{C_{d,t} \times A_t \times \sqrt{2(\Delta P)\rho_a}}{C_{d,0} \times A_0 \times \sqrt{2P_f\,[\Delta P - \rho_f gz)}}$$
$$= 14.496$$

(ii)
$$\dot{m}_f = C_{d,0} \times A_0 \times \sqrt{2\rho_f\,[\Delta P - \rho_f gz)}$$
$$= 2.552 \times 10^{-3} \text{ kg/sec} = 9.189 \text{ kg/hr}$$

(iii) For critical velocity $m_{f\,\text{sec}}$ = 0

i.e.
$$(\Delta P)_{\text{critical}} = \rho_f gz = 44.145 \text{ N/m}^2$$
$$(C_{2,\text{critical}})_{\text{air}} = \sqrt{\frac{2 \times (\Delta P)_{\text{critical}}}{\rho_a}} = 8.577 \text{ m/sec}$$

Q.11 Determine the air-fuel ratio at 6000 m altitude in a carburettor adjusted to give an air-fuel ratio of 15 : 1 at sea level where air temperature is 27°C and pressure is 1.013 bar.

The temperature of air decreases with altitude and is given by the expression $t = t_s - 0.0065\,h$ where h is height in metres and t_s is the sea level temperature in °C.

The air pressure decreases with altitude as per the relation

$$h = 19220 \log_e\left(\frac{1.013}{P}\right), \text{ where } P \text{ is in bar.}$$

[CSE (Mains) 2004 : 20 Marks]

Solution:

Given: Height, h = 6000 m, Air fuel ratio, AFR = 15 : 1, Ambient temperature, T_1 = 300 K, Ambient pressure, P_1 = 1.013 bar = 101.3 kPa

The temperature of air decreases with altitude as given by the relation.

$$t_h = t_s - 0.0065\,\text{h} = -12°\text{C}$$
$$\Rightarrow \quad T_h = 273 - 12 = 261 \text{ K}$$

The air pressure decreases with altitude as given by the relation.

$$h = 19220 \log_e\left(\frac{1.013}{P_h}\right)$$

$$\Rightarrow \quad 6000 = 19220 \log_e\left(\frac{1.013}{P_h}\right)$$

$$\Rightarrow \quad P_h = 0.73185 \text{ bar} = 73.185 \text{ kPa}$$

$$\text{AFR } \alpha \sqrt{\text{density of air}}$$

$$\Rightarrow \quad \text{AFR } \alpha \sqrt{\rho_{air}}$$

$$\Rightarrow \quad \frac{\text{AFR}_h}{\text{AFR}_s} = \sqrt{\frac{\rho_{air,h}}{\rho_{air,h}}} = \sqrt{\frac{\frac{P_h}{RT_h}}{\frac{P_s}{RT_s}}} = 0.91127$$

$$\text{AFR}_h = 13.669$$

Q.12 An 8-cylinder, 4-stroke diesel engine has a power output of 368 kW at 800 RPM. The fuel consumption is 0.238 kg/kW-hr. The pressure in the cylinder at the beginning of injection is 35 bar and the maximum cylinder pressure is 60 bar. The injector is adjusted to operate at 210 bar and the maximum pressure in the injector is set at 600 bar. Calculate the orifice area required per injector if the injection takes place over 12° crank angle. Assume the coefficient of discharge for the injector = 0.6, specific gravity of fuel = 0.85 and the atmospheric pressure = 1.013 bar. Take the effective pressure difference to be the average pressure difference over the injection period.

[CSE (Mains) 2004 : 30 Marks]

Solution:

Given: No. of cylinders, k = 8, 4-stroke (Diesel engine), Brake power, BP = 368 kW, Speed, N = 800 rpm $\Rightarrow \omega = 2\pi N/60 = 83.775$ rad/s

Brake specific fuel consumption bsfc = 0.238 kg/kw-hr = 6.611×10^{-5} kg/kJ

Brake specific fuel consumption = Mass flow rate of fuel required to produce unit power.

$$\Rightarrow \quad (\text{bsfc}) = \frac{\dot{m}_f}{\text{BP}}$$

$$\Rightarrow \quad \dot{m}_f = bsfc \times \text{BP} = 0.0243 \text{ kg/s}$$

Pressure in cylinder at the beginning of injection, $P_{cyl,\,1}$ = 35 bar

Maximum pressure in cylinder at the beginning of injection, $P_{cyl,\,2}$ = 60 bar

Mean pressure in the cylinder, $P_{cyl,\,m} = \dfrac{P_{cyl,\,1} + P_{cyl,\,2}}{2} = 47.5$ bar = 4750 kPa

Operating pressure in injector, $P_{inj,\,1}$ = 210 bar

Maximum pressure in injector, $P_{inj,\,2}$ = 600 bar

Mean pressure in injector, $P_{inj,\,m} = \dfrac{P_{inj,\,1} + P_{inj,\,2}}{2} = 405$ bar = 40500 kPa

Crank angle at the time of injection, θ_{inj} = 12°

Coefficient of discharge for injector, C_d = 0.6

Specific gravity of fuel = 0.85

Density of fuel, ρ_f = 850 kg/m^3

Atmosphere pressure = 1.013 bar

Let area of injector orifice, A_0

Fuel flow rate during injection, = $\dot{m}_{inj} = C_d \times \rho_f \times A_0 \times C_{2f}$

Apply Bernoulli's equation between the injector and cylinder

$$\frac{P_{inj,m}}{\rho_{fg}}+\frac{C_1^2}{2g}+Z_1 = \frac{P_{cyl,m}}{\rho_{fg}}+\frac{C_2^2}{2g}+Z_2$$

Assume constant datum ⇒ $Z_1 = Z_2$

Let inlet velocity is negligible ⇒ $C_1 \simeq 0$

⇒ $$C_2 = \sqrt{\frac{2(P_{inj,m}-P_{cyl,m})}{\rho_f}} = 290 \text{ m/s}$$

⇒ $$\dot{m}_{inj} = 0.6\times 850\times A_0\times 290 = 147{,}900\,A_0$$

$$\text{Time of injection} = t_{inj} = \frac{\theta_{inj}\times\frac{\pi}{180}}{\omega} = 2.5\times10^{-3}\text{ s}$$

$$\text{Cycle time} = t_{cycle} = \frac{2\times360°\times\frac{\pi}{180}}{\omega} = 0.158$$

[As crank completes 2 rotations in a cycle in 4-stroke engine]

Applying mass conservation, $\dot{m}_{inj}\times t_{inj} = \dot{m}_f\times t_{cycle}$

⇒ $147{,}900\,A_0\times 2.5\times 10^{-3} = 0.0243\times 0.15$

⇒ $A_0 = 9.858\times 10^{-6}\text{ m}^2 = A_0 = 9.858\text{ mm}^2$

∴ Area of injector orifice required $= 9.858\text{ mm}^2$

Q.13 Describe the different phases of combustion in a S.I. engine. How should the following factors be changed to reduce the tendency of knock in a S.I. engine:

(i) Compression ratio

(ii) Distance of flame travel

(iii) Inlet mixture temperature

(iv) Turbulence

Justify your answer with proper reasoning.

[CSE (Mains) 2004 : 30 Marks]

Solution:

Different phases of combustion in S.I. engine are:

First stage (Ignition lag): It is referred to as ignition lag or preparation phase in which the growth and development of a self-propagating nucleus of flame takes place. This is a chemical process depending upon both temperature and pressure, nature of fuel and the proportion of the exhaust residual gases. It also depends upon the relationship between the temperature and rate of reaction.

Second stage (Propagation of flame): This stage is marked by spread of the flame throughout the combustion chamber. The starting point of the stage is where the first measurable rise of pressure is seen on the indicator diagram.

During the second stage, the flame propagates at a constant velocity. Heat transfer to the cylinder wall is low, because only a small part of the burning mixture comes in contact with the cylinder wall during this period. During this stage, the combustion chamber volume remains constant, hence rate of pressure rise is proportional to the rate of heat release.

Third stage (Afterburning): The onset of this stage is taken as the instant at which maximum pressure is reached on the indicator diagram.

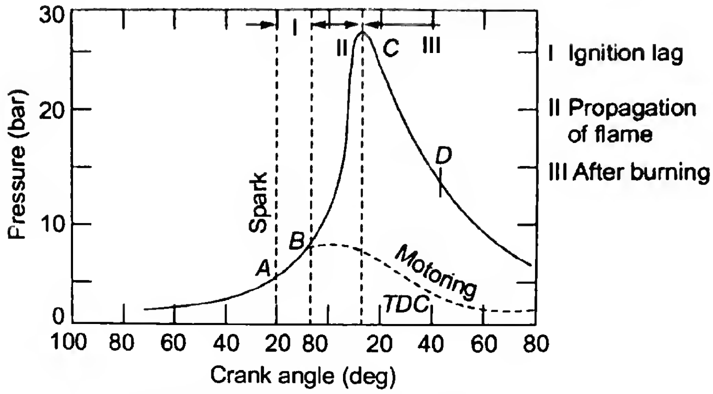
The flame velocity decreases during this stage.

The rate of combustion becomes low due to lower flame velocity and reduced flame front surface.

No pressure rise takes place during this stage as the expansion stroke starts before this stage of combustion.

Effect of variables on Knock:

(i) **Compression Ratio:** When compression ratio increases, pressure and temperature increases and an overall increase in the density of the unburned mixture is observed. The increase in temperature reduces the delay period of end gas which increases the knock tendency.



Stages of Combustion in S.I. Engine

(ii) **Distance of flame travel:** If the distance of flame travel is more, the possibility of knocking increases as the time of exposure of the unburned mixture to auto-ignition conditions increases.

Hence, combustion chambers are made as spherical as possible to minimize length of flame travel.

(iii) **Inlet mixture temperature:** An increase in the inlet temperature of mixture makes the compression temperature higher, thus increasing the temperature of unburnt mixture and raising the possibility of knock.

(iv) **Turbulence:** Increase of turbulence increases the flame speed and reduces the time available for the end charge to reach auto-ignition condition. This reduces the knocking tendency.

Q.14 A 4-stroke petrol engine has a swept volume of 2.0 litres and is running at 4000 r.p.m. The volumetric efficiency at this speed is 0.75 and the air-fuel ratio is 14 :1. The venturi throat diameter of the carburettor fitted to the engine is 30 mm. Estimate the air velocity at the throat if the discharge coefficient for air is 0.9. The ambient conditions are: pressure = 10 bar, temperature = 20°C. Calculate the diameter of the fuel jet if the fuel density is 760 kg/m³. For air C_p = 1.005 kj/kg K and R = 287 J/kg K. Assume C_{df} = 1.0.

[CSE (Mains) 2005 : 30 Marks]

Solution:

Given: 4-stroke, petrol engine, Swept volume, $V_s = 2 \times 10^{-3}$ m³, Speed, N = 4000 rpm,

$$\text{Rate of volume swept, } \dot{V}_s = \frac{V_s \times N}{2 \times 60} = 0.0667 \text{ m}^3/\text{s}$$

$$\text{Volumetric efficiency, } \eta_{vol} = 0.75$$

$$\eta_{vol} = \frac{\text{Rate of actual air flow}}{\text{Rate of volume swept}} \Rightarrow \eta_{vol} = \frac{\dot{V}_a}{\dot{V}_s}$$

$$\Rightarrow \quad \dot{V}_a = \dot{V}_s \times \eta_{vol} = 0.05 \text{ m}^3/\text{s}$$

$$\text{Ambient pressure, } P_1 = 100 \text{ kPa}$$

We know, Ambient temperature, $T_1 = 20 + 273 = 293$ K

Ideal gas equation, $$P_1\dot{V}_a = \dot{m}_a R T_1$$

$$\Rightarrow \quad \dot{m}_a = \frac{P_1\dot{V}_a}{RT_1} = 0.059 \text{ kg/s}$$

$$\Rightarrow \quad \dot{m}_a = 214.054 \text{ kg/h}$$

$$\text{Air fuel ratio} = \text{AFR} = \frac{\dot{m}_a}{\dot{m}_f} = \frac{14}{1}$$

$$\Rightarrow \quad \dot{m}_f = \frac{\dot{m}_a}{\text{AFR}} = 15.289 \text{ kg/h}$$

$$\text{Density of fuel, } \rho_f = 760 \text{ kg/m}^3$$

$$\text{Density of air, } \rho_a = \frac{\dot{m}_a}{\dot{V}_a} = 1.18 \text{ kg/m}^3$$

Let cross-section area of fuel jet = A_f

Velocity of fuel jet = $C_{2,f}$

Coefficient of discharge of fuel jet = $C_{d,f} (= 1)$

$$\dot{m}_f = C_{d,f} \times \rho_f \times A_f \times C_{2,f}$$

$$\Rightarrow \quad A_f = \frac{\dot{m}_f}{C_{d,f} \times \rho_f \times C_{2,f}} \Rightarrow A_f = 1.797 \times 10^{-6}\ m^2$$

$$\Rightarrow \quad \frac{\pi}{4} d_f^2 = A_f$$

$$d_f = 1.513\ mm$$

$\Rightarrow$ Diameter of fuel jet = 1.513 mm

Coefficient of discharge of air, $C_{da} = 0.9$

Venturi throat diameter, d_v = 30 mm = 0.03 m

Area of venturi throat, $A_v = \frac{\pi}{4} d_v^2 = 7.068 \times 10^{-4} m^2$

Let air velocity at throat, $C_{2a} = \left[\dot{m}_a = C_{d,v} \times \rho_a \times A_v \times C_{2,a} \Rightarrow C_{2,a} = \frac{\dot{m}_a}{C_{dv} \times \rho_a \times A_v}\right]$

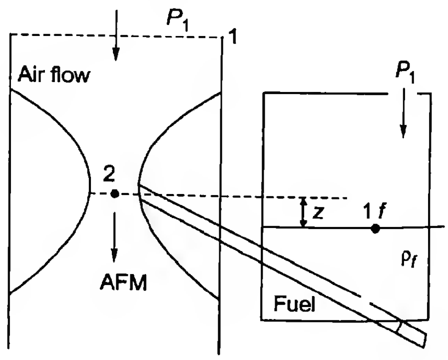
$\Rightarrow$ C_{2a} = 78.595 m/s

$\therefore$ Velocity of air at venuri throat = 78.595 m/s

Q.15 Derive an expression for air-fuel ratio delivered by a simple carburetor, neglecting the effect of compressibility. Discuss the limitations of simple carburetor. What are the modifications incorporated for its use in automotive vehicles?

[CSE (Mains) 2006 : 30 Marks]

Solution:



Assumption: 1. Incompressible flow of air ($\rho_1 = \rho_2 = \rho_a$)

$Z \rightarrow$ difference of head between fuel surface and fuel discharge.

I. Air Flow: Applying Bernoulli's equation in air-flow

$$\frac{P_1}{\rho_a g} + \frac{C_{1,a}^2}{2g} + Z_1 = \frac{P_2}{\rho_a g} + \frac{C_{2,a}^2}{2g} + Z_2$$

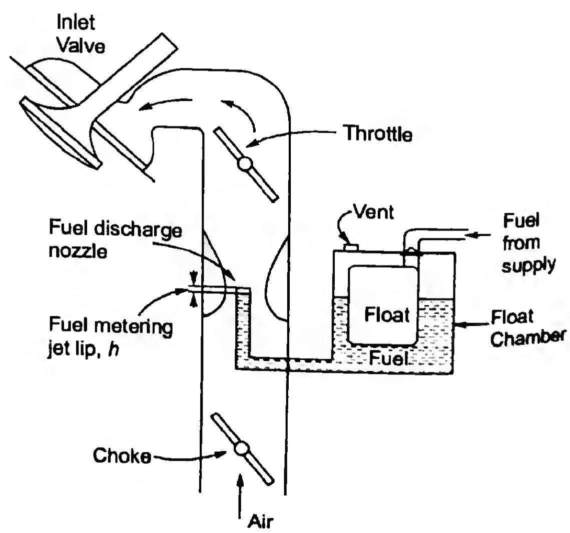
As, $C_1 << C_2$, $Z_1 - Z_2 \rightarrow 0$

$$\frac{C_{2,a}^2}{2g} = \frac{P_1 - P_2}{\rho_a g} \Rightarrow C_{2,a} = \sqrt{\frac{2(P_1 - P_2)}{\rho_a}}$$

$$\dot{m}_a = C_{d,v} \times \rho_a \times A_t \times C_{2,a} = C_{d,v} \times \left(\frac{\pi}{4} \times d_t^2\right) \times \sqrt{2\rho_a (P_1 - P_2)}$$

$C_{d,v} \rightarrow$ coefficient of discharge of venturi

A_t, $\rightarrow$ Area of throat; d_t = diameter of throat



II. Fuel Flow:

Applying Bernoulli's equation

$$\frac{P_1}{\rho_f g} + \frac{C_{1,f}^2}{2g} + Z_{1,f} = \frac{P_2}{\rho_f g} + \frac{C_{2,f}^2}{2g} + Z_{2,t}$$

$$\Rightarrow \quad \frac{C_2^2}{2g} = \frac{P_1 - P_2}{\rho_f g} + (Z_{1,f} - Z_{2,f})$$

$$\Rightarrow \quad C_{2,f} = \sqrt{2\frac{[(P_1 - P_2) - \rho_f g Z]}{\rho_f}}$$

$$\dot{m}_f = C_{d,0} \times \rho_f \times A_0 \times C_{2,f}$$

$$= C_{d_0} \times \left(\frac{\pi}{4} d_0^2\right) \times \sqrt{2\rho_f\left[(P_1 - P_2) - \rho_f g z\right]}$$

$C_{d,0} \rightarrow$ coefficient of discharge of fuel orifice

$A_0 \rightarrow$ area of fuel orifice; $d_0 \rightarrow$ diameter of fuel orifice

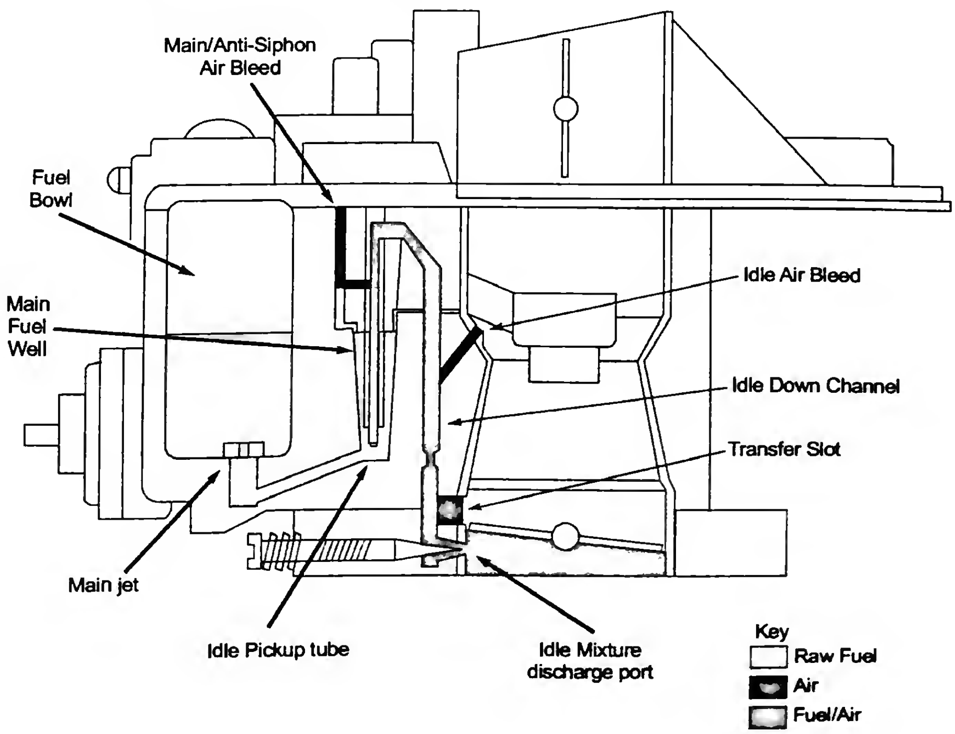
$$\text{AFR} = \frac{\dot{m}_a}{\dot{m}_f} = \frac{C_{d,v} \times A_t \times \sqrt{2\rho_a(P_1 - P_2)}}{C_{d,0} \times A_0 \times \sqrt{2\rho_f\left[(P_1 - P_2) - \rho_f g z\right]}}$$

Limitations of a Simple Carburettor

1. A simple carburettor provides the required air-fuel ratio only at one throttle position. At all other throttle positions, the mixture is either leaner or richer depending on whether the throttle is opened less or more.
2. The throttle opening changes the velocity of air. It also changes the pressure differential between the float chamber and venturi throat and regulates the fuel flow through the nozzle.
3. At very low speeds, the mixture supplied by a simple carburettor is so weak that it does not ignite properly.
4. The working of simple carburettor is affected by changes of atmospheric pressure and temperature. If the initial conditions of the device are set according to a particular ambient temperature, the air fuel mixture may become rich or lean owing to changes in the density of air .
5. In simple carburettor, the mixture is weakened when the throttle is suddenly opened because of inertia effect of the fuel which prevents the proper quantity of fuel from flowing immediately.
6. It gives the proper mixture at only one engine speed and load, therefore, suitable only for engines running at constant speed. Any increase or decrease, the quantity of fuel issuing out will change and not match the velocity of air flowing through the venturi and proper mixing will not take place.

   On city roads, where the vehicle can be operated only between 25 to 60% of throttle opening, the carburettor must be able to supply nearly constant air fuel mixture. However, the tendency of simple carburettor is to progressively enrich the mixture as the throttle starts opening. The main metering system will not be sufficient to meet these needs. Therefore, certain compensating devices are added in the carburettor along with the main metering system to supply a mixture with the required air fuel ratio.

Modification incorporated in the Carburettor:



1. **Air Bleed Jet** Air bleeds, sometimes referred to as "air jets" or "air bleeders" play a vital role in the operation of a carburettor. Air bleeds are responsible for determining the amount of air that will mix with each circuit in the metering block.

   Idle Air Bleed : The idle air bleed could be the hardest working among them all. Air to be mixed with idle fuel is provided by the idle air bleed. The idle mixture screws rely on air provided by this bleed.

   Intermediate Bleed: The intermediate bleed is found on 3 circuit carburettors only. The intermediate bleed provides air for the 3rd circuit. On most large flange carburetors this would be the bleed found in the middle.

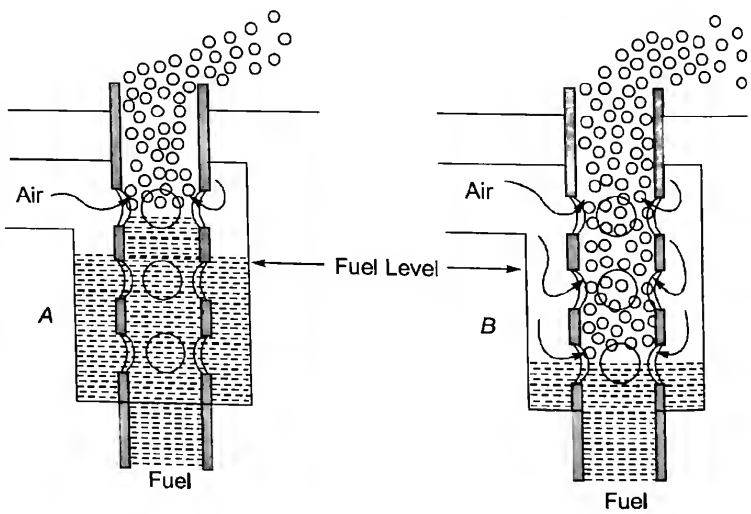
   High Speed Bleed: The high speed air bleed or also referred to as the "main bleed" correlates to the main system. The high speed air bleed controls how much air is fed to the emulsion channels of the metering block. The high speed air bleed is generally located closest to the squirter when looking at most race carburettors.

2. **Compensating Jet:** As throttle valve is opened more and more, an extra air valve, which is mounted on carburettor, opens against the force of a spring and supplies extra air to mixture. Thus throughout the economy range, the strength of mixture is kept reasonably constant.

   Restricted Air Bleed Compensation method:

   A jet tube, which has opening at its periphery, is provided with carburettor in this type of compensation. A restricted air bleed opening connects main air passage to the outer enclosure of the jet tube. At starting time and at slow speed of engine, there is small pressure due to the effect of viscosity and surface tension of the fuel, the more quantity fuel flows into the venturi gives a rich mixture when throttle valve opens more at high speed. The effect of viscosity considerably diminishes and there is a higher pressure drop in the venturi due to which more fuel is to be drawn and sprayed by the nozzle but at this stage the air bubbles starts bleeding through the jet tube opening, making a lean mixture.

3. **Emulsion Tube:** Emulsion tube in a carburettor is used to maintain the air fuel ratio at all speeds. It consists of a well with main metering jet at its bottom. The jet has holes on its sides. It is in communication with atmospheric air. Initially air is drawn through the holes into the well and petrol is emulsified. When the throttle valve is opened, the reduced throat pressure causes the emulsified petrol and mixes with incoming air and reduces richness of mixture. As speed increases, the holes in the central tube are progressively uncovered, thus maintaining air/fuel ratio.



In some carburettors compensating air is fed around an emulsion tube, in which the fuel level drops as throat velocity increases and brings extra sets of bleed holes into action.

Q.16 A simple carburetor has a venturi throat diameter of 20 mm and the coefficient of flow is 0.8. The diameter of the fuel orifice is 1.14 mm and the coefficient of fuel is 0.65. The gasoline surface is 5 mm below the throat. Calculate:

(i) the air-fuel ratio for a pressure drop of 0.08 bar when the nozzle tip is neglected;

(ii) the air-fuel ratio when the nozzle tip is taken into account;

(iii) the minimum velocity of air or critical air velocity required to start the fuel flow when the nozzle tip is provided.

Assume the density of air and fuel to be 1.20 kg/m^3 and 750 kg/m^3 respectively.

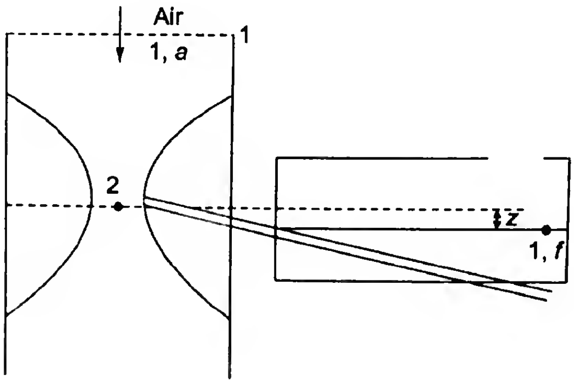
[CSE (Mains) 2007 : 20 Marks]

Solution:

Given: Venturi-throat diameter, d_v = 20 mm = 0.02 m, Coefficient of discharge through venturi, C_{dv} = 0.8, Fuel orifice diameter, d_0 = 1.14 mm = 1.14 × 10^{-3} m, Coefficient of discharge of fuel orifice, C_{d0} = 0.65
Z = 5 mm = 0.005 m

$$\Delta P = P_1 - P_2 = 0.08 \text{ bar}$$

Density of fuel, ρ_f = 750 kg/m³



For air flow, applying Bernouli's equation between 1 and 2

Density of air, ρ_a = 1.2 kg/m³

$$\frac{P_1}{\rho_a g} + \frac{C_1^2}{2g} + Z_1 = \frac{P_2}{\rho_a g} + \frac{C_2^2}{2g} + Z_2$$

$$Z_1 = Z_2 \qquad C_1 \to 0$$

$$\Rightarrow \quad C_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho_a}}$$

$$\Rightarrow \quad \dot{m}_a = C_{dv} \times A_v \times \rho_a \times C_2 = C_{dv} \times A_v \times \sqrt{2(P_1 - P_2)\rho_a}$$

(i) When nozzle tip is neglected:

$$\dot{m}_f = C_{d0} \times A_0 \times \sqrt{2(P_1 - P_2)\rho_f}$$

$$A_v = \frac{\pi}{4} d_v^2$$

$$A_0 = \frac{\pi}{4} d_0^2$$

$$\Rightarrow \quad \text{AFR} = \frac{\dot{m}_a}{\dot{m}_f} = \frac{C_{dv} \times A_v \sqrt{2(P_1 - P_2)\rho_a}}{C_{d0} \times A_0 \times \sqrt{2(P_1 - P_2)\rho_f}} = \frac{C_{dv}}{C_{d0}} \times \frac{dv^2}{d_0^2} \sqrt{\frac{\rho_a}{\rho_f}} = 15.15$$

(ii) When nozzle tip is taken into account
Apply Bernoulli between 1, f and 2

$$\Rightarrow \quad \frac{P_{1f}}{\rho_{fg}} + \frac{C_1^2}{2g} + Z_1 = \frac{P_{2f}}{\rho_{fg}} + \frac{C_2^2}{2g} + Z_2$$

$$Z_1 = 0;\ Z_2 = Z$$

$$\Rightarrow \quad C_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho_f} - gz} \qquad [\text{As } C_1 <<< C_2]$$

$$\dot{m}_f = C_{d0} \times A_0 \times \sqrt{2((P_1 - P_2) - \rho_f gz)\rho_f}$$

$$\Rightarrow \quad \text{AFR} = \frac{\dot{m}_a}{\dot{m}_f} = \frac{C_{dv} \times A_v \times \sqrt{2(P_1 - P_2)\rho_a}}{C_{d0} \times A_0 \times \sqrt{2((P_1 - P_2) - \rho_f gz)\rho_f}}$$

$$\text{AFR} = \frac{C_{dv} \times d_v^2}{C_{d0}\, d_0^2} \times \sqrt{\frac{(P_1 - P_2)\rho_a}{((P_1 - P_2) - \rho_f gz)\rho_f}} = 15.19$$

(iii) At critical velocity,

$$\dot{m}_f = 0$$

$$(\Delta P)_{critical} = \rho_f gz = 36.787 \text{ Pa}$$

$$C_{2,\,critical} = \sqrt{\frac{2 \times (\Delta P)_{critical}}{\rho_{air}}} = 7.83 \text{ m/sec}$$

Q.17 (i) Discuss the requirements of an injection system of a diesel engine.

(ii) With the help of a sketch discuss the working of common rail injection system.

[CSE (Mains) 2010 : 20 Marks]

Solution:

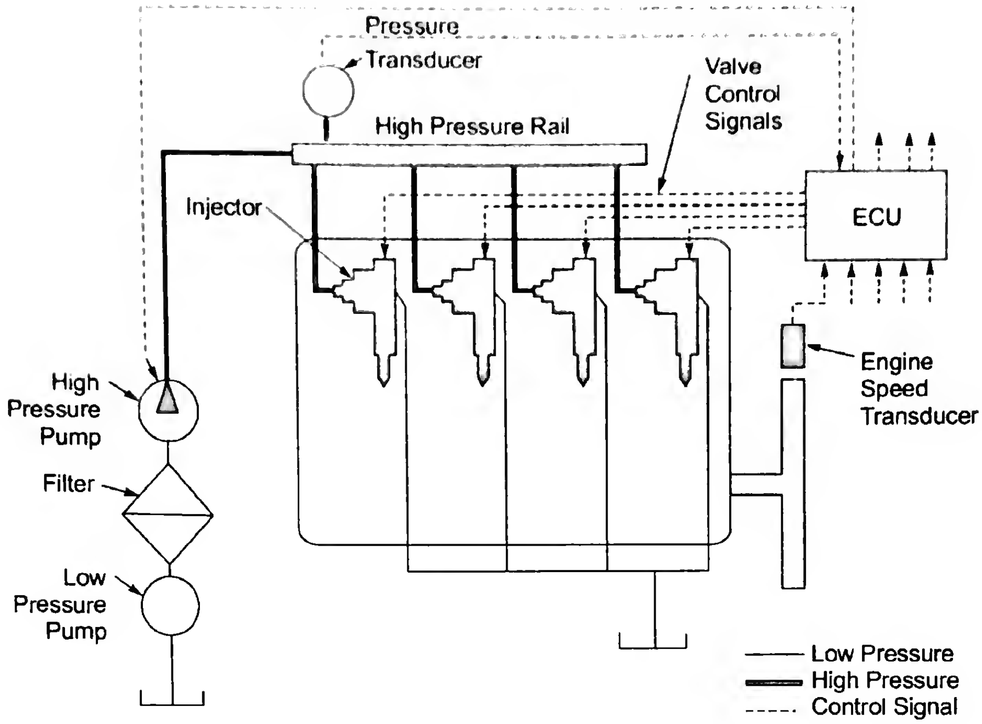
(i) Requirements of an injection system of a diesel engine are as follows:

1. The injection system should have accurate metering of fuel per cycle as small quantities of fuel are being handled. Metering errors may cause drastic variation from the desired output.
2. The quantity of fuel metered should vary to meet the varying speed and load requirements of the system.
3. The timing of fuel injection should be correct in the cycle so that maximum power is obtained ensuring fuel economy and clean burning.
4. It should provide proper control over injection rate so that the desired heat-release pattern is achieved during combustion.
5. It should atomise fuel into very fine droplets.
6. It should have proper spray pattern to ensure rapid mixing of fuel and air.
7. It should be able to provide uniform distribution of fuel droplets in the combustion chamber.
8. It should supply equal quantities of metered fuel to all cylinders in case of multi -cylinder engines.
9. It should have no lag in the beginning and end of injection to eliminate dribbling of fuel droplets into the cylinder.

(ii) **Common Rail Fuel Injection Sytems:** The development of common rail systems gained impetus in recent times due to implementation of stringent emission standards and demand of better fuel economy and engine performance.

The following are the features of this system:

1. In this system, the generation of high fuel pressure and effective fuel injection are separated.
2. The fuel pressure is independent of engine speed and load, unlike the mechanical in-line jerk and distributor pumps.
3. The high-pressure fuel is fed to a rail (or manifold) from where the fuel is supplied to the individual injectors.
4. A typical system consists of four main components:
   (a) high-pressure pump
   (b) high-pressure distribution rail and pipes
   (c) injectors
   (d) electronic control unit (ECU)
5. A mechanical pump pressurizes the fuel and feeds the common rail with the fuel at high pressure.
6. The common rail is connected to the injectors by short pipes.
7. A solenoid value in each injector controls the injection process.
8. The amount of fuel delivery is controlled by the injection pressure and opening period.
9. This system divided into two main classes:
   (a) Non-intensified systems: In non-intensified systems, the rail pressure is same as the injector pressure and no pressure intensification takes place in the injector.
   (b) Intensified systems: In intensified systems, the rail pressure is lower than the injection pressure and a stepped piston in the injector body is used to multiply the fuel pressure by a factor ranging from 3 : 1 to 10 : 1.
10. The main advantage of the common rail system over the conventional in line jerk pumps is that the injection pressure is constant and independent of engine speed and load.



Q.18 What are the effects of the following additives that are added to petrol to improve its combustion and other characteristics?

(i) Antioxidant
(ii) Corrosion inhibitor
(iii) Anti icing agent
(iv) Antiknock agent
(v) Metal deactivator

[CSE (Mains) 2012 : 10 Marks]

Solution:

(i) **Antioxidants**: These are the molecules that inhibit oxidation of other molecules. Gasoline can contain a number of unstable species such as olefins that can polymerise to form gums. Gums in the fuel are transported through fuel system and can lead to malfunction and breakdown. In the fuel system, gums are responsible for injection system fouling and are implicated in intake valve deposit formation.

Unstable species in gasoline produce free radicals which react with oxygen to produce further free radicals in a chain reaction and react with olefinic compounds to form gums. Antioxidants work by disrupting the chain propagating steps, by decomposing peroxides and by acting as free radical traps.

(ii) **Corrosion inhibitor:** The presence of sulphur in gasoline has a corrosive effect on non-ferrous metals. Corrosion inhibitors such as copper and silver act as barriers by forming a film on the metal surface and preventing the sulphur from reaching and reacting with the surface.

(iii) **Anti-icing agent:** These are additives which lower the freezing point of water based liquids. Since water has good properties as coolant, water plus antifreeze is used in IC engines to overcome the shortcomings of water as a heat transfer fluid, which can tolerate a wide temperature range. The purpose of anti-icing agent is to prevent a rigid enclosure from barsing due to expansion when water freezes. It is also used in aviation fuel to prevent formation of ice in fuel lines.

(iv) **Anti-knock agents:** In earlier times, TEL(tetra -ethyl lead)was used for reducing the knocking tendency of petrol fuel by decreasing the chance of formation of flame front from the end charge. Nowadays, TEL is replaced by EBD(ethylene-di-bromide) due to the problem of lead oxide formation using TEL.

(v) **Metal deactivator:** The presence of metals in fuels, particularly copper is associated with reduced oxidation stability. Soluble metal salts in fuel catalyse oxidation reaction with subsequent gum formation and deposit build up in fuel system or on intake valves. Metal deactivators have the ability to chelate or cage dissolved metal ions. In the stable chelate form, the metal has no pro-oxidant effect.

Q.19 (i) What is the purpose of shrouding the inlet valve in compression ignition engines?

(ii) With the help of cross-section figures and schematics show the shrouded inlet valve and masked cylinder head for producing net in-cylinder angular momentum viz., swirl in compression ignition engine.

(iii) Give two disadvantages and their effects of open combustion chambers.

[CSE (Mains) 2012 : 4 + 12 + 4 = 20 Marks]

Solution:

(i) **Purpose of shrouding the inlet valve in compression ignition engines:** Shrouding intake valve is done to generate a swirling motion of air. A suitable level of swirl is produced by rotating the valve about its axis. A shroud intake valve, increases the turbulence intensity of in-cylinder flow. When the turbulence intensity increases, the fuel air mixture becomes easier to ignite and flame propagation and flame speed is increased.

Thus, it reduces the knocking tendency by reducing the time available for end charge to auto-ignite.

(iii) **Open Combustion Chamber:** CI engine combustion chamber serves an important function of proper mixing of fuel and air through an organized air movement called air swirl which provides a high relative velocity between the fuel droplets and the air.

The air motion and turbulence in the chamber is guided by the shape of combustion chamber.

An open combustion chamber, is defined as one in which the combustion space is essentially a single cavity with little restriction from one part of the chamber to the other and hence with little pressure difference between parts of the chamber during the combustion process.

The drawbacks of these combustion chamber are:

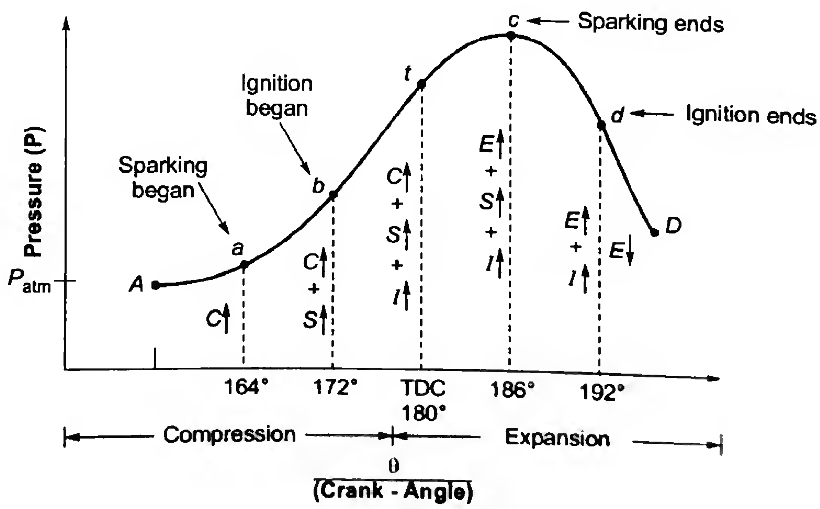
(i) High fuel-injection pressure required and hence complex design of fuel-injection pump.

(ii) Necessity of accurate metering of fuel by the injection system, particularly for small engines.

Q.20 In the pressure crank angle diagram of normal combustion SI engine, show the point of ignition, point of combustion, angle of advance, ignition lag and combustion period.

[CSE (Mains) 2013 : 10 Marks]

Solution:



C : Compression

E : Expansion

S : Sparking

I : Ignition

Point (b) : Point of ignition

Point (a) : Point of combustion

Sparking begins at 164° BTDC → angle of advance

164° BTDC to 192° ATDC → combustion period.

Q.21 How do the following parameters influence knocking in SI engine combustion?

(i) Self-ignition temperature of the fuel **(ii) Air-fuel ratio**

(iii) Dilution by residual gas **(iv) Shrouded inlet valve**

(v) Combustion chamber design

[CSE (Mains) 2013 : 10 Marks]

Solution:

(i) **Self-Ignition Temperature of the fuel:** In an SI engine, combustion is initiated between spark plug electrodes and spreads across the combustible mixture. A flame front propagates from the spark plug to the other end of the combustion chamber. Heat release due to combustion increases the temperature and pressure of the burned part of the mixture which expands and compresses the unburned mixture adiabatically, thereby increasing its temperature and pressure.

If the temperature of the unburnt mixture exceeds the self-ignition temperature of the fuel, auto-ignition might occur which leads to collision of the opposite travelling flame fronts resulting in knocking. It is required that the fuel must have a high self-ignition temperature to prevent auto-ignition and reduce the knocking tendency.

(ii) **Fuel-Air Ratio:** The maximum tendency to knock takes place for the fuel-air ratio which gives maximum reaction time. Also, the flame speeds and flame temperature are affected by fuel-air ratios. Maximum flame temperature is obtained when equivalence ratio (ϕ) $\simeq$ 1.1 to 1.2, whereas $\phi = 1$ gives minimum reaction time for auto-ignition.

In general, enriching the fuel-air ratio alters the chemical reactions during combustion, reduces the combustion temperature and increases the margin above detonation.

(iii) **Dilution by residual gas:** Energy during combustion is absorbed and if the mixture is diluted by exhaust gases, the flame propagation is reduced. As a result, the ignition lag is prolonged. Thus, longer time is required for the flame front to burn through the unburned charge and knocking tendency is reduced.

(iv) **Shrouded Inlet Valve:** Shrouded intake valve is used to generate swirling motion of air and it permits suitable level of swirl by rotating the valve about its axis. Thus, on using the shroud intake valve, turbulence intensity of the in-cylinder flow can be increased. When turbulence intensity increases, the fuel-air mixture becomes easier to ignite and flame propagation and flame speed is increased.

This results in less time available for the end charge to attain auto-ignition conditions, thereby decreasing the tendency to knock.

(v) **Combustion Chamber Design:** Compact combustion chamber results in shorter flame travel and combustion time and hence better antiknock characteristics. Therefore, combustion chambers are made as spherical as possible to minimize the length of flame travel for a given volume. The combustion chamber, in general is shaped in a way to promote turbulence as higher turbulence results in higher combustion rate and consequently less combustion time and knocking tendency.

Q.22 Differentiate between normal and abnormal combustion in SI engines. List out the three major knock limited parameters and explain its use in the engine design.

[CSE (Mains) 2015 : 10 Marks]

Solution:

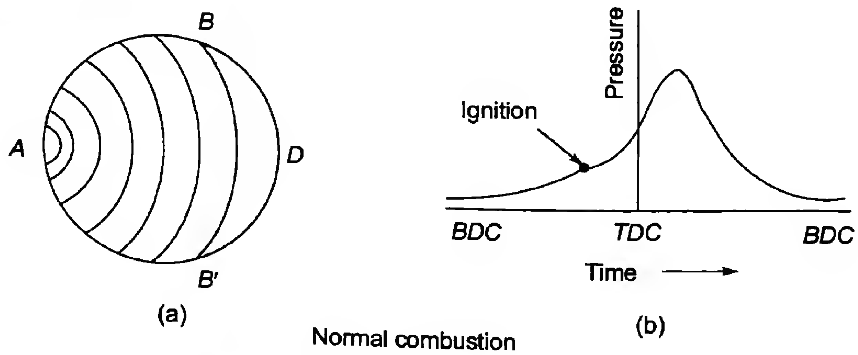
Normal Combustion: In normal combustion, spark-ignited flame moves steadily across the combustion chamber until the charge is fully consumed.

Abnormal combustion: Under certain operating conditions, the combustion deviates from its normal course leading to loss of performance and possible damage to the engine.

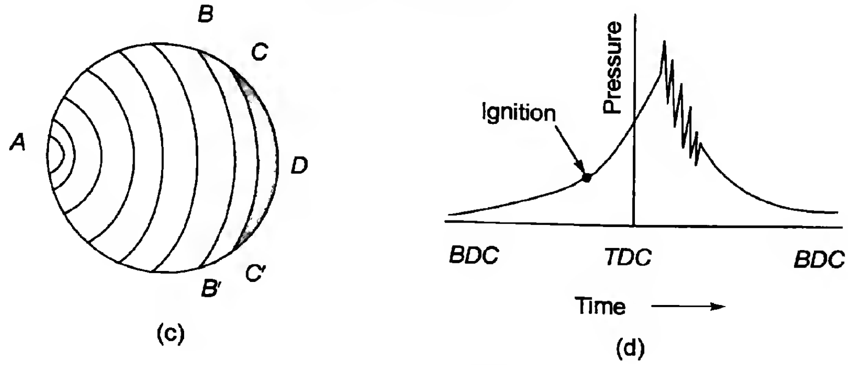
Factors responsible for abnormal combustion are: fuel composition, engine design, operating parameters and combustion chamber deposits.

There are two types of abnormal combustion:

(a) Knock (b) Surface Ignition



Normal combustion



Combustion with detonation

Three major knock limited parameters and their use in engine design:

1. **Knock Limited Compression Ratio:** The knock limited compression ratio is obtained by increasing the compression ratio on a variable compression ratio engine until incipient knocking is observed. Any change in operating conditions such as fuel-air-ratio or in the engine design that increases the knock-limited compression ratio is said to reduce the tendency towards knocking.
2. **Knock Limited Inlet Pressure:** The inlet pressure can be increased by opening the throttle or increasing supercharger delivery pressure until incipient knock is observed. An increase in knock limited inlet pressure indicates a reduction in knocking tendency.
3. **Knock Limited Indicated Mean Effective Pressure (KLIMEP):** A useful measure of knocking tendency called the performance number is defined as the ratio of KLIMEP with the fuel in question to (KLIMEP) with iso-octane when the inlet pressure is kept constant. This performance number is related to octane number and one of the advantages of this is that it can be applied to fuels with knocking. Characteristics superior to that of iso-octane i.e. extending beyond 100.

   The concept of relative performance number, rpn, is also defined as:

$$\text{rpn} = \frac{\text{Actual Performance Number}}{\text{Performance number corresponding to the imep of 100}}$$

Q.23 An engine having a single-jet carburettor consumes 6.0 kg/*h* of fuel. The density of fuel is 750 kg/m^3. The level in the float chamber is 3 mm below the top of the jet when the engine is not running. The ambient conditions are:

Pressure = 1.013 bar and Temperature = 21°C

The jet diameter is 1.2 mm and its discharge coefficient is 0.65. The discharge coefficient of air is 0.80. The air/fuel ratio is 15.3 : 1. Determine:

(i) critical air velocity; (ii) depression at the throat in mm of H_2O;

(iii) effective throat diameter.

Neglect the compressibility of air.

[CSE (Mains) 2016 : 20 Marks]

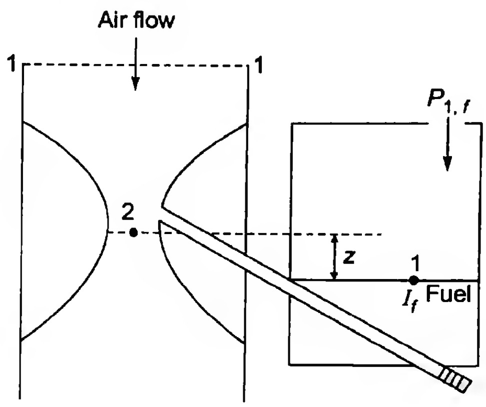
Solution:

Given: Fuel flow rate = $\dot{m}_F$ = 6 kg/h = 1.67×10^{-3} kg/s, Density of fuel = ρ_f = 750 kg/m^3, Z = 3 mm = 0.003 m, Ambient conditions Pressure = P_1 = 1.013 bar = 101.3 kPa, Temperature = T_1 = 273 + 21 = 294 k, Diameter of fuel jet = d_f = 1.2 mm = 1.2×10^{-3} m;

$$\text{Cross-section area of fuel jet, } A_F = \frac{\pi}{4}d_F^2 = 1.131 \times 10^{-6}\text{ m}^2$$

coefficient of discharge of fuel surface, $c_{df} = 0.65$

coefficient of discharge of virtual throat, $c_{d,a} = 0.8$



$$\text{Air-fuel ratio} = \text{AFR} = 15.3 = \frac{\dot{m}_a}{\dot{m}_f}$$

$\Rightarrow$ $$\dot{m}_a = AFR \times \dot{m}_f = 91.8\text{ kg/h}$$

$$\dot{m}_a = 0.0255\text{ kg/s}$$

Neglecting compressibility effect of air;

(i) For critical air velocity ; $$\dot{m}_f = 0$$

$\Rightarrow$ $$(\Delta P)_{cr} = \rho_f g z = 22.072\text{ Pa}$$

$\Rightarrow$ $$C_{2a} = \sqrt{2\left(\frac{(\Delta P)_{cr}}{\rho_a}\right)}$$

where $$\rho_a = \frac{P_1}{RT_1} = 1.2\text{ kg/m}^3$$

$\Rightarrow$ $$C_{2'a} = 6.064\text{ m/s}$$

(ii) $$\dot{m}_f = C_{df} \times \rho_F \times A_F \times C_{2'}f$$

$$C_{2f} = \sqrt{2\left(\frac{\Delta P - \rho_F g z}{\rho_f}\right)}$$

$\Rightarrow$ $$1.67 \times 10^{-3} = 0.65 \times 750 \times 1.13 \times 10^{-6} \times \sqrt{2\left(\frac{\Delta P - 750 \times 9.81 \times 0.003}{750}\right)}$$

$$\Delta P = 0.03449\text{ bar} = P_{atm} - P_t = 0.03449$$

$\Rightarrow$ $$P_t = 0.9785\text{ bar}$$

$\Rightarrow$ $$0.9785 = \rho_{H_2O} \times g \times h_{H_2O}$$

$\Rightarrow$ $$h_{H_2O} = 9.975\text{ m of }H_2O$$

$\therefore$ Head in terms of water column = 9.975 m

(iii) $$\dot{m}_a = C_{d,a} \times \rho_a \times A_t \times C_{2,a}$$

$$C_{2'a} = \sqrt{\frac{2\Delta P}{\rho_a}} = \sqrt{\frac{2 \times 0.03449 \times 10^5}{1.2}} = 75.82\text{ m/s}$$

$\Rightarrow$ $$0.0255 = 0.8 \times 1.2 \times A_t \times 75.82$$

$\Rightarrow$ $$A_t = 3.503 \times 10^{-4}\text{ m}^2$$

$\Rightarrow$ $$\frac{\pi}{4}d_v^2 = 3.503 \times 10^{-4}\text{ m}^2$$

$\Rightarrow$ $$d_v = 0.02112\text{ m} = 21.12\text{ mm}$$

Diameter of virtual throat = 21.12 mm

3. Fuel and Emission Control

Q.24 Explain Cetane rating of a fuel. Discuss its importance for the selection of fuels. Justify the range of Cetane numbers of commercial diesel fuels.

[CSE (Mains) 2002 : 20 Marks]

Solution:

Cetane Rating of Fuel: indicates the percentage by volume of normal cetane in a mixture of n-hexadecane or cetane ($C_{16}H_{34}$) and α - methyl naphthalene($C_{11}H_{10}$) which exhibit the same ignition characteristics as the test fuel when combustion is carried out under specified operating conditions.

It is an inverse function of fuel's ignition delay and the time period between start of injection and first identifiable pressure rise during combustion of fuel. Since it is a measure of ignition delay and an indicator of combustion speed of diesel fuel, it plays an important role in fuel selection.

Importance of cetane number for fuel selection: Diesel is injected in the hot compressed air in the cylinder of a diesel engine which burns to produce power. If there is a large time gap between injection and ignition, there is an unwanted accumulation of fuel in the cylinder which suddenly burns at a time with a pressure wave, producing diesel knock. Hence it is important to select a fuel with high cetane numbers to have shorter ignition delay.

Cetane number is the most widely accepted measure of fuel ignition quality as high cetane number (CN) implies:

1. Short ignition delay period: It results in more complete combustion of fuel charge in combustion chamber as ignition delay is the primary factor for auto-ignition or knock.
2. Easy ignition of fuel: It results in smoother running and better engine power and less harmful emissions
3. Cetane number also relates to how well diesel engine starts in cold weather. In diesel engine, in absence of spark plug to start ignition, the fuel with higher CN reaches ignition point more easily and makes the engine start more easily in cold weather conditions.
4. It reduces tendency of diesel knock.

   CN requirements mainly depend on:

   (a) engine design and size
   (b) speed of operation
   (c) load variations
   (d) atmospheric conditions (to some extent)

A model fuel is prepared using n-hexadecane or cetane ($C_{16}H_{34}$), which is assigned CN 100 and α-methyl naphthalene ($C_{11}H_{10}$), with CN 0. The percent by volume of n-hexadecane in a model fuel is the cetane number of the test diesel whose knocking performance matches with the model fuel when tested in a specified engine. Most modern fuels have CN between 45 and 55. It is the most optimum range to avoid diesel knock as fuels with higher cetane number which have shorter ignition delays provide more time for the fuel combustion process to be completed. Hence, higher speed diesels operate more effectively with higher cetane number fuels. There is no performance or emission advantage when the CN is raised past approximately 55.

The ascending order of cetane rating of hydrocarbons is given by:

Aromatics < iso-paraffins < naphthalene< olefins< *n*-paraffins

Following are the cetane numbers of varying grades and types of compression ignition diesel fuels:

- Regular diesel--48
- Premium diesel--55
- Biodiesel (B100)--55
- Biodiesel blend (B20)--50
- Synthetic diesel--55

Q.25 What are the different sources of emissions from an I.C. engine? Discuss the mechanism of formation of oxides of nitrogen and carbon monoxide. Explain various methods that can be used for the control of emissions of oxides of nitrogen.

[CSE (Mains) 2004 : 30 Marks]

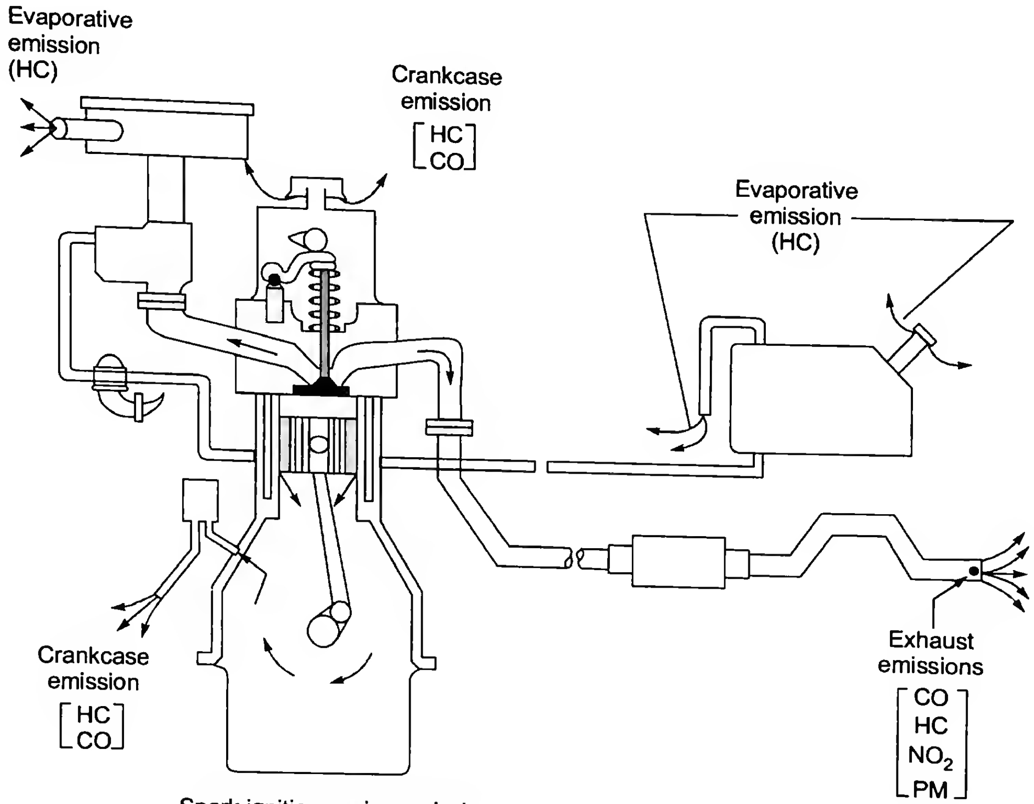
Solution:

Sources of emissions

Sources of pollutants from SI engine:

The following are the three main sources from which pollutants are emitted from the SI engine:

- **The crankcase:** Where piston blow-by fumes and oil mist are vented to the atmosphere.
- **The fuel system:** Where evaporative emissions from the carburetor or petrol injection air intake and fuel tank are vented to the atmosphere.
- **The exhaust system:** Where the products of incomplete combustion are expelled from the tail pipe into the atmosphere.



Spark ignition engine emissions

Crankcase Emission: The piston and its rings are designed to form a gas-tight seal between the sliding piston and cylinder walls. However, in practice there will always be some compressed charge and burnt fumes escape during compression and power stroke to crankcase. These gases are usually unburnt air-fuel mixture hydrocarbons, or burnt (or partially burnt) products of combustion, CO_2, H_2O (steam) or CO. These products also contaminate the lubricating oils.

Evaporative Emission: Evaporative emissions account for 15 to 25% of total hydrocarbon emission from a gasoline engine. The following are two main sources of evaporative emissions:

- The fuel tank
- The carburettor

(i) **Fuel tank losses:** The main factors governing the tank emissions are fuel volatility and the ambient temperature but the tank design and location can also influence the emissions as location affects the temperature. Insulation of tank and vapour collection systems have all been explored with a view to reduce the tank emission.

(ii) **Carburettor losses:** Although most internally vented carburettors have an external vent which opens at idle throttle position, the existing pressure forces prevent outflow of vapours to the atmosphere. Internally vented carburettor may enrich the mixture which in turn increases exhaust emission.

Exhaust Emission: The different constituents which are exhausted from S.I. engine and different factors which will affect percentages of different constituents are discussed below:

Hydrocarbons (HC): The emission amount of HC (due to incomplete combustion) is closely related to design variables, operating variables and engine condition. When the mixture supplied is lean or rich, the flame propagation becomes weak which causes in turn incomplete combustion and results in HC emission.

Carbon Mono-oxide (CO): If the oxidation of CO to CO_2 is not complete, CO remains in the exhaust. Theoretically, the petrol engine exhaust can be made free from CO by operating it at A/F ratio = 15. CO emissions are lowest during acceleration and at steady speeds. They are high during idling and reach maximum during deceleration.

Oxides of nitrogen (NO_x): Oxides of nitrogen occur mainly in the form of NO which are generally formed at high temperature. The maximum NO_x levels are observed with A/F ratios of about 10 percent above stoichiometric. Also, NO increases with increasing manifold pressure, engine load and compression ratio. This characteristic is different from HC and CO emission which is nearly independent of engine load except for idling and deceleration.

Lead emission: Lead emissions come only from S.I. engines. In the fuel, lead is present as antiknock agents in SI Engine. It may not be possible to eliminate lead completely from all petrols immediately because a large number of existing engines rely upon the lubrication provided by a lead film to prevent rapid wear of exhaust valve seats.

Mechanism of formation of NOx:

- NOx includes nitric oxide (NO) and nitrogen dioxide (NO_2).
- In SI engines, the dominant component is NO.
- It forms as a result of dissociation of molecular nitrogen and oxygen.
- Since the activation energy of the critical elementary reaction is high, the reaction is very temperature dependent.
- Therefore NO is only formed at high temperature and the reaction rate is relatively slow and becomes extremely slow at temperatures below 2000 K.
- Since the cylinder temperature changes throughout the cycle, the NO reaction rate also changes.
- Each fluid element burns to its adiabatic flame temperature (AFT) based on its initial temperature, elements that burn first near the spark plug achieve a higher temperature.
- Once the element temperature reaches 2000 K, the reaction rate becomes so slow that the NO concentration effectively freezes at a value greater than the equilibrium value.

  Reactions Involved

$$N + O_2 \longrightarrow 2\,NO$$

$$N_2 + 2\,H_2O \longrightarrow 2\,NO + 2\,H_2O$$

Mechanism of formation of CO

- CO is an intermediate product of combustion which remains in the exhaust if oxidation of CO to CO_2 is not complete.
- CO is generally formed when the mixture is rich in fuel and its amount increases as the air-fuel mixture becomes more and more rich in fuel.
- A small amount of CO comes out of the exhaust even when the mixture is lightly lean in fuel, due to the fact that equilibrium is not established when the products pass to the exhaust.

  At high temperature developed during combustion, the products formed are unstable and the following reactions take place before equilibrium is established.

  $$2H_2O + O_2 \longrightarrow 2(1-y)H_2O + 2yH_2 + yO_2$$

  where y is the fraction of H_2O dissociated.

  $$C + O_2 \longrightarrow CO_2 \longrightarrow (1-x)CO_2 + xCO + \frac{x}{2}O_2$$

As the products cool down to exhaust temperature, a major part of CO reacts with oxygen to form CO_2. A relatively small amount of CO remains in exhaust, its concentration increasing with rich mixtures.

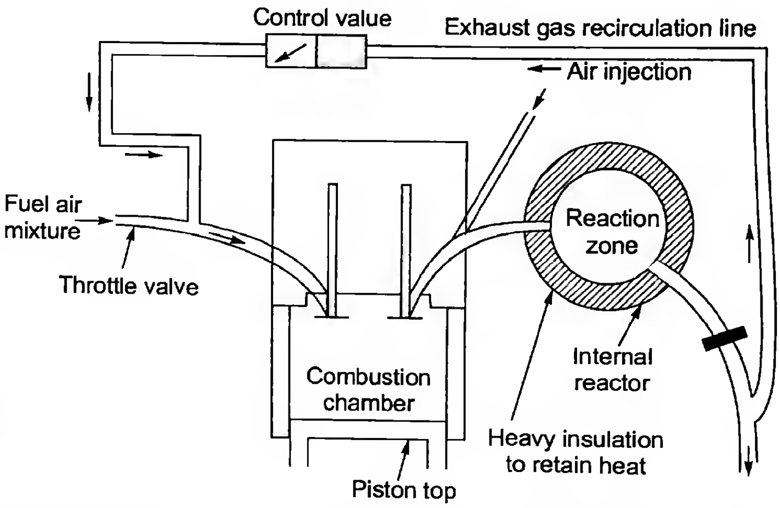
Control of emission of oxides of nitrogen: The concentration of oxides of nitrogen in the exhaust is closely related to the peak cycle temperature. The following are the three method for reducing peak cycle temperature and thereby reducing NO emission.

1. Exhaust Gas Recirculation (EGR)
2. Catalyst
3. Water injection

1. **Exhaust Gas Recirculation:** Commonly used method to reduce NOx in petrol as well as diesel engines. In SI engines, about 10 percent recirculation reduces NOx emission by 50 percent.

EGR Arrangement:

EGR works by recirculating a portion of an engine's exhaust gas back to the engine cylinder. This dilutes the O_2 in the incoming air stream and provides gases inert to combustion to act as absorbents of combustion heat to reduce peak in-cylinder temperatures.



In SI engines, the inert exhaust displaces the amount of combustible matter in the cylinder. In CI engine, the exhaust gas replaces some of the excess oxygen in the pre-combustion mixture.

This results in lower combustion chamber temperature which reduces the amount of NOx generated during combustion.

Gas reintroduced from EGR systems contain near equilibrium concentrations of NOx and CO, the small fraction initially within the combustion chamber inhibits the total net production of these.

In a typical SI engine, 5% to 15% of exhaust gas is routed back to the intake as EGR. A properly operating EGR can theoretically increase the efficiency of petrol engines via reduced throttling losses, reduced heat injection and reduced chemical dissociation.

In diesel engines, the lower oxygen exhaust gas into the intake, makes combustion less efficient, compromising economy and power.

In general, the poor combustion directly increases HC emission and calls for mixture enrichment to restore combustion regularity which a further indirect increase of both HC and CO.

2. **Catalyst:** Emission control catalysts enlisted below are being used to reduce NOx emission.

(a) **Lean NOx catalyst:** does selective catalytic reduction by hydrocarbons (HC–SCR) which targets CO and HC in addition to NOx.

(b) **NOx absorber catalyst:** absorbs (traps) NOx from lean exhaust, followed by release and catalytic reduction under rich conditions. It is commercially used on lean burn petrol engines and some light-duty diesel engines.

(c) **SCR catalyst:** selective catalytic reduction of NOx takes place by ammonia governed by the following reactions.

Hydrolysis of urea is carried out at first on a catalyst on board of the vehicle to produce ammonia and carbon dioxide.

$$(NH_2)_2CO + H_2O \longrightarrow CO_2 + 2NH_3$$

Ammonia then, reacts on the SCR catalyst with the NOx and converts it to nitrogen.

$$4\,NO + 4NH_3 + O_2 \longrightarrow 4\,N_2 + 6\,H_2O$$

$$6NO_2 + 8NH_3 \longrightarrow 7N_2 + 12H_2O$$

Q.26 Justify in brief, the following statements:

(i) By advancing the spark timing, the possibility to knock in a S.I. engine increases.

(ii) Willans line method for estimating frictional power can be used only in case of unthrottled engines.

(iii) Exhaust hydrocarbon emissions increase with increase in surface to volume ratio of an engine.

(iv) Carbon monoxide emissions are low for fuel lean mixtures.

[CSE (Mains) 2005 : 20 Marks]

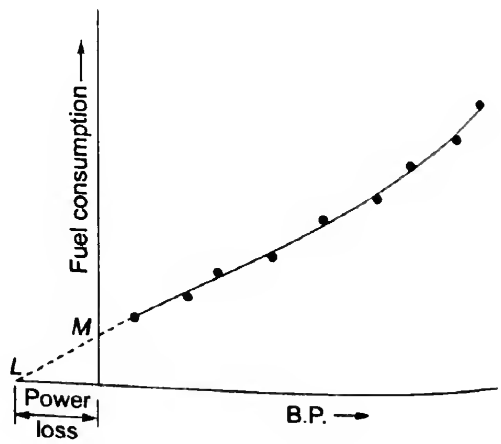
Solution:

(i) "Timing advance" refers to the number of degrees before top dead center (TDC) that the spark will ignite the air-fuel mixture in the combustion chamber during the compression stroke.

Timing advance is required because it takes time to burn the air-fuel mixture. Igniting the mixture before the piston reaches TDC will allow the mixture to fully burn soon after the piston reaches TDC. If the air-fuel mixture is ignited at the correct time, maximum pressure in the cylinder will occur sometime after the piston reaches TDC allowing the ignited mixture to push the piston down the cylinder with the greatest force. Ideally, the time at which the mixture should be fully burnt is about 20 degrees at TDC. This will maximize the engine's power producing potential. If the ignition spark occurs at a position that is too advanced relative to piston position, the rapidly expanding air-fuel mixture can actually push against the piston still moving up, causing knocking (pinging) and possible engine damage. However, if the spark occurs too retarded relative to the piston position, maximum cylinder pressure will occur after the piston is already traveling too far down the cylinder. This might reduce the knocking tendency but results in brake power losses, overheating tendencies, high emissions, and unburned fuel.

(ii) Willans line method is used only in case of unthrottled engine because of the following reason : The Willans line is plotted for fuel consumption versus load (from no load to full load) at constant speed. The intersection of this line on the negative side of X-axis gives the friction power of the engine at that speed. The friction power (F.P.) is assumed constant from no load to full load at that constant speed. The FP includes mechanical friction and pumping power.

For a throttled engine if such a trial is carried out, the throttle position has to be varied from almost closed at no load to full open at maximum load, to keep the engine speed constant. Therefore the pumping load will be bigger at no load, and reduce gradually as the load is increased. As a result, the pumping power and FP will not remain constant as in the assumption in Willans line method.

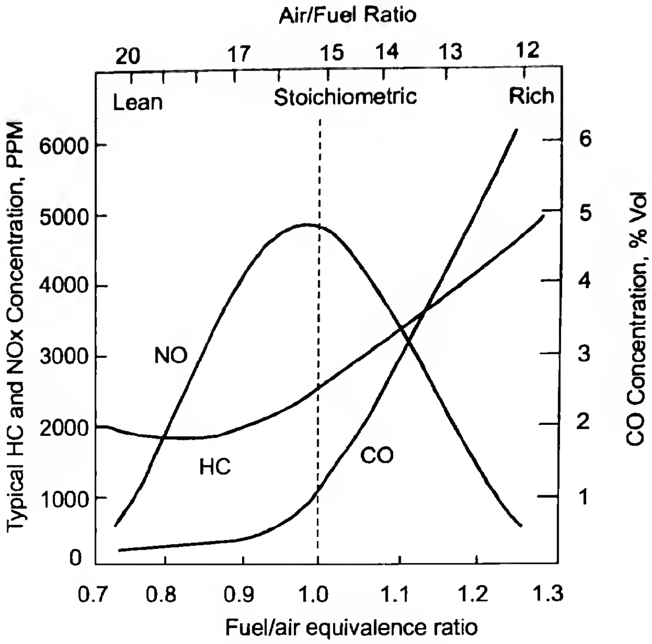


(iii) The emission amount of HC (due to incomplete combustion) is closely related to design variables, operating variables and engine conditions. The surface to volume ratio(S/V) greatly affects the HC emission.

High surface to volume ratio results from a high compression ratio combustion chamber which also results in lower exhaust gas temperatures. Higher the S/V ratio, higher the HC emission irrespective of whether mixture is rich or lean.

When the mixture supplied is lean or rich, the volume of flame quenching regions increases and the flame propagation becomes weak which in turn causes incomplete combustion, thus resulting in higher HC emissions. The problem is further enhanced as due to lower exhaust gas temperatures oxidation of the unburned HC is reduced during exhaust process. These factors result in an increase in HC emissions with increase in surface to volume ratio of an engine.

(iv) CO results from incomplete oxidation of fuel carbon when insufficient oxygen is available to completely oxidize the fuel. CO rises steeply as the air-fuel (A/F) ratio is decreased below the stoichiometric A/F ratio as the availability of oxygen required to convert CO to CO_2 decreases. Air-fuel ratio is one of the most important parameter that affects the engine exhaust emissions.



The SI engine is operated close to stoichiometric air-fuel ratio as it provides a smooth engine operation. Carbon monoxide and HC emissions reduce with increase in the air-fuel ratio as more oxygen gets available for combustion. Lean engine operation to a certain critical value of air-fuel ratio tend to reduce all the three pollutants. Further leaning of mixture results in poor quality of combustion and eventually in engine misfiring causing an erratic engine operation and sharp increase in HC emissions. Generally an engine is operated on lean mixtures that would give low CO and HC, and moderate NOx emissions, The variation in emissions with air-fuel ratio for premixed charge SI engines is shown in above figure.

Q.27 Explain the necessary modifications that have to be made to convert a bus running on diesel fuel to Compressed Natural Gas (CNG).

[CSE (Mains) 2005 : 20 Marks]

Solution:

Diesel to natural gas conversion requires careful engineering on the base engine modifications as well as the control system. Following is an overview of modifications required for a successful conversion:

Compression Ratio: A typical diesel engine has a compression ratio between 16 and 18. CNG usually works best between 10 and 12; so new or modified pistons are required, with an appropriately shaped combustion chamber to allow proper air-fuel mixing.

Spark Plugs: Diesel engines have diesel fuel injectors instead of spark plugs. A diesel conversion replaces the injector with a spark plug and may also require an insert to go through the valve cover - depending on the engine. Spark plug wear is a common problem, and the high compression ratio and use of gaseous fuel requires higher spark voltage than a petrol car.

Valves: Natural Gas is a dry fuel so valve seats in a converted engine need to be hardened to prevent abnormal wear. Older engines need valve guide seals to prevent engine vacuum from drawing oil into the combustion chamber.

Thermal Issues: Spark ignited engines run hotter than diesels. Such engines may require upgraded thermal management components, including larger oil coolers, larger radiators, and heat shields around exhaust components.

Catalytic Converter: A catalyst will generally be required to meet emission regulations. The exception is lean-burn engines, which, if carefully engineered, can meet certain emissions targets without a converter.

Engine Management System: The choice of system will depend on the exhaust emissions requirements, efficiency targets, durability expectations, technology level of the vehicle and peripheral device control requirements such as cruise control, power take-off, automatic transmissions etc.

Q.28 List the advantages and disadvantages of hydrogen as an alternative IC engine fuel. Explain two methods by which hydrogen can be used in CI engine.

[CSE (Mains) 2008 : 10 Marks]

Solution:

In recent years, the concern for cleaner air, along with stricter air pollution regulation and the desire to reduce the dependency on fossil fuels have rekindled the interest in hydrogen as a vehicular fuel.

The properties that contribute to its use as a combustible fuel are its:

- wide range of flammability
- low ignition energy
- small quenching distance
- high auto ignition temperature
- high flame speed at stoichiometric ratios
- high diffusivity or very low density.

Advantages:

- Hydrogen -air mixture burns nearly 10 times faster than gasoline -air mixture.
- It has high self -ignition temperature but requires very little energy to ignite it.
- Hydrogen octane rating is 106 RON making it very suitable for spark-ignited engines.
- The laminar flame speed of hydrogen is 3 m/s, about 10 times that of gasoline and methane.
- Hydrogen on combustion produces water and there are no emissions of carbon containing pollutants such as HC, CO and CO_2 and air toxics. Trace amounts of HC, CO and CO_2 may be emitted as a result of combustion of lubricating oil leaking into engine cylinder.
- Hydrogen fuelled engines produces almost no CO_2 and its global warming potential is insignificant.
- It is an efficient fuel as there are no throttling losses associated.

Disadvantages:

1. Pre-Ignition Problems: The primary problem in the development of operational hydrogen engines is premature ignition. Premature ignition is a much greater problem in hydrogen fueled engines than in other IC engines, because of Hydrogen's lower ignition energy, Wider flammability range Shorter quenching distance.

   Premature ignition occurs when the fuel mixture in the combustion chamber becomes ignited before ignition by the spark plug, and results in an inefficient, rough running engine. Backfire conditions can also develop if the premature ignition occurs near the fuel intake valve and the resultant flame travels back into the induction system. Pre-ignition in hydrogen engines may be caused due to the following reasons:
   (a) hot spots in the combustion chamber, such as on a spark plug or exhaust valve, or on carbon deposits.
   (b) Backfire when there is overlap between the openings of the intake and exhaust valves
   (c) Pyrolysis (chemical decomposition brought about by heat) of oil suspended in the combustion chamber or in the crevices just above the top piston ring.
2. Its adiabatic flame temperature is higher by about 110º C compared to gasoline. If inducted along with intake air, the volume of hydrogen is nearly 30% of the stoichiometric mixture decreasing maximum engine power
3. NOx is the only pollutant of concern from hydrogen engines. Very low NOx emissions can be obtained with extremely lean engine operation ($f < 0.05$) and/or injection of water into intake manifold or exhaust gas recirculation which in this case consists primarily of water vapours
4. Problems of fuel handling and storage.

Methods by which hydrogen can be used in CI engines are as follows:

1. **Direct injection:** The direct injection of hydrogen constitutes a very promising method of hydrogen use in CI engines. The method can be divided into two different concepts:
   - Method using moderate hydrogen pressures
   - Method using high hydrogen pressures

   In both methods, the injection takes place only when the cylinder valves are closed. In the first mode, the fuel is injected at low pressure early in the compression process and ignition takes place only when the final temperature of compression is reached, making the ignition angle slightly erratic and difficult to control. With the high pressure direct injection method, hydrogen is injected only when the final compression temperature is above the self-ignition temperature of the hydrogen charge. (Similarly as in a standard diesel engine.)
2. **Homogeneous Charge Compression Ignition (HCCI):** Hydrogen HCCI can be achieved using a high compression ratio diesel engine, typically above 20:1. With such a high compression ratio, the final temperature of compression will be sufficiently high to ignite the cylinder charge. This method typically uses timed injection of hydrogen at a low pressure in the engine inlet manifold. Hydrogen is injected only during the engine induction stroke, while the exhaust valve is already closed. This method has interesting characteristics, including a potential for high engine thermal efficiency and extremely low exhaust gas emissions. There are, however, some problems which need to be solved, such as engine load and speed control, mechanical component loading, and also the possibility of air manifold explosions.

Q.29 What are the main pollutants emitted by petrol and diesel engine? Discuss their effect on human and biological life.

[CSE (Mains) 2011 : 20 Marks]

Solution:

Principal Engine Emissions

a. exhaust emissions b. non-exhaust emissions

Exhaust emissions:

SI engines : CO, HC and NOx CI engines: CO, HC, NOx and PM

CO = Carbon monoxide, HC = Unburned hydrocarbons,

NOx = Nitrogen oxides mainly mixture of NO and NO_2,

PM = Particulate matter

Other engine emissions include

1. Aldehydes such as formaldehyde 2. Acetaldehyde primarily from the alcohol fuelled engines,
3. Benzene 4. polyaromatic hydrocarbons (PAH).

Non-exhaust emissions : Unburnt hydrocarbon from fuel tank and crankcase blowby.

Hydrocarbon emissions: they may be saturated hydrocarbons or unsaturated ones which form aldehydes and ketones upon atmospheric reactions. Following are some of the causes of hydrocarbon emission:

1. Incomplete combustion. 2. Crevice volume and flow in crevices.
3. Leakage past exhaust valve 4. Deposition on wall
5. Oil on combustion chamber walls.

Carbon monoxide emissions: It is a colorless, odourless but poisonous gas which is generated in an engine when it is operated with a fuel-rich equivalence ratio. Due to insufficient oxygen, incomplete combustion takes place and CO is formed. It is an undesirable emission which also leads to loss of chemical energy. Typical exhaust of SI engine has 0.2 to 5% CO.

Oxides of Nitrogen: Exhaust gases of an engine can have up to 2000 ppm of oxides of nitrogen, most of which is nitrogen oxide (NO), with small amounts of Nitrogen dioxide(NO_2) and traces of other nitrogen-oxygen combinations.

It is created mostly from N_2 in the air but can also be found in fuel blends.
High combustion temperature favours the formation of NO_x .It also depends upon pressure and air-fuel ratio.
Maximum NO_x is formed at an equivalence ratio of 0.95.
In SI engine highest concentration of NO_x is near the spark plug .
CI engines have higher compression ratios and higher temperature and pressures, which tends to generate higher levels of NO_x.

Particulates: These are solid carbon soot particles which are generated in the fuel-rich zones within the cylinder during combustion in CI engines.
Maximum emission takes place when the engine is under load, and maximum fuel is injected to supply maximum power resulting in rich mixture and poor economy.
Nearly 25% of the carbon in soot comes from vaporising lubricating oil components which react during combustion.
0.2 to 0.5 % carbon comes from the fuel
Particulate generation can be reduced by engine design and control of operating conditions.

Aldehydes: A product of incomplete combustion formed generally when alcohol fuel is used.
Sulphur: many fuels used in CI engines contain small amounts of sulphur . They are exhausted in the form of SO_2 and SO_3 (SO_x).
Unleaded gasoline contains 150-550 ppm by weight of sulphur while diesel fuels may contain 5500 ppm by weight.

Formaldehyde: Forms Carbon monoxide, hydroperoxyl radicals upon atmospheric reactions
Effect of emissions on human and biological life:

Emission component	Biological Impact
Carbon Monoxide	• Highly toxic to humans; blocks oxygen intake.
Nitrogen Oxides	• Nitrogen dioxide is a respiratory tract irritant and major ozone precursor. • Nitric acid contributes to acid rain
Sulfur dioxide	• Respiratory tract irritation. • Contributor to acid rain.
Carbon dioxide	• Major contributor to global warming
Saturated hydrocarbons (Alkanes, < C19)	• Respiratory tract irritation. • Reaction products are ozone precursors (in the presence of NOx).
Unsaturated hydrocarbons (Alkenes < C5)	• Respiratory tract irritation. • Some alkenes are mutagenic and carcinogenic. • Reaction products are ozone precursors (in the presence of NOx).
Formaldehyde	• Formaldehyde is a probable human carcinogen and an ozone precursor (in the presence of NOx).
Higher aldehydes (e.g., acrolein)	• It causes respiratory human tract and eye irritation. • It causes plant damage.
Monocylic aromatic compounds (e.g. benzene, toluenece)	• Benzene is toxic and carcinogenic in humans. • Some reaction products are mutagenic in bacteria (Ames assay).
PAHs (< 5 rings) (e.g. phenanthrene, fluoranthene)	• Some of these PAHs and nitro-PAHs are known mutagens and carcinogens.
Elemental carbon	• The Nuclei of organisms absorb organic compounds; • Its size permits transport deep into the lungs (alveoli).
Inorganic sulphates	• Respiratory tract irritation
PAHs (4 rings and more) (e.g., pyrene, benzo(a)pyrene)	• Larger PAHs are major contributors of carcinogens in combustion emissions. • Many nitro-PAHs are potent mutagens and carcinogens.
PAHs (3 rings and more) (e.g., nitropyrenes)	• Many nitro-PAHs are potent mutagens and carcinogens. • Some reaction products are mutagenic in bacteria

Q.30 What do you understand by catalytic converter? Explain the principle and the working of 3-way catalytic converter.

[CSE (Mains) 2011 : 20 Marks]

Solution:

Catalytic converter is an after treatment for reducing engine emissions, found on most automobiles and other modern engines of medium or large size.

Principle and working of 3-way catalytic converters: A catalyst is a substance that accelerates a chemical reaction by lowering the energy needed for it to proceed. Catalytic converters are chambers mounted in the flow system through which exhaust gases pass through. These chambers contain catalytic material, which promotes the oxidation of the emissions contained in the exhaust flow.

They are called three-way converters because they are used to reduce the concentration of CO, HC and NO_x in the exhaust. It is usually a container made of stainless steel mounted along the exhaust pipe of the engine. The container contains a porous ceramic structure through which the exhaust gas flow.

The ceramic structure is generally of 2 types:

(a) A single honeycomb structure with many flow passages, used commonly.

(b) A loose granular ceramic with the gas passing between the packed spheres.

The surface of the ceramic passages contains small embedded particles of catalytic material that promote the oxidation reactions in the exhaust gas as it passes.

Aluminium oxide (Alumina) is the base ceramic material used for most catalytic converters because of certain favourable properties like ability to withstand high temperature, chemically neutral, low thermal expansion etc.

Platinum-promotes oxidation of HC

Palladium-promotes oxidation of CO

Rhodium-promotes reduction of NOx,

$$CO + \frac{1}{2}O_2 \xrightarrow{\text{Palladium}} CO_2$$

$$C_\lambda Hy + zO_2 \xrightarrow{\text{Platinum}} xCO_2 + H_2O$$
$(Z = \lambda + 0.25\, y)$

$$NO + CO \longrightarrow \frac{1}{2}N_2 + CO_2$$

$$2NO + 5CO + 3H_2O \longrightarrow 2NH_3 + 5CO_2$$

$$2NO + CO \longrightarrow N_2O + CO_2$$

$$2NO + 5\,H_2 \longrightarrow 2\,NH_3 + 2\,H_2O$$

$$NO + H_2 \longrightarrow \frac{1}{2}N_2 + H_2O$$

$$2NO + H_2 \longrightarrow N_2O + H_2O$$

Cerium oxide is used to promote the water gas shift as shown by the reaction:

$$CO + H_2O \longrightarrow CO_2 + H_2$$

This reduces CO by using water vapour as an oxidant instead of O_2, which is important when engine is running rich.

The efficiency of catalytic convertor depends (a) on equivalence ratio (b) temperature.

Optimum engine temperature is important to avoid engine malfunctions and overheating of converters.

Effective control of HC and CO occurs with stoichiometric or lean mixtures while control of NOx requires near stoichiometric conditions.

Impurities present in fuel, lubricating oil and air and use of leaded gasoline can poison the catalyst material, makes it ineffective.

Q.31 What are the measures required for the reduction of the following emissions from diesel engine?

(i) Particulate Emission **(ii) Smoke Emission**

(iii) NO_x Emissions **(iv) HC Emissions**

(v) SO_x Emissions

[CSE (Mains) 2012 : 10 Marks]

Solution:

(i) **Particulate Emission:** Soot or smoke made up of particles in micrometer size range are formed due to improper combustion and are mainly in highly rich AFM condition, which can be reduced by using exhaust gas recirculation by making proper combustion.

(ii) **Smoke Emission:** Smoke emission can be reduced by using Diesel Particular Filter (DPF), which is a device that traps the exhaust smoke particles by means of physical filtration.

(iii) **NOx Emission:** 'NOx' formation are mainly due to the high combustion temperature which enabled the reaction between 'N_2' and 'O_2' at slightly leaner mixture. It can be reduced by using.

I > Exhaust gas recirculation.

II > Water injection and water emulsion which also reduces the temperature of combustion.

III > Use of catalytic convertor that may reduces the NOx formation by reducing it to 'N_2' gas.

(iv) **HC Emissions:** It is due to improper combustion, which can reduced by using catalytic convertor by proceeding to oxidation reaction and also by the use of thermal convertors.

(v) **SOx Emissions:** SOx are formed due to presence of sulphur in fuel and it can be reduced by the use of water, separate SOx from exhaust gas and also by fuel additive like penta-ethyle Napthalene.

4. Performance and Testing of I.C. Engines

Q.32 A four cylinder engine of a truck has been converted to run on propane fuel. A dry analysis of the engine exhaust gives the following volumetric percentages:
CO_2 – 4.90; CO – 9.79 and O_2 – 2.45.
Calculate the equivalence ratio at which the engine is operating.

[CSE (Mains) 2001 : 30 Marks]

Solution:

Stoichiometric Reaction

$$C_3H_8 + 5\left\{O_2 + \frac{79}{21}N_2\right\} \xrightarrow[Comb^n]{Oxid^n} 4H_2O + 3CO_2 + 5\times\frac{79}{21}\times N_2$$

$$(FAR)_{stoichiometric} = \frac{3\times12+8\times1}{5\times\left\{1+\frac{79}{21}\right\}\times29} = 0.0637$$

$$c.C_3H_8 + a\left\{O_2 + \frac{79}{21}\times N_2\right\} \longrightarrow bH_2O + 4.9CO_2 + 9.79CO + O_2 2.45 + N_2\times82.86$$

Balancing N_2: →

$$a\times\frac{79}{21} = 82.86 \Rightarrow a = 22.026$$

Balancing O_2: →

$$b + 4.9\times2 + 9.79 + 2.45\times2 = 22.026\times2$$

$$\Rightarrow \quad b = 19.562$$

Balancing H_2: → $\quad 8\times c = b\times2 \Rightarrow c = 4.8905$

Divide moles by $C = 4.8905$

$$C_3H_8 + 4.5\left\{O_2 + \frac{79}{21}\times N_2\right\} \longrightarrow 4H_2O + CO_2 + 2CO + \frac{1}{2}O_2 + 17N_2$$

$$(FAR)_{actual} = \frac{3\times12+8\times1}{45\times\left\{1+\frac{79}{21}\right\}\times29} = 0.0708$$

$$\phi = \frac{(FAR)_{actual}}{(FAR)_{stoichiometric}} = 1.111$$

Q.33 A six cylinder, four stroke petrol engine with a bore of 120 mm and stroke of 180 mm under test, is supplied with petrol of composition: C = 82% and H_2 = 18% by mass. The Orsat gas analysis indicated that CO_2 = 12%, O_2 = 4% and N_2 = 84% by volume. Determine

(i) that air-fuel ratio and (ii) the percentage of excess air

Also calculate the volumetric efficiency of engine based on intake conditions when the mass flow rate of petrol is 32 kg/min at 1600 RPM. Intake conditions are 1 bar and 17°C. Consider the density of petrol vapour to be 3.5 times that of air at same temperature and pressure. Air contains 23% oxygen by mass.

[CSE (Mains) 2003 : 30 Marks]

Solution:

Given: No. of cylinders, k = 6, 4-stroke (petrol engine), Bore, D = 120 mm = 0.12 m, Stroke, L = 180 mm = 0.18 m, Speed, N = 1600 rpm

$$\text{Swept volume, } \dot{V}_s = \frac{\pi}{4}D^2\times L\times\frac{N}{60\times2}\times k = 0.1628 \text{ m}^3/\text{s}$$

Petrol Compositon				
Element	Percentage composition	Mass	Element Quantity	Element composition
C	82%	12	0.82 × 12 = 9.84	9.84/9.84 = 1
H_2	18%	2	0.36	0.36/9.84 = 0.0366

$$kC + z\times0.0366\,H_2 + x\left(O_2+\frac{79}{21}\times N_2\right)\rightarrow y\cdot H_2O + 12CO_2 + 84N_2 + 4O_2$$

$$x\times\frac{79}{21} = 84 \Rightarrow x = 22.329$$

$$2x = y + 12\times2 + 4\times2$$

$$\Rightarrow \quad y = 12.658$$

$$2y = z\times0.0366 \Rightarrow z = 691.694$$

$$k = 12$$

$$C + 2.1\,H_2 + 1.86\left(O_2+\frac{79}{21}\times N_2\right)\rightarrow 1.05H_2 + CO_2 + 7N_2 + 0.33O_2$$

$$\Rightarrow \quad AFR = \frac{1.86\times\left(1+\frac{79}{21}\right)\times29}{12+2.1\times2} = 15.855$$

Determining Stoichiometric

$$\underset{12\text{ kg}}{C} + \underset{32\text{ kg}}{O_2} \longrightarrow \underset{44\text{ kg}}{CO_2} \qquad 1\text{ kg of C} \longrightarrow \frac{32}{12}\text{ kg of } O_2$$

$$\text{for 1 kg of fuel} \longrightarrow \frac{32}{12}\times0.82 = 2.187 \text{ kg of } O_2$$

$$\underset{2\text{ kg}}{H_2} + \underset{16\text{ kg}}{\frac{1}{2}O_2} \longrightarrow \underset{18\text{ kg}}{H_2O} \qquad 1\text{ kg of } H_2 \longrightarrow 8\text{ kg of } O_2$$

For 1 kg of fuel $\longrightarrow$ $8 \times 0.18 = 1.44$ kg of O_2

$$\text{Total } O_2 \text{ required for 1 kg of fuel} = 3.627 \text{ kg of } O_2$$

$$\text{Total air required/kg of fuel} = \frac{3.627}{0.23} = 15.769$$

$$\text{Percentage of excess air} = \frac{15.855 - 15.769}{15.769} \times 100 = 0.55\%$$

$$\text{AFR} = \frac{\dot{m}_a}{\dot{m}_f} = 15.855 \Rightarrow \dot{m}_a = 507.36 \text{ kg/h}$$

$$P_1\dot{V}_a = \dot{m}_a R T_1 \Rightarrow \dot{V}_a = 0.1173 \text{ m}^3/\text{s}$$

$$\text{Volumetric efficiency} = \eta_{vol} = \frac{\dot{V}_a}{\dot{V}_s} \times 100 = 72.02\ \%$$

Q.34 An eight cylinder automobile engine of 80 mm diameter and 90 mm stroke with a compression ratio of 7, is tested at 4000 RPM on a dynamometer of 600 mm arm length. During a ten minutes test period at a dynamometer scale reading of 450 N, 4.8 kg of gasoline having a calorific value of 45000 kJ/kg was burnt and air at 27°C and 1.0 bar was supplied to the carburettor at the rate of 6.6 kg/min. Find

(i) the brake power delivered,

(ii) the brake mean effective pressure,

(iii) the brake specific fuel consumption,

(iv) brake thermal efficiency,

(v) the volumetric efficiency and

(vi) the air-fuel ratio.

[CSE (Mains) 2003 : 40 Marks]

Solution:

Number of cylinders, $k = 8$, Diameter of cylinder, $D = 80$ mm $= 0.08$ m, Stroke length, $L = 90$ mm $= 0.09$ m

Compression Ratio, $r = 7$, Speed, $N = 4000$ rpm, Swept volume, $\dot{V}_s = \left(\frac{\pi}{4} D^2 \cdot L \frac{N}{2 \times 60} \times K\right)$

$$\Rightarrow \qquad \dot{V}_s = 0.12064 \text{ m}^3/\text{s}$$

Dynamometer Test ($t = 10$ min): Dynamometer arm length $= r_d = 600$ mm $= 0.6$ m

Load, $F_b = 450$ N

$$\Rightarrow \qquad \text{Braking Torque, } \tau_b = F_b r_d = 270 \text{ N-m}$$

$$\text{Mass flow rate of fuel, } \dot{m}_f = \frac{m_f}{t} = \frac{4.8}{10 \times 60} = 0.008 \text{ kg/s}$$

$$\text{Calorific value of fuel, } (CV)_f = 45{,}000 \text{ kJ/kg}$$

$$\text{Rate of Heat Addition, H.A./sec} = (\dot{m}_f) \times (CV)_f = 360 \text{ kW}$$

Initial conditions:

Inlet temperature, $T_1 = 27°\text{C} = 300$ K

Inlet pressure, $P_1 = 1$ bar $= 100$ kPa

Rate of air supply $= \dot{m}_a = 6.6$ kg/min $= 0.11$ kg/s.

(i) $$\text{Brake power delivered} = \text{B.P.} = T_b \times \omega = T_b \times \frac{2\pi N}{60} = 113.097 \text{ kW}$$

(ii) Brake mean effective pressure (bmep)

$$\dot{V}_s \times bmep = \text{B.P.}$$

$$\Rightarrow \qquad \text{bmep} = \frac{B.P.}{\dot{V}_s} = 937.5 \text{ kPa} = 9.375 \text{ bar}$$

(iii) Brake specific consumption (bsfc)

The mass flow rate of fuel required to deliver unit brake power.

$$\text{bsfc} = \frac{\dot{m}_f}{BP} = 0.2546 \text{ kg/kWhr}$$

(iv) Brake thermal efficiency (η_{BT})

$$\eta_{BT} = \frac{\text{Brake Power}}{\text{Rate of Heat Addition}} = \frac{BP}{\text{HA/sec}} = 0.31416 = 31.416\%$$

(v) Volumetric efficiency (η_{vol})

$$\eta_{vol} = \frac{\text{Actual volume}}{\text{Swept volume}} = \frac{\dot{V}_a}{\dot{V}_s}$$

$$\dot{V}_a = \frac{\dot{m}_a R T_1}{P_1} = 0.09471 \text{ m}^3/\text{s}$$

$$\eta_{vol} = 0.7851 = 78.51\%$$

(vi) Air fuel ratio (AFR)

$$\text{AFR} = \frac{\dot{m}_a}{\dot{m}_f} = 13.75 : 1$$

Q.35 An I.C. engine fuel has the following composition:

$C = 89\%$, $H_2 = 5\%$, $O_2 = 4\%$ and rest N_2.

Determine the chemically correct air-fuel ratio. If 40% excess air is supplied, find the percentage of dry products of combustion by volume.

[CSE (Mains) 2005 : 20 Marks]

Solution:

Fuel composition: C = 89%, H_2 = 5%, O_2 = 4%, N_2 = (100 – 89 – 5 – 4)% = 2%

Oxygen required for burning of carbon, hydrogen only,

Basis: 1 kg of fuel burned.

$$\underset{12}{C} + \underset{32}{O_2} \longrightarrow \underset{44\text{ kg}}{CO_2} \qquad C: 0.89 \times \frac{32}{12} \text{ kg} = 2.373 \text{ kg of } O_2$$

$$\underset{2\text{ kg}}{H_2} + \underset{16\text{ kg}}{\frac{1}{2}O_2} \longrightarrow \underset{18\text{ kg}}{H_2O} \qquad H: 0.05 \times \frac{16}{2} - 0.04 = 0.36 \text{ kg of } O_2$$

Note: 1 kg of H_2 consumes 8 kg of O_2 and 0.04 kg of O_2 present initially in fuel.

Total 'O_2' required ⇒ 2.373 + 0.36 = 2.733 kg

General formula, O_2 required = 2.667 × C + (H × 8 – 0) + 5 kg

Air – 77% of N_2 and 23% of O_2 (mass basis)

$$N_2 \text{ required} = 2.733 \times \frac{0.77}{0.23} = 9.1496 \text{ kg}$$

i.e. Stoichiometric air to fuel ratio is: (9.149 + 2.733) = 11.883 : 1

Q.36 Explain the effect of the following factors on the performance of an SI engine:

(i) spark timing **(ii) engine speed**

(iii) mass of inducted charge **(iv) heat losses**

[CSE (Mains) 2008 : 20 Marks]

Solution:

In SI engine, combustion which is initiated between the spark plug electrodes spreads across the combustible mixture. A definite flame front which separates the fresh mixture from the products of combustion travels from the spark plug to the other end of the combustion chamber. Heat-release due to combustion increases the temperature and pressure of the burned part of the mixture above those of unburned mixture. To effect pressure

equalization, burned part of the mixture expands and compresses the unburned mixture adiabatically increasing its pressure and temperature. If the temperature exceeds the self-ignition temperature of the fuel and remains above it during ignition lag, auto-ignition occurs at various pin-point locations. This phenomenon is called knocking which may lead to engine failure. The following factors directly affect knocking tendency and hence performance of the engine:

(a) **Spark timing:** By retarding the spark timing from the optimized timing i.e. having the spark closes to TDC, the peak pressures are reached further down on the power stroke and are thus of lower magnitude. This might reduce knocking and may bear effect on brake torque and power output of the engine.

(b) **Engine Speed:** An increase in engine speed increases the turbulence of the mixture considerably resulting in increased flame speed and reduces the time available for pre flame reactions. Hence knocking tendency is reduced improving engine efficiency.

(c) **Mass of Inducted charge:** A reduction in the mass of the inducted charge into the cylinder of an engine by throttling or by reducing the amount of supercharging reduces both temperature and density of the charge at the time of ignition. This decreases the tendency of knocking.

(d) **Heat Losses:** Heat transfer in the inlet decreases volumetric efficiency. In the cylinder, heat losses to the wall is a loss of availability. Heat transfer also influences inlet mixture temperature, chamber, cylinder head, linear, piston and valve temperatures and therefore end-gas temperatures which affect knock. Heat transfer also affects build-up of in-cylinder deposit which affects knock.

Q.37 The following set of observations refer to a trial on a Single-cylinder, Four-stroke, Solid injection diesel engine of 200 mm bore and 400 mm stroke:

Gross mean effective pressure = 6.2 bar	**Pumping mean effective pressure = 0.44 bar**
Speed of the engine = 262 rpm	**Brake torque = 668 N-m**
Fuel supply rate = 4.5 kg/hr	**Calorific value of the fuel = 52,000 kJ/kg**
Cooling water flow rate = 6 kg/min	**Cooling water temperature gain = 47°C**

Calculate the Indicated Power, Brake Power and Mechanical efficiency of the engine.

Draw up a heat balance sheet for the trial expressing various quantities in kJ/min, if the fuel contains 13.5% H_2 (by mass) and air supply to the engine is 2.71 kg/min at 17°C. The exhaust gases leave the engine at 400°C. The following data may be used:

Mean specific heat of exhaust gases = 1 kJ/kgK

Specific heat of steam = 2.1 kJ/kgK Latent heat of steam = 2250 kJ/kg

Estimate the heat carried away by steam in exhaust gases.

[CSE (Mains) 2008 : 50 Marks]

Solution:

No. of cylinder, $k = 1$, 4-Stroke (Diesel engine), Bore, $D = 200$ mm, Stroke, $L = 400$ mm, Speed, $N = 262$ rpm

$$\text{Rate of volume swept} = \dot{V}_s = \frac{\pi D^2}{4} \times L \times k \times \frac{N}{60 \times 2} = 0.0274 \text{ m}^3\text{/sec}$$

Gross mean effective pressure, $P_{ig} = 6.2$ bar

Pumping mean effective pressure, $P_{pg} = 0.44$ bar

Indicated mean effective pressure, $P_{im} = P_{ig} - P_{pg} = 5.76$ bar

$$\text{Indicated Power, IP} = P_{im} \times \dot{V}_s = 15.7824 \text{ kW}$$

$$\text{Brake Power, BP} = T_b \times \omega = 668 \times \left(\frac{2\pi N}{60}\right) = 18.327 \text{ kW}$$

Fuel flow rate, $\dot{m}_f = 4.5$ kg/hr

Calorific value of fuel, $(CV)_f = 52{,}000$ kJ/kg $\rightarrow$ Heat Addition Rate, HA/sec = 65 kW = $\dot{m}_F \times C \cdot V$

(i) Heat taken by cooling water jacket,

$$\dot{Q}_{WJ} = m_C \times C_{P,W} \times (\Delta T)_{WJ} = 6 \times 4.187 \times 47 = 1180.734 \text{ kJ/min}$$

$$\dot{m}_a = 2.71 \text{ kg/min}$$

$$\text{Exhaust gases flow rate, } \dot{m}_{exh} = \dot{m}_a + \dot{m}_f = 2.71 + \frac{4.5}{60} = 2.785 \text{ kg/min}$$

$$\underset{1\,kg}{H_2} + \underset{8\,kg}{\frac{1}{2}O_2} \longrightarrow \underset{9\,kg}{H_2O} \Rightarrow 1 \text{ kg of } H_2 \text{ produced 9 kg of } H_2O$$

$$\text{Mass of } H_2O \text{ produced/kg of fuel} = 9 \times \frac{13.5}{100} = 1.215 \text{ kg}$$

∴ Mass of H_2O (steam) per min $= 1.215 \times \dot{m}_f = 0.09125$ kg/min

Mass of dry exhaust gas/min = mass of wet exhaust/min – mass of steam/min = 2.6939 kg/min

(ii) Heat lost to dry exhaust gas/min $\left(\dot{Q}_{ex,\,dry}\right)$

$$= 2.6939 \times 1 \times (400 - 17) = 1031.754 \text{ kJ/min}$$

$$= \dot{m}_{exh,\,dry} \times C_{p,\,exh} \times (T_{ex} - T_{room})$$

⇒ Assuming steam in exhaust is superheated steam at atmospheric pressure, P_{atm} = 1.01325 bar and exhaust gas temperature.

Enthalpy of exhaust steam $h_s = h_w + (LH) + h_{sup}$

$$h_s = C_{P,W} \times (100 - 17) + 2250 + C_{P,\,steam} \times (400 - 100)$$

$$h_s = 3227.521 \text{ kJ/kg}$$

(III) Heat lost to steam/min $(\dot{Q}_{ex,wet})$ = (mass of steam/min) × h_s = 294.108 kJ/min

(IV) Heat lost to radiation, errors, (Unaccounted Heat Loss)

$$= \frac{HA}{\min} - \left[\frac{B.P.}{\min} + \dot{Q}_{WJ} + \dot{Q}_{ex,\,dry} + \dot{Q}_{ex,\,wet}\right]$$

$$= 3900 - [1099.62 + 1180.734 + 1031.754 + 294.108]$$

$$= 293.784 \text{ kJ/min}$$

Note: As, I.P. is obtained during the trial is less than B.P. of the engine, which is dynamically impossible; hence, there is an error in observation for indicated mean effective pressure measurement.

Note: HA/min kJ/min: 3900 kJ/min

Processes

(i) Heat lost cooling jacket, $\dot{Q}_{WJ} \rightarrow$ 1180.734 kJ/min; percentage contribution → 30.27%

(ii) Heat loss to dry exh., $\dot{Q}_{ex,\,dry} \rightarrow$ 1031.754 kJ/min; percentage contribution → 26.45%

(iii) Heat lost to steam, $\dot{Q}_{steam}$ = 294.108 kJ/min; percentage contribution → 7.54%

(iv) Unaccounted heat loss, $\dot{Q}_{UN}$ = 293.784 kJ/min; percentage contribution → 7.53%

Q.38 During a test on a two stroke engine on full load, the following observations were recorded;

Speed = 350 rpm	**Net brake load = 590 N**
Mean effective pressure = 2.8 bar	**Fuel oil consumption = 4.3 kg/h**
Cooling water required = 500 kg/h	**Rise in cooling water temperature = 25°C**
Air used per kg of fuel = 33 kg	**Room temperature = 25°C**
Exhaust gas temperature = 400°C	**Cylinder diameter = 220 mm**
Stroke length = 280 mm	**Effective brake diameter = 1 m**
C.V of fuel oil = 43900 kJ/kg	**Proportion of hydrogen in fuel = 15%**

Mean specific heat of exhaust gases = 1.0 kJ/kg-K
Specific heat of steam = 2.09 kJ/kg-K
Calculate the following:
(i) Indicated power
(ii) Brake power
(iii) Draw heat balance sheet on the basis of kJ/min.

[CSE (Mains) 2010 : 20 Marks]

Solution:

Given: 2-stroke engine, Speed, $N = 350$ rpm, Brake load, $F_b = 590$ N, Indicated mean effective process, $P_{im} = 2.8$ bar, Fuel flow rate, $\dot{m}_f = 4.3$ kg/hr, Calorific value of fuel $(CV)_f = 43900$ kJ/kg, HA/sec $= \dot{m}_f \times (CV)_f$ $= 52.436$ kW, Bore, $D = 220$ mm, Stroke, $L = 280$ mm $= 0.28$ m

$$\dot{V}_s = \frac{\pi}{4} D^2 \times L \times k \times \frac{N}{60} = 0.0621 \text{ (m}^3\text{/sec)}$$

Effective brake radius, $r_d = 0.5$ m

$$\text{Indicated Power, I.P.} = P_{im} \times \dot{V}_s = 17.385 \text{ kW}$$

$$\text{Brake power, B.P.} = \tau_b \times \omega = (F_b \times r_d) \times \left(\frac{2\pi N}{60}\right) = 10.812 \text{ kW}$$

1. Heat taken by cooling water jacket,

$$\dot{Q}_c = \dot{m}_w \times C_{P,W} \times (T_0 - T_i) = \frac{500}{60} \times 4.187 \times 25 = 872.291 \text{ kJ/min}$$

$$\text{Air flow rate, } \dot{m}_a = 33 \times \dot{m}_f = 141.9 \text{ kg/hr}$$

$$\text{Exhaust gas flow rate, } \dot{m}_{exhaust} = \dot{m}_a + \dot{m}_f = 146.2 \text{ kg/hr}$$

$$\underset{1\,kg}{H_2} + \underset{8\,kg}{\frac{1}{2}O_2} \longrightarrow \underset{9\,kg}{H_2O}$$

1 kg of H_2 produces 9 kg of H_2O.

Mass of H_2O produced/kg of fuel $= 9 \times 0.15 = 1.35$ kg

$\therefore$ Mass of H_2O(steam) per min $= 1.35 \times \dot{m}_f = 0.09675$ kg/min

Mass of dry exhaust gas/min = mass of wet exhaust/min – mass of steam in wet/min

$$= \frac{146.2}{60} - 0.09675 = 2.3399 \text{ kg/min}$$

2. $\therefore$ Heat lost to dry exhaust gas/min = (mass of dry exhaust) $\times C_{P,ex} \times (T_{ex} - T_{room})$
$= 2.3399 \times 1 \times (400 - 25) = 877.469$ kJ/min

Assuming steam in exhaust is superheated steam at atmospheric pressure ($P_{atm} = 1.01325$ bar) and at exhaust gas temperature

$$h = h_w + (LH) + h_{sup}$$
$$h = C_{P,w} \times (100 - 25) + 2250 + C_{P,steam} \times (400 - 100)$$
$$h = 3191.025 \text{ kJ/kg}$$

3. Heat lost to steam/min = (mass of steam/min) $\times$ (enthalpy of 1 kg of steam)
= 308.732 kg/min

4. Heat lost due to radiation, errors (Unaccounted heat loss)

$$= 52.436 \times 60 - [B.P. + \dot{Q}_{wj} + \dot{Q}_{ex,dry} + \dot{Q}_{ex,wet}]$$

$$= 438.948 \text{ kJ/min.}$$

Heat Balance Sheet : Heat Supplied by oil → 3146.16 kJ/min kW

(i) B.P. → 648.72 kJ/min

(ii) $\dot{Q}_{water}$ → 872.291 kJ/min

(iii) $\dot{Q}_{Ex,w}$ → Heat lost to dry exhaust has → 877.469 kJ/min

(iv) $\dot{Q}_{Ex,gas}$ → Heat carried away by steam → 308.732 kJ/min

(v) $\dot{Q}_{unacounted}$ → 438.948 kJ/min

Q.39 **In a 4-stroke, 2-cylinder diesel engine, the following data was collected:**

Piston stroke = 60 cm	Diameter of the cylinder = 40 cm
Speed of the engine = 250 r.p.m.	Indicated mean effective pressure = 8 bar
Brake power of the engine = 220 kW	Fuel consumption = 80 kg/hr
CV of fuel used = 43000 kJ/kg	Hydrogen content in fuel = 13% and remaining is carbon
Air consumption = 30 kg/min	Cooling water circulated = 90 kg/min
Rise in temperature of cooling water = 38°C	
Piston cooling oil used = 45 kg/min	Rise in temperature of cooling oil = 23°C
C_p of water = 4.18 kJ/kg-K	C_p of cooling oil = 2.2 kJ/kg-K
C_p of exhaust gases = 1.1 kJ/kg-K	C_p of superheated steam = 2 kJ/kg-K
Latent heat of steam = 2520 kJ/kg	Exhaust gas temperature = 450°C
Ambient temperature = 27°C	

Find the following quantities per minute:

(i) Heat converted to useful brake power (BP).

(ii) Heat carried away by cooling water.

(iii) Heat carried away by cooling oil.

(iv) Heat carried away by dry exhaust gases.

(v) Heat carried away by steam formed.

(vi) Heat supplied by fuel.

Draw up also a heat balance sheet on minute basis and percentage basis.

[CSE (Mains) 2013 : 40 Marks]

Solution:

Given: No. of cylinders, $k = 2$, 4-stroke (Disel Engine), Stroke length, $L = 60$ cm, Bore, $D = 40$ cm, Speed, $N = 250$ rpm

$$\text{Rate of volume swept, } \dot{V}_s = \frac{\pi}{4} \times D^2 \times L \times k \times \frac{N}{60 \times 2} = 0.314 \text{ m}^3\text{/sec}$$

$$\text{Indicated mean effective pressure, } P_{im} = 8 \text{ bar}$$

$$\text{Indicator power, } I.P. = P_{im} \times \dot{V}_s = 251.327 \text{ kW}$$

$$\text{Brake power, B.P.} = 220 \text{ kW}$$

$$\text{Mechanical efficiency, } \eta_{mech} = \frac{B.P.}{I.P.} = 87.53\%$$

$$\text{Fuel flow rate, } \dot{m}_f = 80 \text{ kg/hr}$$

$$\text{Calorific value of fuel, } (CV)_f = 43000 \text{ kJ/kg}$$

$$\text{HA/min} = 57333.33 \text{ kJ/min}$$

$$H_2 \text{ content in fuel} = 13\%$$

$$H_2 + \frac{1}{2}O_2 \rightarrow H_2O$$

1kg   8kg   9kg

i.e. 1 kg of H_2 = 9 kg of H_2O

In, 1 kg of fuel = 9 × 0.13 = 1.17 kg of H_2O

Hence, in 80 kg/hr of fuel, $\dot{m}_{steam} = 80 \times 1.17 = 93.6$ kg/hr of H_2O

Total mass of exhaust (wet), $\dot{m}_{ex} = \dot{m}_a + \dot{m}_f = \frac{80}{60} + 30 = 31.333$ kg/min

Total mass of dry exhaust, $\dot{m}_{ex,dry} = \dot{m}_{ex} - \dot{m}_{steam} = 29.773$ kg/min

(i) Heat lost ot cooling -water jacket, $\dot{Q}_{wj}$

$$\dot{Q}_{wj} = \dot{m}_w \times C_{P,w} \times (\Delta T)_{CT}$$
$$= 90 \times 4.18 \times 38 = 14295.6 \text{ kJ/min}$$

(ii) Heat lost to Dry exhaust gas,

$$\dot{Q}_{ex,dry} = \dot{m}_{ex,dry} \times C_{p,ex} \times (T_{ex} - T_{room})$$
$$\dot{Q}_{ex,dry} = 13853.377 \text{ kJ/min.}$$

(iii) Heat lost to piston cooling oil, $\dot{Q}_{piston\ oil}$

$$\dot{Q}_{piston\ oil} = \dot{m}_{piston\ oil} \times C_{p,oil} \times (\Delta T)_{cooling\ oil}$$
$$= 2277 \text{ kJ/min}$$

(iv) Assuming, the steam (H_2O) in exhaust in superheated state at exhaust temp (T_{ex}).

$$h_s = h_e + (LH)_S + h_{sup.}$$
$$= C_{P,w} \times (100 - 27) + 2520 + C_P, \text{sup} \times (450 - 100)$$
$$= 3525.24 \text{ kJ/kg}$$

Heat lost in steam(H_2O) of exhaust.

$$\dot{Q}_{ex,\ steam} = \dot{m}_{steam} \times h_s$$
$$= 5499.218 \text{ kJ/min}$$

(v) Unaccounted heat loss in radiation, error in calculation, etc

$$\dot{Q}_{un} = \frac{HA}{\min} - \left[\frac{B.P.}{\min} + \dot{Q}_{mj} + \dot{Q}_{ex,dry} + \dot{Q}_{ex,steam} + \dot{Q}_{piston\ oil}\right]$$
$$\dot{Q}_{un} = 26306.506 \text{ kJ/min}$$

		Percentage Contribution
(i)	B.P. = 220 × 60 = 13,200 kJ/min	23.023%
(ii)	$\dot{Q}_{wj}$ = 14295.6 kJ/min	24.93%
(iii)	$\dot{Q}_{piston\ oil}$ = 2277 kJ/min	3.97%
(iv)	$\dot{Q}_{ex,\ dry}$ = 13853.377 kJ/min	24.16%
(v)	$\dot{Q}_{steam}$ = 5499.218 kJ/min	9.59%
(vi)	HA/min = 57333.33 kJ/min	

Q.40 The products of combustion of an unknown fuel C_xH_y have the following composition as measured by an Orsat apparatus:

$CO_2 = 8.0\%$, $CO = 0.9\%$, $O_2 = 8.8\%$, $N_2 = 82.3\%$

Determine the values of x and y, the air-fuel ratio and % of excess air used.

[CSE (Mains) 2014 : 10 Marks]

Solution:

$$C_xH_y \xrightarrow[\text{combustion}]{\text{oxidation}} aCO_2 + bCO + CO_2 + dN_2$$

Orsat Analysis : $C_xH_y + b\left\{O_2 + \frac{79}{21}\times N_2\right\} \rightarrow aH_2O + 8CO_2 + 0.9\ CO + 8.8\times O_2 + 82.3\ N_2$

Balancing the chemical coefficient:

Balancing C: $X = 8 + 0.9 = 8.9$

Balancing O_2: $2b = a + 16 + 0.9 + 17.6 \rightarrow a = 9.254$

Balancing N_2: $\frac{79}{21}\times b = 82.3 \rightarrow b = 21.877$

Balancing H_2: $Y = 2a \Rightarrow Y = 18.508$

$X = 8.9$

$Y = 18.5$

$$\text{Air-Fuel Ratio} = \frac{21.877\times\left\{1+\frac{79}{21}\right\}\times 29}{8.9\times 12 + 18.5} = 24.11$$

$$C_{8.9}H_{18.5} + 13.525\left\{O_2 + \frac{79}{21}\times N_2\right\} \rightarrow 8.9\ CO_2 + 9.25H_2O + 13.525\times\frac{79}{21}\times N_2$$

$$\%\ \text{Excess Air} = \frac{21.877 - 13.525}{13.525}\times 100 = 61.75\%$$

Q.41 A 2-stroke oil engine was subjected to a test at room temperature of 20°C with fuel oil of calorific value 44000 kJ/kg. Calculate the indicated and brake, power, mechanical and brake thermal efficiency, and draw the heat balance sheet using the following data:

Cylinder bore = 20 cm; Stroke-bore ratio = 1.3 : 1; Speed = 500 r.p.m,; Brake drum diameter = 120 cm; Rope diameter = 3 cm; Net brake load = 460 N; Indicated MEP = 2.8 bar; Oil consumption = 3.7 kg/hr; Jacket cooling water rate = 456 kg/hr with a rise in temperature of 27°C; Exhaust gas temperature entering calorimeter is 320°C and leaving 220°C; Temperature rise in calorimeter water is 8°C with a rate of flow 8 kg/min.

[CSE (Mains) 2014 : 20 Marks]

Solution:

Given: For a 2-stroke oil engine

Cylinder bore, $D = 20$ cmStroke bore ratio, $\frac{L}{D} = 1.3$, Stroke, $L = 26$ cm, Speed, $N = 500$ rpm

$$\text{Swept volume flow rate, } \dot{V}_s = \frac{\pi}{4}D^2\times L\times k\times\frac{N}{60} = 0.0681\ \text{m}^3/\text{sec}$$

Brake drum radius, $r_d = 60$ cm

radius of rope, $r_{rope} = 61.5$ cm

Effective radius, $r_{ef} = 61.5$ cm

Brake torque, $T_b = F_b\times r_{eff} = 282.9$ N-m

Brake load, $F_b = 400$ N

Brake power, B.P. $= T_b\times W = 14.8126$ kW $= 14.813$ kW

Indicated mean effective pressure, $P_m = 2.8$ bar

Indicator power, I.P. $= P_{in}\times\dot{V}_s = 19.068$ kW

Fuel flow rate, $\dot{m}_f = 3.7$ kg/hr

Calorific value of fuel, $(C.V)_f = 44000$ kJ/kg

Heat addition rate, HA/sec $= \dot{m}_f\times(CV)_f = 45.222$ kW

$$\text{Mechanical efficiency, } \eta_{mech} = \frac{B.P.}{I.P.} = 77.685\%$$

$$\text{Brake thermal efficiency, } \eta_{bth} = \frac{B.P.}{HA/\sec} = 32.756\%$$

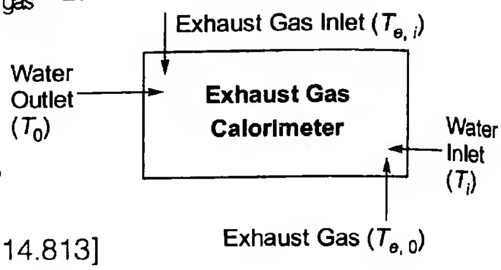
$$\dot{Q}_{\text{cooling water}} = \dot{m}_c \times C_{p,w} \times (\Delta T)_{nse} = 14.319 \text{ kW}$$

Heat absorbed by water in exhaust gas calorimeter,

$$\dot{Q}_{Ex \cdot w} = \frac{8}{60} \times 4.187 \times 8 = 4.466 \text{ kW}$$

Heat carried by exhaust gas is $\dot{Q}_{\text{Ex, gas}} = \frac{HA}{\sec} - \dot{Q}_{\text{cooling}} - \dot{Q}_{\text{Ex, }w} - \dot{Q}_{\text{Ex, gas}} - BP$

$= 2.169$ kW



(Assuming A/F = 20 : 1)

Unaccounted heat lost due to radiation, other losses, etc.

$$\dot{Q}_{\text{unaccounted}} = \frac{HA}{\sec} - \dot{Q}_{\text{cooling}} - \dot{Q}_{Ex,\,w} - \dot{Q}_{Ex,\,\text{gas}} - BP$$

$$\dot{Q}_{\text{unaccounted}} = 45.222 - [14.319 + 4.466 + 2.169 + 14.813]$$

$$= 9.455 \text{ kW}$$

Heat balance sheet

heat supplied by oil = 45.222 kW (100%)

1. Brake power equivalent = 14.813 kW (32.75%)
2. $\dot{Q}_{\text{water}}$ = 14.319 kW (31.66%)
3. $\dot{Q}_{\text{Ex, w}}$ = 4.466 kW (9.875%)
4. $\dot{Q}_{\text{Ex, gas}}$ = 2.169 kW (4.79%)
5. $\dot{Q}_{\text{unaccounted}}$ = 9.455 kW (20.90%)

Q.42 The following data refer to a 4-stroke, 4-cylinder diesel engine:

Cylinder diameter = 36 cm; Stroke = 40 cm;
Speed = 315 r.p.m.; Indicated MEP = 7 bar;
Brake power = 250 kW; Fuel consumption = 80 kg/hr;
Calorific value = 44 MJ/kg; Air consumption = 30 kg/min;
Cooling water circulated = 90 kg/min with rise in temperature 38°C;
Exhaust gas temperature = 324°C and Room temperature = 45°C kJ/kg K; $C_{P_{air}}$ = 1.005 kJ/kg K, $C_{P_{gas}}$ = 1.05 kJ/kg K, $C_{P_{steam}}$ = 2.093 kJ/kg K.
In exhaust gases, partial pressure of steam is 0.03 bar and fuel contains 13% H_2.
Find mechanical efficiency, indicated thermal η, brake specific fuel consumption. Draw heat balance sheet for the engine in hourly basis.

[CSE (Mains) 2014 : 10 Marks]

Solution:

Given: 4-stroke (Diesel Engine)

No. of cylinders, $k = 4$, Bore, $D = 36$ cm, Stroke, $L = 40$ cm, Speed, $N = 315$ rpm

$$\text{Rate of volume swept, } \dot{V}_s = \frac{\pi}{4} D^2 \times L \times k \times \frac{N}{60 \times 2} = 0.4275 \text{ m}^3/\text{s}$$

$$\text{Indicated mean effective pressure, } P_{im} = 7 \text{ bar}$$

$$\Rightarrow \quad \text{Indicated power, I.P.} = P_{in} \times \dot{V}_s = 299.25 \text{ kW}$$

$$\text{Brake power, B.P.} = 250 \text{ kW}$$

$$\text{Fuel flow rate, } \dot{m}_f = 80 \text{ kg/hr}$$

$$\text{Calorific value of fuel, } (CV)_f = 44 \text{ MJ/kg}$$

$$\Rightarrow \quad \frac{HA}{\text{sec}} = 977.78 \text{ kW}$$

$$\text{Air flow rate, } \dot{m}_a = 30 \text{ kg/min}$$

$$\dot{m}_w = 90 \text{ kg/min}$$

$$\text{Temperature difference, } (\Delta T)_W = 38°$$

$$\text{Heat taken by water jacket, } \dot{Q}_{\text{water jacket}} = \dot{m}_w \times C_{p,W} \times (\Delta T)_w = 238.659 \text{ kW}$$

Mass of wet exhaust gas per min. = mass of air per min + mass fo fuel per min

$$= 30 \text{ kg/min} + \frac{80}{60} \text{kg/min} = 31.33 \text{ kg/min}$$

$$\underset{1\,\text{kg}}{2H_2} + \underset{8\,\text{kg}}{O_2} \rightarrow \underset{9\,\text{kg}}{2H_2O}$$

i.e. 1kg of H_2 produced 9 kg of H_2O

$$\text{Mass '}H_2O\text{' burnt/kg of fuel burnt} = 9 \times H_2 = 9 \times 0.13 = 1.17 \text{ kg/kg of fuel}$$

$$\therefore \quad \text{Mass of } H_2O_{(\text{Steam})}\text{Produced/min} = 1.17 \times (\text{mass of fuel/min}) = 1.56 \text{ kg/min}$$

$$\text{Mechanical efficiency } (\eta_m) = \frac{B \cdot P}{I \cdot P}$$

$$\eta_m = \frac{250}{299.25} = 83.54\%$$

$$\eta_{ith} = \frac{I \cdot P}{\dot{m}_f \times C.V} = \frac{299.25}{977.78} = 30.605\%$$

$$\text{BSFC} = \frac{\dot{m}_F}{B \cdot P} = \frac{80}{250} = 0.32 \text{ kg/kW-hr}$$

$$\text{Mass of dry exhaust gases/min} = 31.33 - 1.56 = 29.77 \text{ kg/min}$$

$$\text{Heat lost to dry exhaust gases/minute} = 29.77 \times 1.05 \times (324 - 45)$$

$$= 8721.12 \text{ kJ/min}$$

Assuming that steam in exhaust exists as superheated steam at given conditions.

$$\text{Enthalpy } (h) = h_w + (LH) + h_{\text{sup}}$$

$$h = C_{p,\,w} \times (100 - 45) + 2250 + C_{p,\,\text{steam}} (324 - 100)$$

$$h = 4.18 \times (100 - 45) + 2250 + 2.093 \times (324 - 100)$$

$$h = 2948.732 \text{ kJ/kg}$$

$$\text{Heat lost to steam} = 1.56 \times 2948.732 = 4600 \text{ kJ/min}$$

$$\text{Unaccounted heat loss} = (977.78 \times 60) - [15000 + 14295.6 + 8721.12 + 4600]$$

$$= 16050.08 \text{ kJ/min}$$

Heat balance sheet in hourly basis–

Heat supplied by fuel = 3.52×10^6 kJ/hr

B.P. equivalent = 0.9×10^6 kJ/hr

heat lost to cooling medium = 0.859×10^6 kg/hr

heat lost to dry exhaust gas = 0.523×10^6 kJ/hr

Heat carried away by steam = 0.276×10^6 kJ/hr

Unaccounted losses = 0.963×10^6 kJ/hr

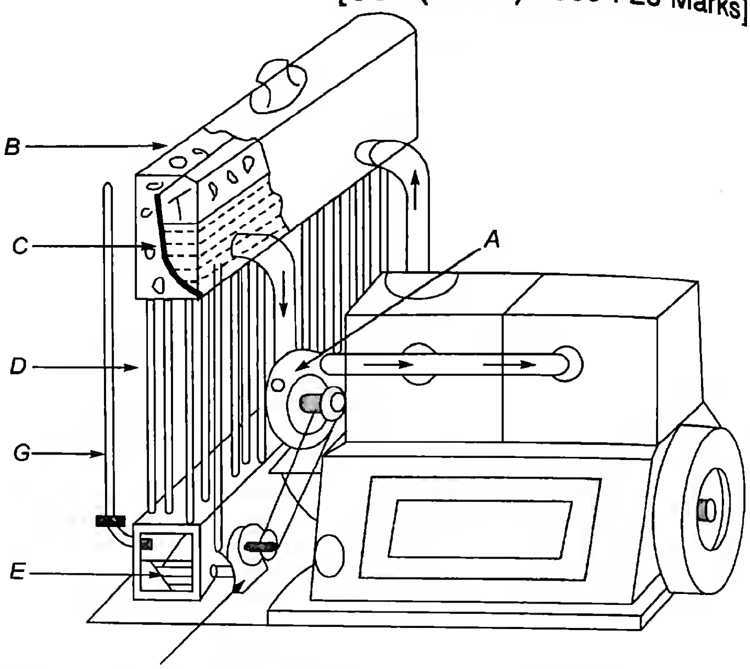
5. Different Systems of I.C. Engines

Q.43 (i) Briefly explain 'Evaporative Cooling System' which is generally used for big-capacity stationary IC engines, with a schematic diagram.
(ii) List four advantages and disadvantages each of a water-cooled system in a CI engine.
[CSE (Mains) 2009 : 20 Marks]

Solution:

Evaporative Cooling System: In this system, the engine is cooled because of the evaporation of the water in the cylinder jackets into steam. The high latent heat of vaporization of water is used to achieve cooling by allowing it to evaporate in the cylinder jackets.

This system is used predominantly in stationary engines and big capacity IC engines.



Evaporative Cooling with air cooled condensor: The figure above illustrates evaporative cooling with air-cooled condensor.

1. Water is circulated by pump *A* and when delivered to the overhead tank *B*, part of it boils out.

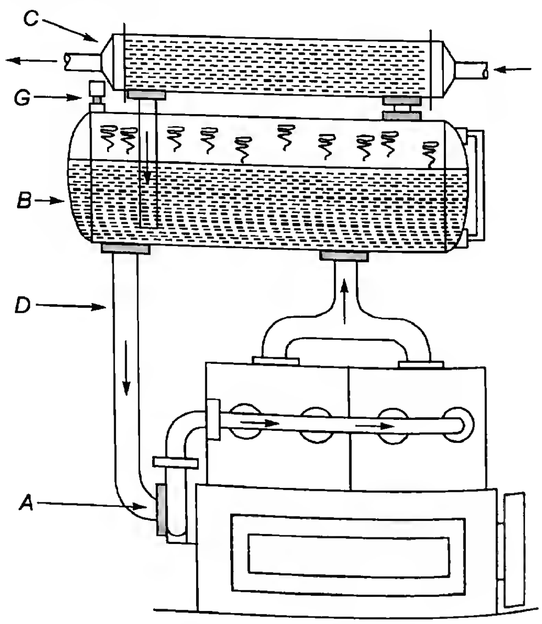
A partition *C* is provided in the tank.

The vapour rises above the partition *C* and the condensing action of the radiator tubes *D* makes the condensate flow into the lower tank *E* from which it is picked up and returned to the tank *B* by the small pump *F*.

The vertical pipe *G* is in communication with the outside atmosphere to prevent collapsing of the tanks *B* and *E* when the pressure inside them due to condensation falls below atmospheric.

Evaporative cooling with water-cooled condensor:

This type of cooling system is illustrated by the following figure:



In this case, condensation of the vapour formed in the overhead tank *B* occurs in the heat exchanger *C* cooled by a secondary water circuit and the water returns to *B* by gravity.

The pump *A* circulates the cooling water to the engine and heated water from the engine is delivered to tank *B* thereby maintaining circulation.

Advantages of water cooled system in CI engine are:

1. Compact design of engines with appreciably smaller frontal area is possible.
2. The fuel consumption of high compression liquid-cooled engines are rather lower than for air cooled ones.
3. Because of the even cooling of cylinder barrel and head due to jacketing makes it possible to reduce the cylinder head and valve seat temperatures.
4. In case of water cooled engines, installation is not necessarily at the front of the mobile vehicles, aircraft etc. as cooling system can be conveniently located wherever required.

Disadvantages:

1. This is a dependent system in which water circulation in jackets is to be ensured by additional means.
2. Power absorbed by the pumb for water circulation is considerable and affects the power output of engine.
3. Cost of the cooling system is considerably high.
4. System requires considerable maintenance of its various parts.

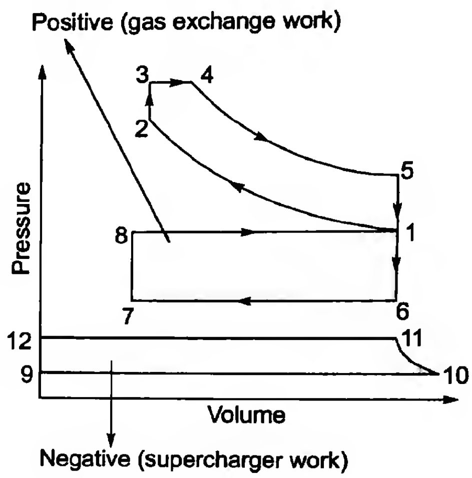
Q.44 (i) Discuss the objectives of supercharging and show the process on p-v diagram,
(ii) Give sketches of two common type of supercharging and turbocharging configurations.
(iii) Discuss parameters affecting engine heat transfer.

[CSE (Mains) 2010 : 20 Marks]

Solution:

(i) **Objectives of supercharging:**

1. To obtain higher power output.
2. To increase induction of charge mass.
3. To achieve better atomization of fuel.
4. To achieve better mixing of fuel and air.
5. To enable better scavenging of products.
6. To achieve better torque characteristic over the whole speed range.
7. To accelerate vehicle quickly.
8. To attain complete and smoother combustion.
9. To have smoother operation and reduction in diesel knock tendency.
10. To reduce exhaust smoke.
11. To have reduced specific fuel consumption in turbocharging.
12. To have improved mechanical efficiency.



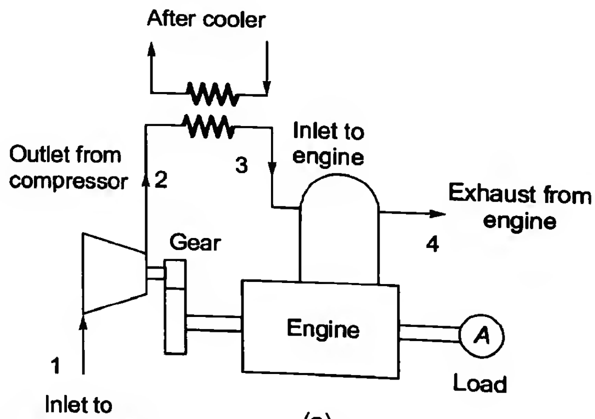
(ii) **Supercharging configurations**

Engine emissions can be classified into two categories:

(a) Gear Driven (b) Exhaust Driven

In gear driven arrangement, the compressor is coupled to the engine with step up gearing to increase the rotational speed of compressor. A certain percentage of engine output is utilized to drive the compressor,

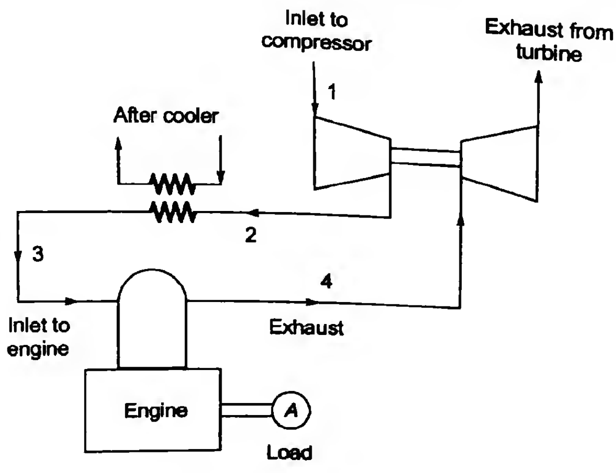
In exhaust driven arrangment, the engines are turbo-supercharged. The exhaust energy of the engine is used to drive the turbine which is coupled to a compressor and there is no mechanical coupling of compressor or turbine with engine.



(a)

Turbocharging configurations: Twin-turbo or bi-turbo designs have two separate turbochargers operating in two configurations:

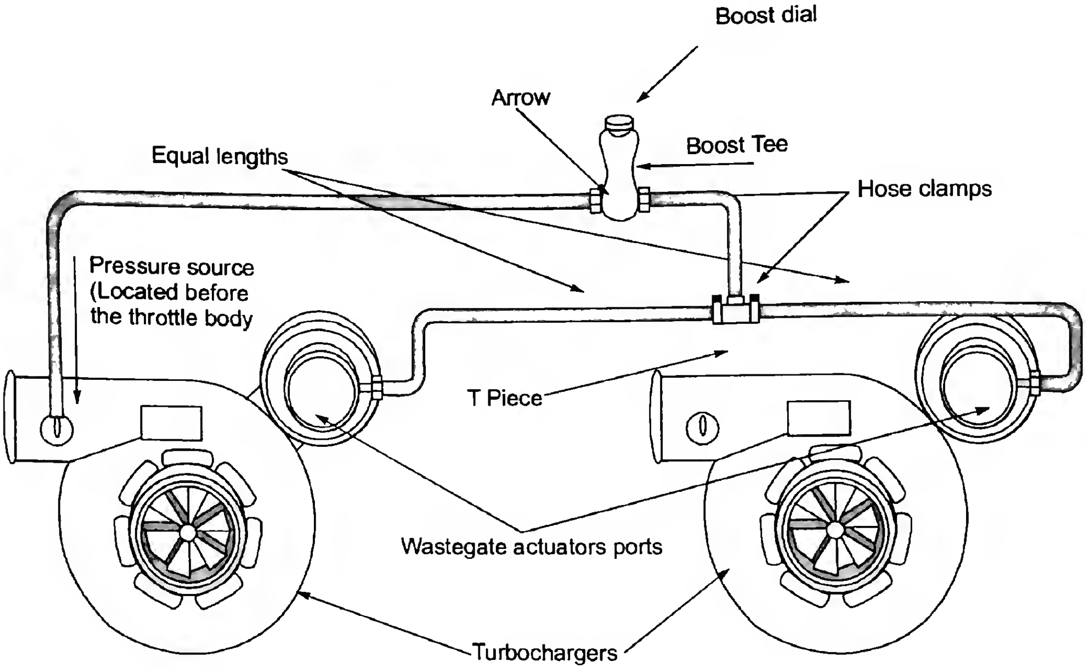
1. **Sequence:**
   - In a sequential setup one turbocharger runs at low speeds and the second turns on at a predetermined engine speed or load.
   - Sequential turbochargers require an intricate set of pipes to properly feed both turbochargers.
   - They reduce turbo lag that is the time required to change power output in response to a throttle change.
2. **Parallel:**
   - In a parallel configuration, both turbochargers are fed one-half of the engine's exhaust.



(b)

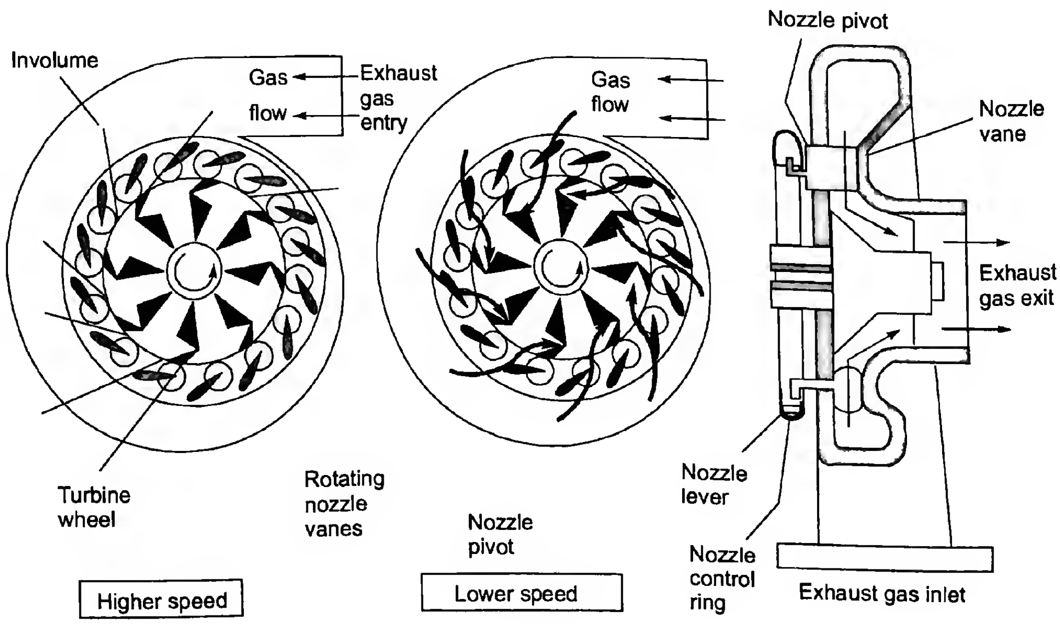
- Two-stage variable twin-turbochargers employ a small turbocharger at low speeds and a large one at higher speeds.

  They are connected in a series so that boost pressure from one turbocharger is multiplied by another. The distribution of exhaust gas is continuously variable, so the transition from using the small turbocharger to the large one can be done incrementally.
- Twin turbochargers are primarily used in Diesel engines.



Variable-geometry or variable-nozzle turbochargers

- They use moveable vanes to adjust the air-flow to the turbine, imitating a turbocharger of the optimal size throughout the power curve.
- The vanes are placed just in front of the turbine like a set of slightly overlapping walls.
- Their angle is adjusted by an actuator to block or increase air flow to the turbine. This variability maintains a comparable exhaust velocity and back pressure throughout the engine's speed range.
- This results in the turbocharger improving fuel efficiency without a noticeable level of turbocharger lag.



(iii) Parameters Affecting Engine Heat Transfer

(i) **Fuel-Air Ratio:** A change in fuel-air ratio changes the temperature of the cylinder gases and affects the flame speed. The maximum gas temperature occurs at a fuel air ratio about 0.075 and maximum heat rejection is found to occur for a mixture slightly leaner than this value.

(ii) **Compression Ratio:** An increase in compression ratio causes a slight increase in gas temperature near the top dead centre. Near bottom dead centre, a considerable reduction in gas temperature is observed. The exhaust gas temperature will be much lower because of greater expansion so that the heat rejected during blowdown will be less. In general, an increase in compression ratio causes marginal reduction in heat rejection.

(iii) **Spark Advance:** A spark advance more than the optimum as well as less than the optimum results in increased heat rejection to the cooling system. This is mainly due to the fact that the spark timing other than MBT value (Minimum spark advance for best torque) will reduce the power output and thereby more heat is rejected.

(iv) **Pre-ignition and knocking:** Effect of pre-ignition is same as advancing the ignition timing, with large spark advance leading to erratic running and knocking. Knocking causes large changes in local heat transfer conditions. Overall effect on heat transfer due to knocking appears negligible.

(v) **Engine Output:** Engines which are designed for high mean effective pressures or high piston speeds, heat rejection will be less. Less heat will be lost for the same indicated power in large engines.

(vi) **Cylinder Wall Temperature:** Since the average cylinder gas temperature is much higher in comparison to cylinder wall temperature, any marginal change in cylinder gas temperature has little effect on the temperature difference and thus on heat rejection.

Q.45 (i) What is meant by firing order in internal combustion engines?
(ii) What are the firing orders used in 4 and 6 cylinder inline engines?
(iii) What are the three purposes of firing order in 'V' engines?

[CSE (Mains) 2012 : 2 + 4 + 6 = 12 Marks]

Solution:

(i) The firing order is the sequence of power delivery of each cylinder in a multi-cylinder reciprocating engine. The number of possibilities of firing order depends upon the number of cylinders and throws of the crankshaft. Proper firing order is achieved by sparking of the spark plugs in a gasoline engine in the correct order, or by the sequence of fuel injection in a diesel engine. When designing an engine, choosing an appropriate firing order is critical to minimizing vibration, to improve engine balance and achieving smooth running, for long engine fatigue life and user comfort, and thus heavily influencing crankshaft design.

(ii) The possible firing orders are given below:

Firing order in 4 cylinder inline engines

1-3-4-2

1-2-4-3

Firing order in 6 cylinder inline engines

1-5-3-6-2-4 (used most commonly)

1-5-4-6-2-3

1-2-4-6-5-3

1-2-3-6-5-4

(iii) The purposes of firing order in V engines are:

i. To evenly distribute the load on the bearings as any imbalance would increase the crankshaft vibrations and would result in failure of the system

ii. To achieve optimum level of cooling, firing order of the engine is important. If two adjacent cylinders are fired consecutively, the portion of the engine between them gets overheated and it puts strain on the cooling system. Proper firing order controls the problem of overheating.

iii. To avoid the development of back pressure, a proper firing order is necessary. The exhaust gases in the exhaust pipe increases the back pressure and the possibility of back flow arises if the firing order is not proper.

Q.46 Discuss the lubrication of the following engine parts with the help of neat sketches:
(i) Main bearings
(ii) Cylinder and small end bearing of connecting rod
(iii) Crank and Gudgeon pin.

[CSE (Mains) 2015 : 15 Marks]

Solution:

(i) **Lubrication of main bearings:** In small stationary engines, the main bearings of the crankshaft are made with rings which are slipped over the shaft and run over the journal, known as ring oilers.

The rings tend to rotate with the rotating shaft. While rotating, the bottom portion of the ring picks up oil from the reservoir and carries it to the top of the journal where it is able to flow into the bearing oil grooves and bearing clearance space, to be distributed to the entire bearing surface.

In large engines, where the crankcase is not very tight, the main bearings are lubricated by positive feed lubricators.

Vertical engines, which have enclosed crank cases, are usually built with pressure lubrication. The oil is drawn by gear pump and delivered to the oil gallery in the crankcase which distributes it to the bearings. Oil from the bearings is picked up by a scavenging pump. It is pumped through a filter to the oil cooler and to a supply tank from which it flows back to the pressure pump.

(ii) **Lubrication of cylinder and small end bearing of connecting rod:** In small horizontal engines, the small end bearing or wristpin bearing is lubricated by a sight feed oiler. A special scraper scoop is provided to scrap the oil dripping from the cylinder walls. the arrangement is shown in the figure.

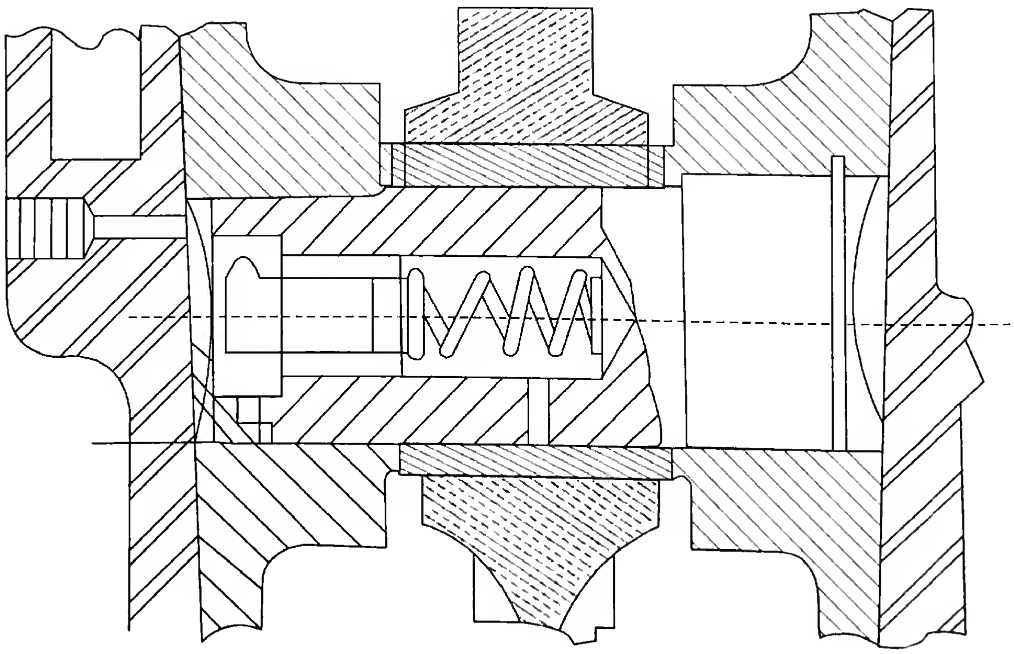
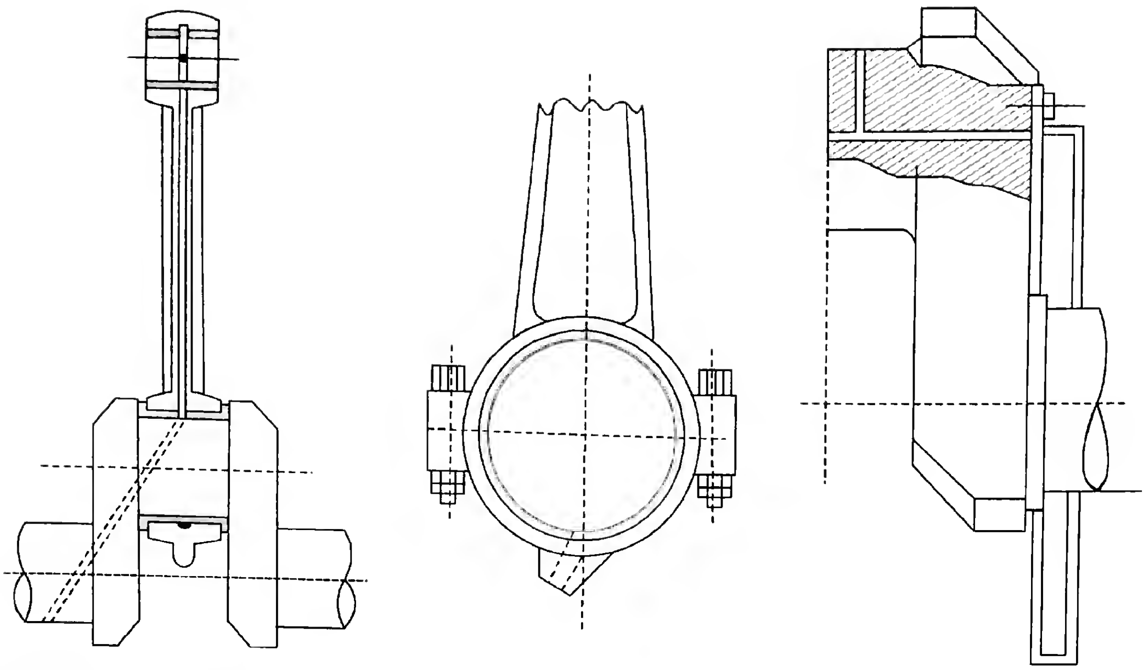


Figure: Lubrication of wrist pin bearings

In vertical engines where lubricating oil is delivered under pressure to the main and crankpin bearings the small end bearing is lubricated by excess oil from the crankpin bearing. A hole is drilled through the connecting rod for this purpose, as shown in the figure.



(iii) **Lubrication of Crank and gudgeon pin:** In small engines, having low bearing pressure, the crankpins are lubricated by splashing oil. The bottom of the connecting rod is attached with a dipper which dips into the oil when the connecting rod comes out.

In small horizontal engines and some two stroke engines, a centrifugal banjo oiler shown by the figure below, is used.

The oil hole is drilled at an angle of 30 degree before the dead centre, so that upper shell receives oil before the ignition at a point of relatively low pressure.

Q.47 What are the supercharging limits of SI and CI engines? What are the modifications recommended for supercharging an IC engine?

[CSE (Mains) 2016 : 10 Marks]

Solution:

Supercharging limitations of SI engines:

1. Knocking tendency increases with increase in temperature, pressure and density of charge. Therefore the fuel must have better anti-knock properties.
2. Compression ratio of engine needs to be reduced in a supercharged engine but the reduced compression ratio reduces the power output and efficiency of engine by increasing the specific fuel consumption.
   In CI engines, the supercharging limits are mainly due to thermal stresses and gas loading. Due to this, heat generation and heat transfer increases and there is greater tendency to burn the piston crown, seat and edges of exhaust valve.

Modifications recommended for supercharging an IC engine:

1. Supercharging increases the thermal load on various engine parts. Hence some engines are provided with a hollow space on piston crown through which oil or water is circulated for cooling.
2. Piston crown, seat and edges of the exhaust valve are made of better materials capable of withstanding high temperatures.
3. Increased gas loading caused by supercharging necessities the use of larger bearing areas and heavier engine components.
4. Increased heat generation and heat transfer during supercharging increases the tendency of burning the piston crown, seat and edges of exhaust valve. Hence valve overlap is increased in supercharged engines during which cooler air flows past the valves and piston crown. The valve overlap may vary from about 80 to 160 degree of crank travel.

■■■■

5 Steam Engineering

1. Economics of Power Generation

Q.1 A power generating station has a maximum demand of 20 MW The daily load on the station is as follows:

Time	Load, MW
6 AM to 8 AM	7
8 AM to 12 NOON	16
12 NOON to 1 PM	6
1 PM to 5 PM	15
5 PM to 7 PM	17
7 PM to 9 PM	20
9 PM to 11 PM	9
11 PM to 6 AM	4

(i) Draw the load curve and load duration curve for the plant.

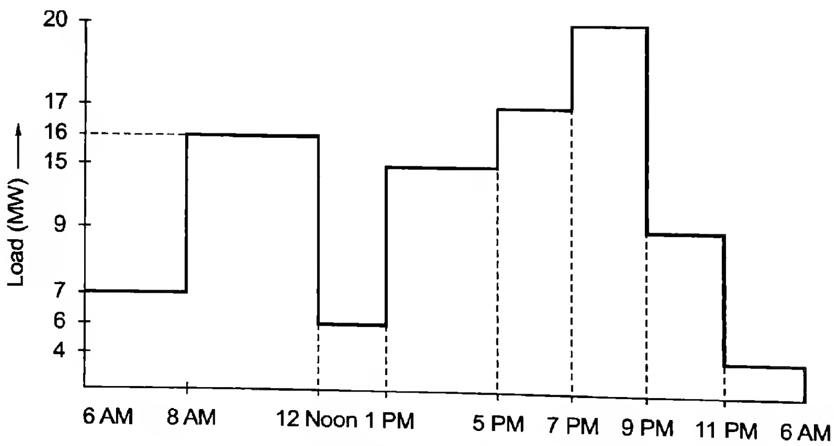
(ii) Decide the capacity and number of units.

(iii) Prepare the operating schedule of the units.

(iv) Determine the load factor, plant capacity factor and plant use factor of the station.

[CSE (Mains) 2001 : 30 Marks]

Solution:

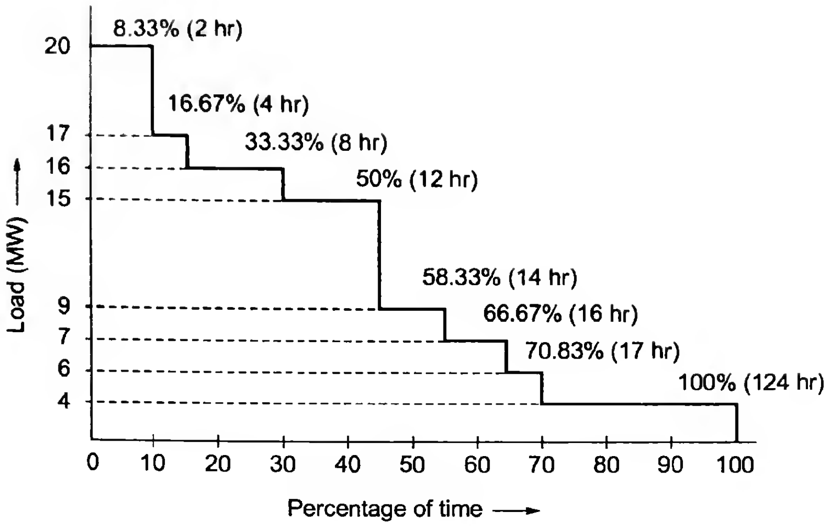


Load Curve

Load	Hours in a day	%time
20 MW	2	8.33
17 MW	2 + 2 = 4	16.67
16 MW	2 + 2 + 4 = 8	33.33
15 MW	2 + 2 + 4 + 4 = 12	50
9 MW	2 + 2 + 4 + 4 + 2 = 14	58.33
7 MW	2 + 2 + 4 + 4 + 2 + 2 = 16	66.67
6 MW	2 + 2 + 4 + 4 + 2 + 2 + 1 = 17	70.83
4 MW	2 + 2 + 4 + 4 + 2 + 2 + 1 + 7 = 24	100

Units generated during 24 hours $= 2 \times 7 + 4 \times 16 + 1 \times 6 + 4 \times 15 + 2 \times 7 + 2 \times 20 + 2 \times 9 + 7 \times 4$

$= \mathbf{264\ MW\ hr}$



Load Duration Curve

From the load variation over the 24 hours (starting from 6 AM) it can be seen that units, each of 5 MW will suffice. Since we want continuity of supply, a reserve capacity equivalent to the largest unit will be taken.

$$\text{Total capacity of the plant} = 5 \text{ MW} \times 5 = \mathbf{25 \text{ MW}}$$

$$\text{Number of units} = \mathbf{5}$$

$$\text{Load factor} = \frac{264}{24 \times 20} = 0.55$$

$$\text{Plant capacity factor} = \frac{264}{25 \times 24} = 0.44$$

Operation Schedule

11 PM to 6 AM	One unit of 5 MW
6 AM to 8 AM	Two unit of 5 MW
8 AM to 12 Noon	Four unit of 5 MW
12 Noon to 1 PM	Two unit of 5 MW
1 PM to 5 PM	Three unit of 5 MW
5 PM to 7 PM	Four unit of 5 MW
7 PM to 9 PM	Four unit of 5 MW
9 PM to 11 PM	Two unit of 5 MW

With the operation schedule listed above, the energy that could have been generated, by the capacity of the plant actually running for the scheduled time will be

$$= 7 \times 5 + 2 \times 10 + 4 \times 20 + 1 \times 10 + 4 \times 15 + 2 \times 20 + 2 \times 20 + 2 \times 10$$
$$= 35 + 20 + 80 + 10 + 60 + 40 + 40 + 20 = 305 \text{ MW}$$

$$\text{Plant use factor} = \frac{264}{305} = 0.8655 \text{ or } \mathbf{86.55\%}$$

Q.2 **A thermal power plant of 200 MW capacity has the maximum load of 160 MW and its annual load factor is 0.65. The coal consumption is kg per kWh of energy generated and the cost of coal is ₹ 800 per ton. Other annual running expenses are ₹ 200 × 10^6. Calculate:**
(i) the annual revenue earned if the energy is sold at ₹ 1.5 per kWh sand
(ii) the capacity factor of the plant.

[CSE (Mains) 2002 : 20 Marks]

Solution:

$$\text{Annual load factor} = \frac{\text{Average load}}{\text{Peak load}}$$

$$\text{Average} = 0.65 \times 160 = 104 \text{ MW}$$

$$\text{Energy generated per year} = 104 \times 24 \times 365 = 911040 \text{ MWh} = 911040 \times 10^3 \text{ kWh}$$

$$\text{Coal required per year} = 911040 \times 10^3 \text{ kg} = 911040 \text{ tonnes}$$

$$\text{Cost of coal} = ₹800 \text{ per ton}$$

$$\text{Cost of coal per year} = 911040 \times 800 = ₹728.832 \times 10^6$$

$$\text{Cost of energy sold} = ₹1.5 \times 911040 \times 10^3 = ₹1366.56 \times 10^6$$

(i) Revenue earned by the power plant per year = cost of energy sold – cost of coal

$$= ₹1366.56 \times 10^6 - ₹728.832 \times 10^6 = ₹637.728 \times 10^6$$

$$= \mathbf{₹63.773 \text{ crore}}$$

(ii) $$\text{Capacity factor} = \frac{\text{Average load}}{\text{Capacity of the plant}} = \frac{104}{200} = \mathbf{0.52}$$

Q.3 A power plant of 210 MW installed capacity has the following particulars:

Capital cost = ₹ 4 crores/MW installed | **Interest and depreciation = 12%**
Annual load factor = 60% | **Annual capacity factor = 54%**
Annual running charges = ₹ 400 × 10^6 | **Energy consumed by power plant auxiliaries = 6%**

Calculate:

(i) The cost of power generation per kWh (ii) The reserve capacity

[CSE (Mains) 2004 : 20 Marks]

Solution:

$$\frac{\text{Load factor}}{\text{Capacity factor}} = \frac{\text{Average load}}{\text{Maximum demand}} \times \frac{\text{Capacity of the plant}}{\text{Average load}}$$

$$\frac{0.60}{0.54} = \frac{210\,\text{MW}}{\text{Maximum demand}}$$

$$\text{Maximum demand} = \frac{210 \times 0.54}{0.60} = 189 \text{ MW}$$

$$\text{Reserve capacity} = 210 - 189 = \mathbf{21 \text{ MW}}$$

$$\text{Average load} = \text{Load factor} \times \text{Maximum demand} = 0.6 \times 189 = 113.4 \text{ MW}$$

$$\text{Energy produced per year} = 113.4 \times 10^3 \times 8760 = 993.384 \times 10^6 \text{ kWh}$$

$$\text{Net energy delivered} = 0.94 \times 993.384 \times 10^6 = 933.781 \times 10^6 \text{ kWh}$$

Annual interest and depreciation (fixed cost)

$$= 0.12 \times 4 \times 10^7 \times 210 = ₹100.8 \times 10^7$$

$$\text{Total annual cost} = \text{Fixed cost} + \text{Running cost}$$

$$= ₹100.8 \times 10^7 + ₹400 \times 10^6 = ₹140.8 \times 10^7$$

$$\text{Cost of power generation} = \frac{₹140.8 \times 10^7}{933.781 \times 10^6} = ₹\ 1.5/\text{KWh}$$

Q.4 Discuss the factors to be considered in the selection of a site for a hydroelectric power plant.

[CSE (Mains) 2004 : 10 Marks]

Solution:

The following factors should be considered while selecting the site for hydroelectric power plant.

1. Availability of water
2. Water storage capacity
3. Available water head
4. Accessibility of the site
5. Distance from the load centre
6. Type of land of site

1. **Availability of water:** The design and capacity of the hydro-plant greatly depends on the amount of water available at the site. The run-off data along with precipitation at the proposed site with maximum and minimum quantity of water available in a year should be made available to
   (a) decide the capacity of the plant,
   (b) set up the peak load plant such as steam, diesel or gas turbine plant,
   (c) provide adequate spillways or gate relief during flood period,

2. **Water storage capacity:** Since there is a wide variation in rainfall all round the year, it is always necessary to store the water for continuous generation of power. The storage capacity can be estimated with the help of mass curve.
3. **Available water head:** In order to generate the desired quantity of power it is necessary that a large quantity of water at a sufficient head should be available. An increase in effective head, for a given output, reduces the quantity of water required to be supplied to the turbines,
4. **Accessibility of the site :** The site should be easily accessible by rail and road. An inaccessible terrain will jeopardize the movement of men and material.
5. **Distance from the load centre:** If the site is close to the load centre, the cost of transmission lines and the transmission losses will be reduced.
6. **Type of the hind of the site:** The land of the site should be cheap and rocky. The dam constructed at the site should have large catchment area to store water at high head. The foundation rocks of the masonry dam should be strong enough to withstand the stresses in the structure and the thrust of water when the reservoir is full.

Q.5 A central power station has annual factors as follows:
Load factor = 0.6, Capacity factor = 0.4 and Use factor = 0.45.
The power station has a maximum demand of 15 MW. Determine:
(i) Annual energy production
(ii) Reserve capacity over and above peak load
(iii) Hours per year the plant is not in service

[CSE (Mains) 2005 : 20 Marks]

Solution:

(i)
$$\text{Load factor} = \frac{\text{Average load}}{\text{Peak load}}$$
$$\text{Average load} = 0.6 \times 15 = 9 \text{ MW}$$
$$\text{Annual energy production} = 9000 \times 8760 = \mathbf{78.84 \times 10^6 \text{ kWh}}$$

(ii)
$$\textbf{Capacity factor} = \frac{\text{Average load}}{\text{Plant capacity}}$$
$$\text{Plant capacity} = \frac{9 \text{ MW}}{0.4} = 22.5 \text{ MW}$$

Reserve capacity over and above the peak load = 22.5 – 15 = **7.5 MW**

(iii)
$$\text{Use factor} = \frac{\text{Energ generator per year}}{\text{Plant capacity} \times \text{hours in operation}}$$
$$\text{Hours in operation} = \frac{78.84 \times 10^6}{22.5 \times 0.45 \times 10^3} = \mathbf{7786.67 \text{ hours}}$$
$$\text{Hours not in service in a year} = 8760 - 7786.67 = \mathbf{973.33 \text{ hours}}$$

Q.6 Sketching 'load curve' and 'load duration curve', explain their purposes. Define also 'demand factor' and 'plant-use factor'.
The loads for certain industries are tabulated below for 24 hours. During load duration and load curve, find power required for 40% of the time of the day. If the capacity of the power plant is 35 MW, find the capacity factor of the power plant:

Time	6 AM to 8 AM	8 AM to 9 AM	9 AM to 11 AM	11 AM to 2 AM	2 AM to 5 AM
Load (in MW)	18	26	30	22	24
Time	5 PM to 8 PM	8 PM to 12 PM	12 PM to 5 PM	5 PM to 6 PM	
Load (in MW)	30	20	15	16	

If the load is supplied by two power plants, one is acting as a base load plant having capacity of 25 MW and other as peak load plant having capacity of 10 MW, find load factor, capacity factor and use factor for both power plants.

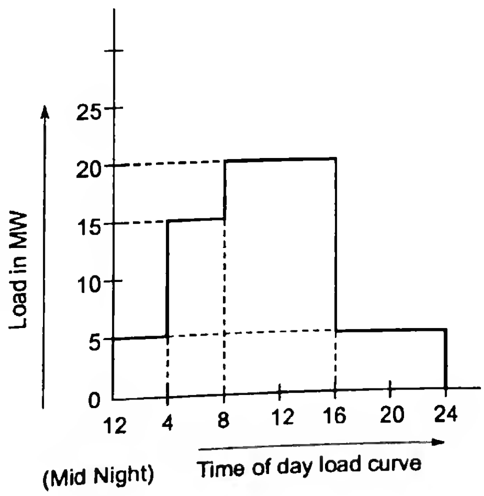
[CSE (Mains) 2007 : 20 Marks]

Solution:

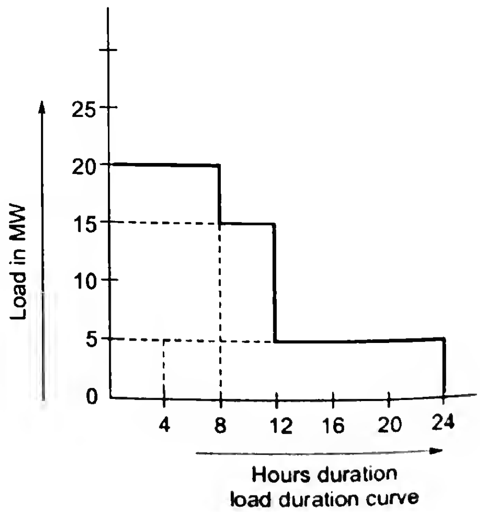
The Load Curve is a Graph, which represents load on the generation station (the load is in kW/MW) recorded at the interval of half hour or hour (time) against the time in chronological order. The Load Curve is defined as the curve which is drawn between loads versus time in sequential order. We have to draw the load curve on daily basis data, weekly, monthly basis data. The Load Curve gives following Information:

- The daily load curve shows the variation of load on the power station during different hours of the day.
- The area under the daily load curve gives the number of unit generated in the day. Unit generated/day = Area (in kWh) under daily load curve.
- The highest point on the daily load curve represents the maximum demand on the station on that day.
- The area under the daily load curve divided by the total number of hours gives the average load on the station in that day.
- The load curves helps in selecting the size & number of generating units.
- The load curve helps in preparing the operation schedule of the station.

When the load elements of a load curve are arranged in the order of descending magnitudes, the curve thus obtained is called a load duration curve. The load duration curve is obtained from the same data as load curve but the ordinate representing the maximum load is represented to the left and the decreasing loads are represented to the right in the descending order.



(i)



(ii)

Demand factor is the ratio of Maximum Demand on the Power Station to its Connected Load. The value of Demand factor is usually less than 1. It is excepted because maximum demand on the power station generally less than the connected load. The knowledge of Demand Factor is vital in determining the capacity of the plant equipments.

Plant use factor is the ratio of kWh generated to the product of plant capacity and the number of hours for which the plant was in operation. Plant use factor indicates how much is the plant capacity utilized, but it does not indicate the time for which the plant remains idle.

Numerical Part:

Energy generated during 24 hours of operation

$$= [18 \times 2 + 26 \times 1 + 30 \times 2 + 22 \times 3 + 24 \times 3 + 30 \times 3 + 20 \times 4 + 15 \times + 16 \times 1] \times 10^3 = 521 + 10^3 \text{ kWh}$$

$$\text{Average load} = \frac{521 \times 10^3}{24} = 21708.33 \text{ kW}$$

$$40\% \text{ of the time of the day} = 0.4 \times 24 = 9.6 \text{ hours}$$

Power required for 40% of the time of the day, = 21708.33 × 9.6 = 208,400 kWh

$$\text{Capacity factor} = \frac{521 \times 10^3}{35 \times 10^3 \times 24} = 0.62$$

$$\text{Installed capacity of base load plant} = 25 \times 10^3 \text{ kW}$$

$$\text{Installed capacity of peak load plant} = 10 \times 10^3 \text{ kW}$$

Base Plant

$$\text{Load factor} = \frac{\text{kWh generated during 24 hours}}{\text{Max. demand} \times \text{Working hours}} = \frac{521 \times 10^3}{25 \times 10^3 \times 24} = 0.868$$

Maximum demand is 30 MW and capacity of base load plant is 25 MW. so, maximum 25 MW can only be met by base load plant.

Q.7 The input-output curve of a 60 MW power station is given by $I = 5 \times 10^6 [8 + 8L + 0.4 L^2]$ kJ/hr.
where, I = Input in kJ/hr, L = Load in MW
Determine:
(i) The heat input per day to the power station if it works for 20 hours at full load and remaining period at no load.
(ii) Saving per kWh of energy produced if the plant works at full load for all 24 hours generating the same amount of energy.

[CSE (Mains) 2008 : 20 Marks]

Solution:

(i) Heat input per day:

Total energy generated by the plant during 24 hours = 20 × 60 + 5 × 0 = 1200 MWh

Input to the plant when the plant is running at full load

$$I_{60} = 5 \times 10^6[8 + 8 \times 60 + 0.4 \times 3600] \times 20$$

$$= 5 \times 10^6 \times 1928 \times 20 \text{ kJ during 20 hours when the plant was running at full load}$$

$$\text{Input at no load, } I_0 = 5 \times 106 \times 8 \times 4$$

$$= 200 \times 10^6 \text{ kJ during 4 hours when the plant was running at no load.}$$

$$\text{Total input to the plant during 24 hours} = I_{60} + I_0 = 5 \times 10^6 \times 1928 \times 20 + 200 \times 10^6 = \mathbf{193 \times 10^9 \text{ kJ/day}}$$

(ii) Saving per kWh:

$$\text{Average heat supplied per kWh generated} = \frac{193 \times 10^9}{1200 \times 10^3} = 160.83 \times 10^3 \text{ kJ/kWh}$$

If the same energy is generated within 24 hours, the average load is given by:

$$\text{Average load} = \frac{1200}{24} = 50 \text{ MW}$$

Heat supplied during 24 hours in this case

$$I_{50} = 5 \times 10^6 [8 + 8 \times 50 + 0.4 \times 2500] \times 24 = 168.96 \times 10^9 \text{ kJ/day}$$

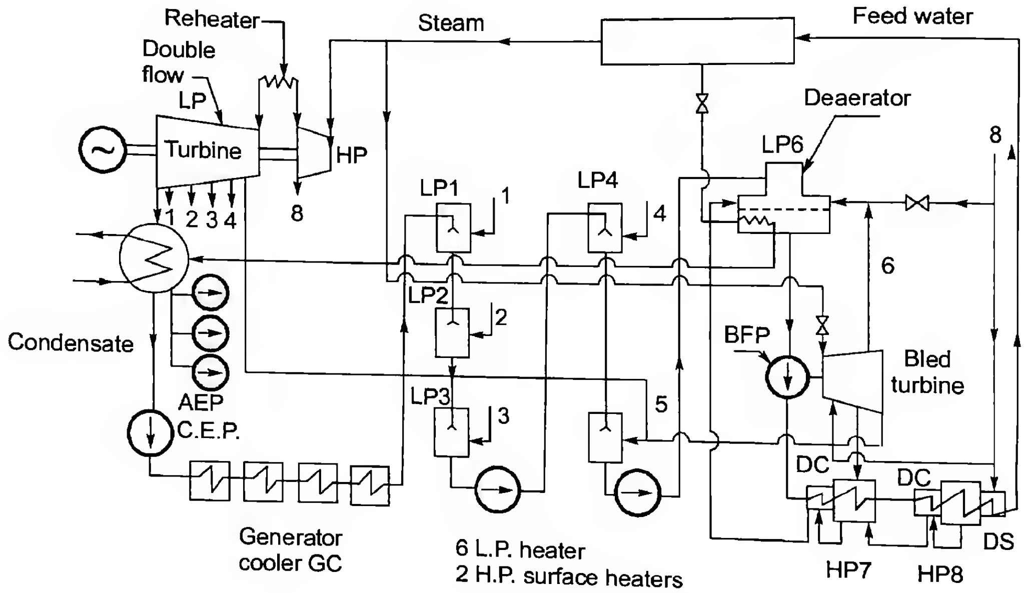
$$\text{Saving per kWh} = \frac{24.04 \times 10^9}{1200 \times 10^3} = 20.03 \times 10^3 \text{ kJ/kWh}$$

Q.8 Give a practical feed heating arrangement of a 660 MW steam power plant by showing steam and feed flow paths. Mention its special features.

[CSE (Mains) 2010 : 10 Marks]

Solution:

Feed Heating System for a 660 MW Unit: Figure shows a typcal feed heating system for a 660 MW steam turbine unit with reheat. The steam conditions at the turbine stop valve are 158 bar and 538°C. The reheat temperature is also 538°C. Eight stages of feed heating is employed to give a final feed water temperature of 254°C.



660 MW Feed Heating System

The salient features of this system are as follows:

(i) There are five direct contact Low Pressure (LP) heaters in two cascades one deaerator and two High Pressure (HP) surface type fded heaters.

(ii) There is a single bank of HP heater instead of two.

(iii) Similar to 350 and 500 MW, boiler feed pump is driven by a separate steam turbine meant for this only.

2. Gas Turbines

Q.9 Differentiate between a cogeneration and a combined cycle power plant.

[CSE (Mains) 2004 : 10 Marks]

Solution:

Combined cycle is the typical process which uses both gas (gas turbine) and steam (steam Turbine) to generate the power which is 50% higher than the normal process.

Combined cycle is a term applied to gas turbine generators in which the exhaust heat from the gas turbine is used to produce steam (in a heat recovery steam generator - HRSG), which is then fed to a steam turbine. The steam turbine may be on the same shaft as the gas turbine generator, or it may be a completely separate steam turbine & electrical generator.

Co-generation Cycle is the method where the consumption of steam required for the process (like sugar plant or paper mill), but the consumption rate is meager and the max portion of steam used to generate the power using seam turbine for their power requirement. most of the captive power plant works in this co-generation terminology.

Cogeneration is when the heat produced from a combustion process is split between electrical generation and industrial process steam.

The 'combustion process' can be either a boiler or a gas turbine with HRSG. The 'industrial process' steam can be for truly industrial chemical processes, or it could be for non-industrial steam plants, such as campus heating & cooling.

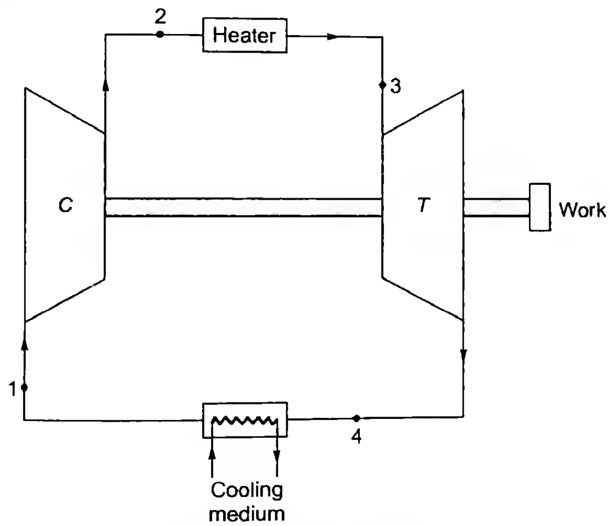
In summary, "combined cycle" refers specifically to a gas turbine generator with an exhaust-heated steam turbine generator to increase overall power plant efficiency.

"Cogeneration" is making heat through whatever means, and dividing the heat energy between electric generation and other process needs.

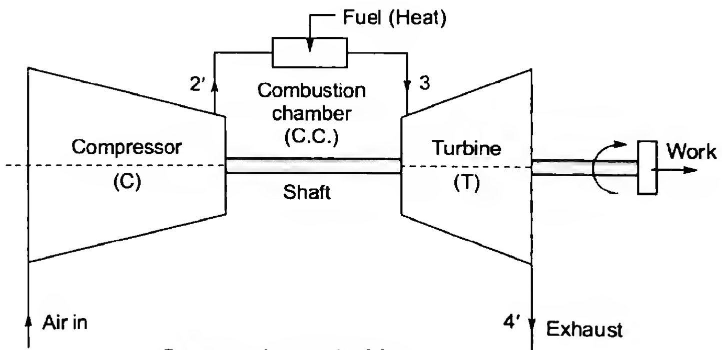
Q.10 **Explain with the help of diagrams the difference of working between an open cycle gas turbine and a closed cycle gas turbine. What advantages are derived by using regeneration for open cycle constant pressure gas turbine? Explain properly with the help of diagram.**

[CSE (Mains) 2008 : 30 Marks]

Solution:



Closed cycles gas turbine

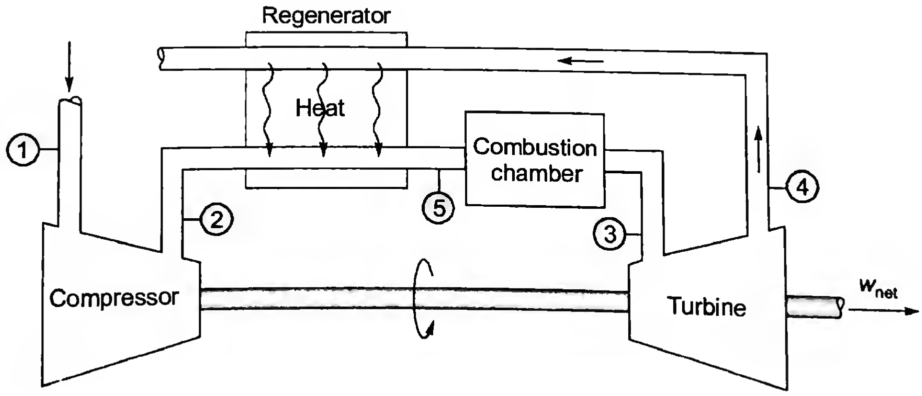


Open cycle gas turbine

S.No.	Criterion	Closed Cycle Gas Turbine	Open cycle gas turbine
1.	Cycle of operation	It works on closed cycle. The working fluid is re-circulated again and again. It is a clean cycle.	It works on open cycle. The fresh charge is supplied to each cycle and after combustion and expansion. It is discharged to atmosphere.
2.	Working fluid	The gases other than the air like Helium or Helium-Carbon dioxide mixture can be used, which has more favourable properties.	Air-fuel mixture is used which leads to lower thermal efficiency.
3.	Type of fuel used	Since heat is transferred externally, so any type of fuel; solid, liquid or gaseous or combination of these can be used for generation of heat.	Since combustion is an integral part of the system thus it requires high quantity liquid or gaseous fuel for burning in a combustion chamber.

4.	Manner of heat input	The heat is transferred indirectly through a heat exchanger.	Direct heat supply. It is generated in the combustion chamber itself.
5.	Quality of heat input	The heat can be supplied from any source like waste heat from some process, nuclear heat and solar heat using a concentrator.	It requires high grade heat energy for generation of power in a gas turbine.
6.	Efficiency	High thermal efficiency for given lower and upper temperature liquids.	Low thermal efficiency for same temperature limits.
7.	Part load efficiency	Part load efficiency is better.	Part load efficiency is less compared to Closed cycle gas turbine
8.	Size of plant	Reduced size per MWh of power output.	Comparatively large size for same power output.
9.	Blade life	Since combustion products do not come in direct contact of turbine blade, thus there is no blade fouling and longer blade life.	Direct contact with combustion products, the blades are subjected to higher thermal stresses and fouling and hence shorter blade life.
10.	Control on power production	Better control on power production.	Poor control on power production.
11.	Cost	Closed cycle gas turbine plant is complex and costly.	Open cycle gas turbine plant is simple and less costly.

Regeneration of Gas Turbine Plant Regeneration process involves the installation of a heat exchanger in the gas turbine cycle. The heat-exchanger is also known as the recuperator. This heat exchanger is used to extract the heat from the exhaust gas. This exhaust gas is used to heat the compressed air. This compressed and pre-heated air then enters the combustors. When the heat exchanger is well designed, the effectiveness is high and pressure drops are minimal. And when these heat exchangers are used an improvement in the efficiency is noticed. Regenerated Gas turbines can improve the efficiency more than 5%. Regenerated Gas Turbine work even more effectively in the improved part load applications.



Advantages of Regeneration cycle:

- Heat supplied to boiler becomes reduced
- Thermal efficiency is increased since the average temperature of heat addition to the cycle is increased.
- Due to bleeding in the turbine, erosion of turbine due to moisture is reduced.

Q.11 Deduce expressions for optimum pressure ratio for
(i) maximum specific output work and
(ii) maximum cycle efficiency for a Brayton cycle in terms of highest and lowest temperatures in the cycle and considering isentropic efficiencies of turbo-compressor and turbo-expander.
[CSE (Mains) 2008 : 30 Marks]

or

Derive the optimum pressure ratio of an ideal gas turbine plant for maximum network. Also, show with the help of T-s diagram that, an optimum pressure ratio exists.
[CSE (Mains) 2015 : 10 Marks]

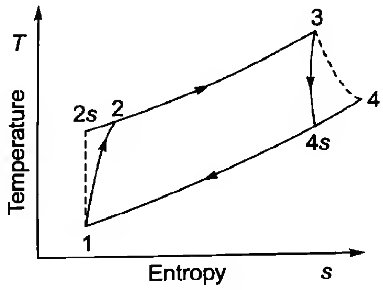
or

A gas turbine operating on actual simple Brayton cycle is to be designed for maximum output. If the maximum and minimum temperatures of the cycle, the efficiencies of compressor and turbine are fixed, derive the expression for optimum actual pressure ratio. What will be the value of optimum pressure ratio for turbine, if the ratio of maximum and minimum temperature is 3 and $\gamma = 2$?
Take efficiencies of turbine and compressor as 0.9 and 0.8 respectively.
[CSE (Mains) 2015 : 20 Marks]

Solution:

$$\eta_c = \frac{T_{2s} - T_1}{T_2 - T_1} \text{ (compressor efficiency)}$$

$$\eta_t = \frac{T_3 - T_4}{T_3 - T_{4s}} \text{ (turbine efficiency)}$$



Turbine work, $W_T = C_P(T_3 - T_4)$

Compressor work, $W_C = C_P(T_2 - T_1)$

$$W_T = C_P(T_3 - T_{4s})\eta_t$$

$$= C_P\eta_t T_3\left(1 - \frac{T_{4s}}{T_3}\right)$$

$$W_C = C_P(T_2 - T_1) = \frac{C_P}{\eta_c}(T_{2s} - T_1) = \frac{C_P T_1}{\eta_c}\left(\frac{T_{2s}}{T_1} - 1\right)$$

Let $\frac{T_3}{T_1} = \frac{T_{max}}{T_{min}} = t$ and $\frac{P_2}{P_1} = r$; $\frac{T_{2s}}{T_1} = \frac{T_3}{T_{4s}} = r^{\frac{\gamma-1}{\gamma}} = C$ (say)

Net work output is given as,

$$W = W_T - W_C = C_P\eta_t T_3\left(1 - \frac{1}{r^{\frac{\gamma-1}{\gamma}}}\right) - \frac{C_P T_1}{\eta_c}\left(r^{\frac{\gamma-1}{\gamma}} - 1\right)$$

$$\frac{W}{C_P T_1} = \eta_t \frac{T_3}{T_1}\left[1 - \frac{1}{r^{\frac{\gamma-1}{\gamma}}}\right] - \left[\frac{1}{\eta_c}\left(r^{\frac{\gamma-1}{\gamma}} - 1\right)\right] = \eta_t t\left[1 - \frac{1}{e}\right] - \frac{1}{\eta_c}[C - 1] \quad \text{...(i)}$$

Equation (i) shows that the specific work output $\frac{W}{C_P T_1}$, upon which the size of the plant depends is a function of not only the pressure ratio, but also of the maximum cycle temperature.

cycle efficiency, $$\eta = \frac{\text{Net work output}}{\text{Heat input}} = \frac{W}{Q_S} = \frac{C_P T_1\left[\eta_t t\left(1-\frac{1}{C}\right)-\frac{1}{\eta_C}(C-1)\right]}{C_P(T_3-T_2)}$$

$$\eta = \frac{C_P T_1\left[\left(\eta_t t-\frac{C}{\eta_C}\right)-\left(\frac{\eta_t t}{C}+\frac{1}{\eta_C}\right)\right]}{C_P T_1\left[\frac{T_3}{T_1}-\frac{T_2}{T_1}\right]} \quad \text{...(ii)}$$

$$\eta_C = \frac{T_{2s}-T_1}{T_2-T_1} = \frac{\left(\frac{T_{2s}}{T_1}-1\right)}{\left(\frac{T_2}{T_1}-1\right)}; \quad \frac{T_2}{T_1} = \frac{\left(\frac{T_{2s}}{T_1}-1\right)}{\eta_C}+1 = \frac{(C-1)}{\eta_C}+1$$

Substituting the value of $\frac{T_2}{T_1}$ in equation (ii), we get

$$\eta = \frac{\left(\eta_t t-\frac{c}{\eta_C}\right)-\left(\frac{\eta_t t}{c}+\frac{1}{\eta_C}\right)}{(t-1)-\frac{(c-1)}{\eta_C}} = \frac{\frac{(\eta_t\eta_C t-c)}{\eta_C}-\frac{(\eta_t\eta_C t+c)}{c\eta_C}}{\frac{\eta_C(t+1)-(c-1)}{\eta_C}}$$

$$\eta = \frac{\eta_t\eta_C tc-c^2-\eta_t\eta_C t-c}{c\eta_C(t-1)-c(c-1)} = \frac{\eta_t\eta_C t(c-1)-c(c+1)}{c\eta_C(t-1)-c(c-1)} \quad \text{...(iii)}$$

First Part:

For optimum pressure ratio for maximum specific work output, we have to differentiate equation (i) as follows:

$$\frac{d}{dc}\left(\frac{W}{C_P T_1}\right) = 0$$

or $$\eta_t t\left(\frac{1}{c^2}\right)-\frac{1}{\eta_C} = 0$$

or $$c^2 = t\eta_t\eta_c$$

$$c = (t\eta_t\eta_c)^{1/2}$$

$$r_{opt}^{\frac{\gamma-1}{\gamma}} = (t\eta_t\eta_c)^{1/2}$$

Optimum pressure ratio, $$r_{opt} = \left(\frac{T_{max}}{T_{min}}\eta_t\eta_c\right)^{\frac{\gamma}{2(\gamma-1)}}$$

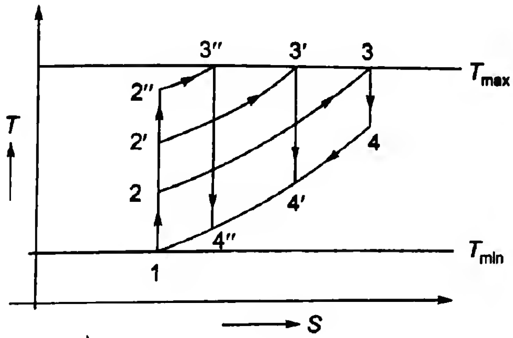
when $$\frac{T_{max}}{T_{min}} = 3,\ g = 2,\ \eta_T = 0.9,\ \eta_c = 0.8$$

$$(r_p)_{optimum} = \left(\eta_T\eta_C\frac{T_{max}}{T_{min}}\right)^{\frac{\gamma}{2(\gamma-1)}}$$

$$(r_p)_{optimum} = (0.9\times0.8\times3)^{\frac{2}{2(2-2)}} = 2.16$$

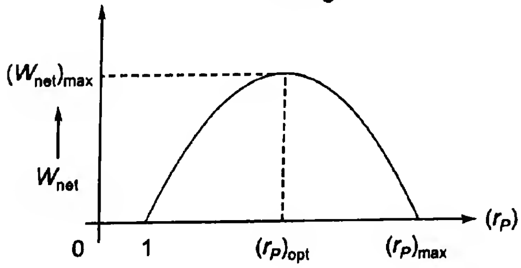
From T-S diagram

It can be seen that the work capacity of cycle, operating between T_{max} and T_{min} is zero, when $r_p = 1$ passes through a maximum, and then again becomes zero when the carnot efficiency is reached. There is an optimum value of pressure ratio $(r_p)_{opt}$ at which work capacity becomes maximum.



Second Part:

For optimum pressure ratio to corresponding to maximum cycle efficiency, we have to differentiate equation (iii) as follows:



$$\frac{d}{dc}(\eta) = 0$$

or
$$\frac{[c\eta_c(t-1)-c(c-1)][\eta_t\eta_c t-2c-1]-[\eta_t\eta_c t(c-1)-c(c+1)][\eta_c(t-1)-2c+1]}{[c\eta_c(t-1)-c(c-1]^2} = 0$$

$$\left[\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2}\right]$$

As $\quad [c\eta_c(t-1)-c(c-1)]^2 \neq 0$

We have

$$[c\eta_c t - c - c^2 + c][\eta_t\eta_c t - 2c - 1] - [\eta_t\eta_c t(c-1) - c(c+1][\eta_c(t-1) - 2c + 1] = 0$$

$$[c\eta_c t - c^2][\eta_t\eta_c t - 2c - 1] - [\eta_t\eta_c t(c-1) - c(c+1][\eta_c(t-1) - 2c + 1] = 0$$

From this, we can deduce the expression for optimum pressure ratio pertaining to maximum cycle efficiency for a Brayton cycle.

Q.12 An open cycle gas turbine takes in air at 300 K and 1 bar and develops a pressure ratio of 20. The turbine inlet temperature is 1650 K, The polytropic efficiency of compressor and turbine each is 90%. The pressure loss in the combustor is 3% and the alternator efficiency is 97%. Take c_{p_a} = 1.005 kJ/ kg-K and c_{p_g} = 1.128 kJ/kg-K for air and gas respectively. The calorific value of fuel is 42 MJ/kg. Work out the following:

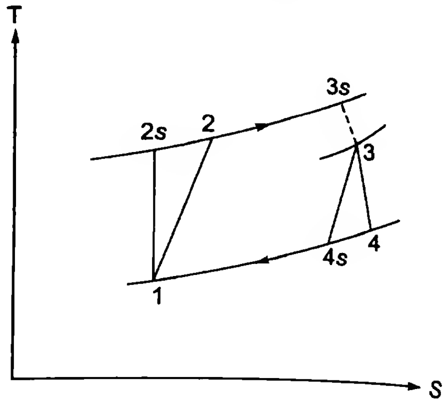
(i) Sketch the system and show the process on T-s diagram.
(ii) The overall efficiency. (iii) The specific power output.
(iv) The fuel to air ratio. (v) The specific fuel consumption.
(vi) Show in general the variation of gas turbine thermal efficiency with compressor ratio for various turbine inlet temperatures.
(vii) What is the reason that thermal efficiency of gas turbine plant increases with decrease in compressor inlet temperature?

[CSE (Mains) 2010 : 20 Marks]

Solution:

Given: T_1 = 300 K, T_3 = 1650 K, P_1 = 1 bar, r_p = 20, $\eta_c = \eta_T$ = 90%, $(\Delta P)_{combustor}$ = 3% of inlet pressure, $\eta_{alternator}$ = 0.97, C_{pa} = 1.005 kJ/kg-K, C_{pg} = 1.128 kJ/kg-K, CV = 42 MJ/kg

$$r_p = \frac{P_2}{P_1} = 20$$



$$P_2 = 20\,\text{bar}$$

$$P_3 = P_2 - (\Delta P) = 20 - (0.03 \times 20) = 19.4\,\text{bar}$$

$$\eta_c = \frac{T_{2s} - T_1}{T_2 - T_1}$$

$$\frac{T_{2s}}{T_1} = (r_P)^{\frac{\gamma_a - 1}{\gamma_a}} = (20)^{0.4/1.4} = 2.35$$

$$T_{2s} = 706\,\text{K}$$

$$\eta_c = 0.9 = \frac{706 - 300}{T_2 - 300}$$

$$T_2 = 751.1\,\text{K}$$

As

$$C_{pg} = 1.128 = \frac{\gamma_g R}{\gamma_g - 1}$$

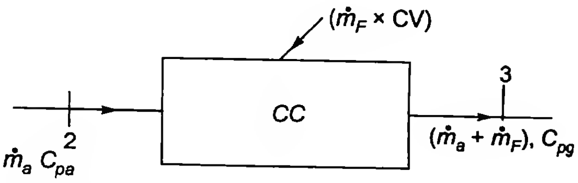
$$\gamma_g = 1.34$$

$$\frac{T_3}{T_{4s}} = \left(\frac{P_3}{P_{4s}}\right)^{\frac{\gamma_g - 1}{\gamma_g}} = \left(\frac{19.4}{1}\right)^{\frac{0.34}{1.34}} = 2.122$$

$$T_{4s} = 777.55\,\text{K}$$

$$\eta_T = 0.9 = \frac{T_3 - T_4}{T_3 - T_{4s}}$$

$$T_4 = 864.4\,\text{K}$$



Energy balance in combustion chamber,

$$\dot{m}_F CV = \dot{m}_g C_{pg} T_3 - \dot{m}_a C_{pa} T_2 = (\dot{m}_a + \dot{m}_F) C_{pg} T_3 - \dot{m}_a C_{pa} T_2$$

$$= (\dot{m}_a + \dot{m}_F) 1.128 \times 1650 - \dot{m}_a \times 1.005 \times 751.1$$

$$42000 \dot{m}_F = 1106.34 \dot{m}_a + 1861.2 \dot{m}_F$$

Fuel to air ratio, $\dfrac{\dot{m}_F}{\dot{m}_a} = 0.0275$

$$\dot{m}_g = \dot{m}_a + \dot{m}_F = \dot{m}_a + 0.0275 \dot{m}_a = 1.0275 \dot{m}_a$$

Power output of turbine $(W_T) = \dot{m}_g C_{pg}(T_3 - T_4) = (1.0275\dot{m}_a)(1.128)(1650 - 864.8)$

$$W_T = 910 \dot{m}_a \text{ kW}$$

$$W_C = \dot{m}_a C_{pa}(T_2 - T_1) = \dot{m}_a (1.005)(751.1 - 300) = 453.35 \dot{m}_a \text{ kW}$$

$$W_{net} = W_T - W_C = 456.65 \dot{m}_a \text{ kW}$$

and Power output $= W_{net} \times \eta_{alternator}$

Specific power output $= \dfrac{(456.65 \dot{m}_a) \times 0.97}{\dot{m}_F} = \dfrac{456.65 \times 0.97}{0.0275} = 16107.3$

$$= 16107.3\,\text{kW-sec/kg}$$

$$\eta_{overall} = \frac{P_{net}}{Q_1} = \frac{(456.65 \times 0.97\dot{m}_a)}{\dot{m}_F \times 42000} = 0.3835 \text{ or } 38.35\%$$

$$\text{Specific fuel consumption} = \frac{\dot{m}_F}{w_{net}} = \frac{\dot{m}_F}{456.65\dot{m}_a} = \frac{0.0275}{456.65} = 0.216 \text{ kg/kW-hr}$$

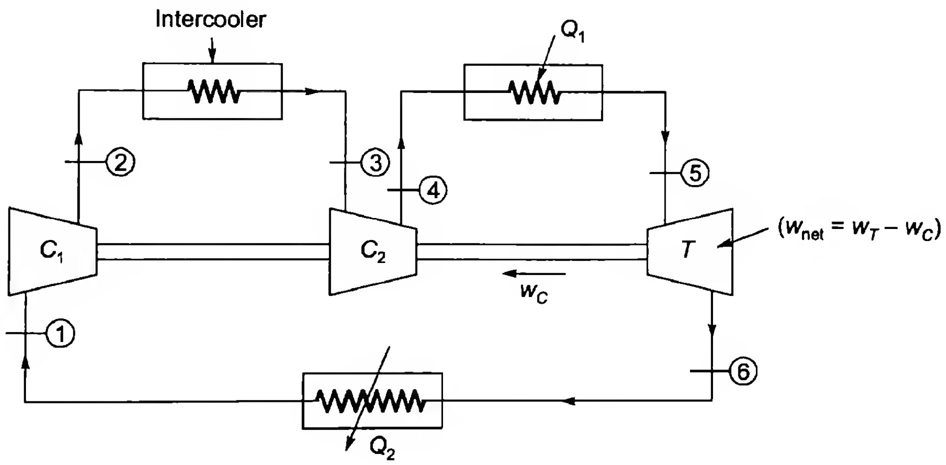
Thermal efficiency of gas turbine plant increases with decrease in compressor inlet temperature. As the compressor work is dependent on the inlet temperature of air. The compressor work increases with the increase in inlet temperature as more energy is required to compress the warm air. This increases the power consumption of compressor and reduces the net power output of cycle. Thus, efficiency also decreases with increase in inlet temperature of incoming air.

Q.13 Draw the schematic arrangement of a simple cycle with intercooled and heat exchanger and explain briefly the working principle. Also draw the P-V and T-S diagrams of the cycle. Further, derive expressions for specific work output and the efficiency of a simple cycle with intercooled and heat exchanger. Draw their trends as a function of pressure ratio.

[CSE (Mains) 2012 : 20 Marks]

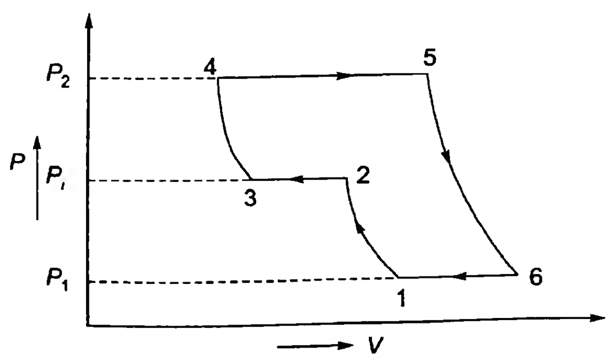
Solution:

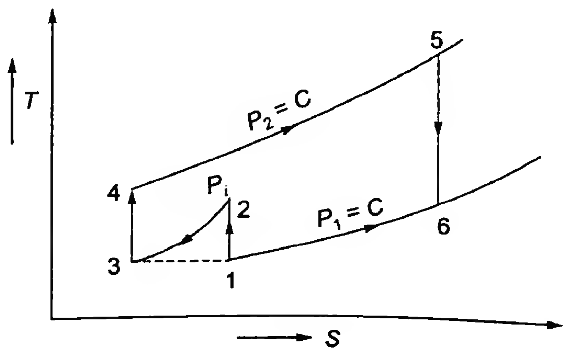
Simple cycle with intercooled and Heat Exchanger.



The efficiency of simple cycle is increased by the use of staged compression with intercooling. Let the compression process be divided into two stages. Air after being compressed in first stage, is cooled to the initial temperature in a heat exchanger, called an intercooler and then compressed further in the second stage. Heat is then added by combustion of gases in the combustion chamber. The hot gases then expands in a turbine producing power output, a fraction of which is utilized in running compressors.

The gases rejects (Q_2) heat in a heat exchanger and cycle is repeated.





work output $= W_T - W_C$

Compressor work, $W_C = W_{C_1} + W_{C_2} = C_P[T_2 - T_1] + C_P[T_4 - T_3] = C_P T_1\left[\frac{T_2}{T_1} - 1 + \frac{T_4}{T_1} - \frac{T_3}{T_1}\right]$

For perfect intercooling, $T_1 = T_3$

$$W_C = C_P T_1\left[\frac{T_2}{T_1} + \frac{T_4}{T_3} - 2\right]$$

Also,
$$\frac{T_2}{T_1} = \left(\frac{P_i}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

Let $\frac{\gamma - 1}{\gamma} = x$

$$\frac{T_2}{T_1} = \left(\frac{P_i}{P_1}\right)^x$$

And,
$$\frac{T_4}{T_3} = \left(\frac{P_2}{P_i}\right)^x$$

$$W = CT_1\left[\frac{P_i^x}{\rho_1^x} - \frac{P_2^x}{P_i^x} - 2\right]$$

For W_C to be minimum, $\frac{dW_C}{dP^c} = 0$, $P_i = \sqrt{P_1 P_2}$ and $W_{C1} = W_{C2}$

$$W_C = W_{C_1} + W_{C_2} = 2W_{C_1}$$
$$= 2\,C_P[T_2 - T_1]$$
$$W_T = C_P[T_5 - T_6]$$
$$W_{net} = W_T - W_C$$
$$W_{net} = C_P[T_5 - T_6] - 2C_P[T_2 - T_1]$$
$$W_{net} = C_P[T_5 - T_6 - 2T_2 + 2T_1]$$

$$\text{Efficiency } \eta = \frac{W_{net}}{Q_1}$$

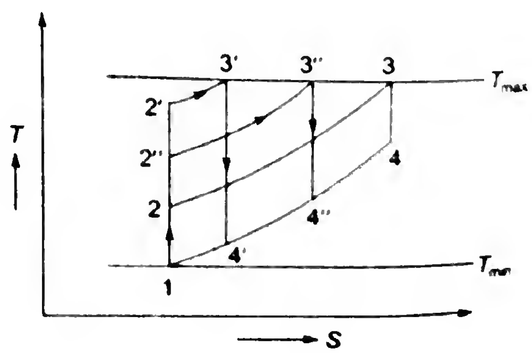
Where,
$$Q_1 = C_P(T_5 - T_4)$$

$$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \left(\frac{T_6 - T_1}{T_5 - T_4}\right)$$

$$\frac{T_5}{T_6} = \left(\frac{P_2}{P_1}\right)^x = (r_P)^x \;;\; \text{and } \frac{T_4}{T_3} = \frac{T_4}{T_1} = \left(\frac{P_2}{P_i}\right)^x$$

$$\frac{T_4}{T_1} = \frac{T_4}{T_3} = \left(\frac{P_2}{P_1}\right)^{x/2} \Rightarrow \left(\sqrt{r_P}\right)^x$$

$$\eta = 1 - \left\{\frac{T_6 - T_1}{(r_p)^x T_6 - \left(\sqrt{r_p}\right)^x T_1}\right\}$$



Effect of Pressure ratio on simple cycle.

Note : As r_p (increases), η(efficiency) is also increases.

Q.14 Hot gases enter the blades of a gas turbine with a velocity of 550 m/s and leave with a velocity of 120 m/s. There is an increase in the enthalpy of the gases in the blade passages to the extent of 5.1 kJ/kg. The rate of gas flow is 98 kg/min. Determine the power produced.

[CSE (Mains) 2013 : 15 Marks]

Solution:

Given: V_1 = 550 m/sec, V_2 = 120 m/sec, Δh = increase in enthalpy = $h_2 - h_1$ = 5.1 kJ/kg,

$$\dot{m}_g = 98 \text{ kg/min} = 1.63 \text{ kg/sec}$$

Assuming the flow of hot gases through the gas turbine to be adiabatic process,

$$\dot{Q} = 0$$

$$\because \quad \dot{Q} - \dot{W} = \dot{m}\left[(h_2 - h_1) + \left(\frac{V_2^2 - V_1^2}{2}\right)\right]$$

$$\dot{W} = \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right)\right]$$

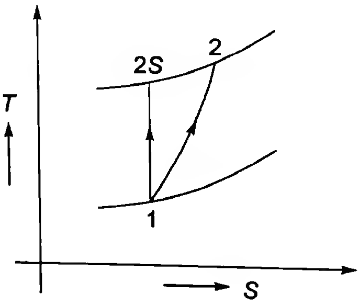
$$\dot{W} = 1.63\left[(-5.1) + \left(\frac{550^2 - 120^2}{2000}\right)\right] = 226.4885 \text{ kW}$$

Power produced is $\dot{W}$ = 226.4885 kW.

Q.15 A centrifugal compressor running at 1200 r.p.m, delivers 800 m³/min of free air. The air is compressed from 1 bar, 30°C to 4.8 bar with isentropic efficiency of 0.84. The impeller blades are radial at outlet and the flow velocity of 80 m/s may be assumed constant throughout. The outer radius of the impeller is twice the inner. The slip factor may be assumed as 0.9. The blade area coefficient is equal to 0.9 at inlet.
(i) Draw inlet and outlet velocity triangles for the impeller, and show the process on a T-s diagram.
(ii) Calculate the input power needed, if mechanical efficiency is 95%.
(iii) Calculate the impeller diameters at inlet and outlet.
(iv) Calculate the impeller and diffuser blade angles at inlet.

[CSE (Mains) 2014 : 20 Marks]

Solution:



Given: N = 1200 rpm, Q = 800 m³/min, (FAD) = 13.33 m³/sec

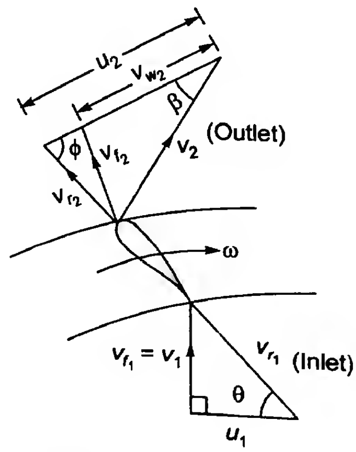
(i) Inlet conditions: P_1 = 1 bar, T_1 = 30°C = 303 K,

Outlet conditions: P_2 = 4.8 bar, η_{isent} = 0.84 $V_{F_1} = V_{F_2} = V_F$ = 80 m/sec,

($D_2 = 2D_1$), ϕ_s(slip factor) = 0.9, k_b (blade area coefficient) = 0.9

$$\frac{T_{2S}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{4.8}{1}\right)^{\frac{1.4-1}{1.4}} = 1.565$$

$$(T_{2S} = 474.3 \text{ k})$$



$$\eta_{isent} = \frac{T_{2S} - T_1}{T_2 - T_1}$$

$$(T_2 - T_1) = \frac{474.3 - 30.3}{0.84} = 203.928$$

$$T_2 = (203.928 + 303) \text{ k}$$

$$T_2 = 506.9 \text{ k}$$

$$\text{Power (ideal)} = \dot{m}C_p(T_2 - T_1)$$

$$P_1\dot{v}_1 = \dot{m}RT_1$$

$$\dot{m} = \frac{1\times 10^5 \times 13.33}{(287)(303)} = 15.32 \text{ kg/sec}$$

$$\text{Power (ideal)} = 15.32 \times 1.005 \times (506.9 - 303) = 3141.94 \text{ kW}$$

$$\eta_{mechanical} = 95\%$$

(ii)

$$\text{Power Input needed} = \frac{3141.94}{0.95} = 3307.3 \text{ kW}$$

$$P = 3.307 \text{ mW}$$

Also,

$$\text{Power input} = \dot{m}\phi_s u_2^2$$

$$(15.32) \times 0.9 \times u_2^2 = (3.307 \times 10^6) \text{ W}$$

$$u_2 = 489.75 \text{ m/sec}$$

Also,

$$u_2 = \frac{\pi D_2 N}{60}$$

(iii)

$$D_2 = \frac{60 \times 489.75}{\pi(1200)} = 7.798 \text{ m}$$

$$D_1 = \frac{D_2}{2} = 3.89 \text{ m}$$

(iv)

$$\tan\phi = \frac{V_{f_1}}{u_1} = \frac{80}{\left(\frac{u_2}{2}\right)} = \frac{80}{244.875}$$

$$\theta = \text{blade angle at inlet} = 18.09^\circ$$

$$\tan\beta = \frac{V_{F_2}}{V_{W_2}}$$

$$\phi_S = \frac{V_{W_2}}{u_2} \Rightarrow (V_{W_2} = \phi_S u_2)$$

$$\tan\beta = \frac{V_{f_2}}{\phi_S \cdot u_2} = \frac{80}{0.9 \times 489.75} = 10.287^\circ$$

$$\beta = \text{Blade angle at diffuser inlet} = 10.287^\circ$$

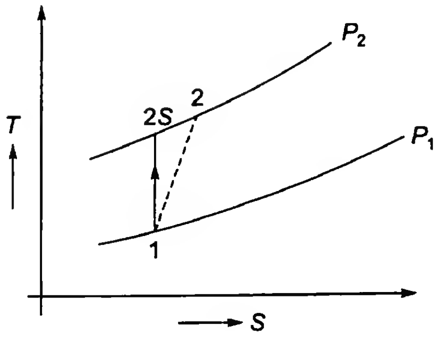
Q.16 A centrifugal compressor running at 16000 rpm takes in air at 17°C and compresses it through a pressure ratio of 4 : 1 with an isentropic efficiency of 82%. The blades are radially inclined and the slip factor is 0.85. Guide vanes at inlet give the air an angle of pre-whirl of 20° to the axial direction. The mean diameter of the impeller eye is 200 mm and the absolute air velocity at inlet is 120 m/s. Calculate the impeller tip diameter. Take $C_p = 1.005\frac{\text{kg}}{\text{kg K}}$; $\gamma = 1.4$. Also draw the velocity triangles at inlet and impeller exit.

[CSE (Mains) 2015 : 20 Marks]

Solution:

Given: For a centrifugal compressor

$N = 16000$ rpm, $r_p = 4$, $\eta = 82\%$, $\phi_s = 0.85$, $T_1 = 17°C = 290$ k



$$\frac{T_{2S}}{T_1} = (r_P)^{\frac{\gamma-1}{\gamma}} = (4)^{\frac{1.4-1}{1.4}} = 1.487$$

$$T_{2S} = 290 \times 1.487 = 431 \text{ k}$$

$$w_c = (h_2 - h_1) = mc_p(T_2 - T_1)$$

$$w_c = mc_p\left(\frac{T_{2s} - T_1}{\eta_{isent}}\right)$$

$$w_c = 1 \times 1.005 \times \left(\frac{431 - 290}{0.82}\right)$$

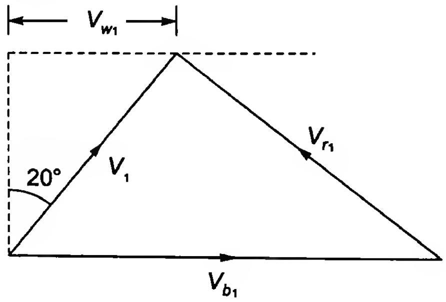
$$w_c = \text{power input per kg} = 172.81 \text{ kJ/kg}$$

Absolute velocity at inlet, $v_1 = 120$ m/sec

$$v_{b1} = \text{blade velocity} = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.2 \times 16000}{60}$$

$$v_{b1} = 167.55 \text{ m/sec}$$

Velocity Triangle at inlet

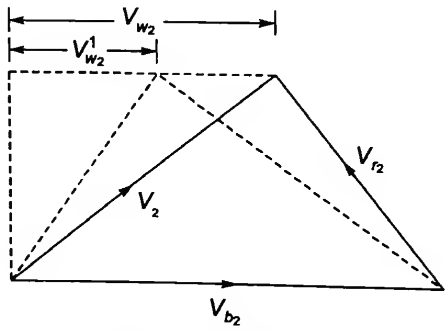


Pre-whirl Angle = 20°

$$v_{w_1} = v_1 \sin 20° = 120 \sin 20°$$

$$= 41.04 \text{ m/sec}$$

At exit of vanes



$$v_{w_2} = v_{b_2} \quad \text{[Radially inclined]}$$

$$\text{slip factor } \phi_s = \frac{v'_{w_2}}{v_{b_2}} = 0.85$$

$$v'_{w_2} = 0.85 v_{b_2}$$

$$\text{Power input per kg} = v_{b_2} v'_{w_2} - v_{b_1} v_{w_1}$$

$$172.81 \times 10^3 = v_{b_2} \times (0.85\, v_{b_2}) - 167.55 \times 41.05 = 0.85 v_{b_2}^2 - 6877.93$$

$$v_{b_2} = 459.78 \text{ m/s} = \frac{\pi D_2 \times 16000}{60}$$

$$\text{Tip diameter } (D_2) = 549 \text{ mm}$$

3. Rankine Cycle Nozzles

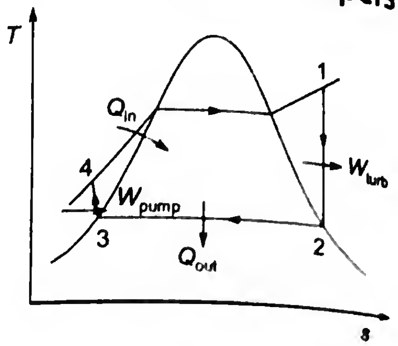
Q.17 For a Rankine cycle, using usual notations, show that the thermal efficiency can be expressed as:

$\eta_{th} = 1 - \dfrac{h_2 - h_3}{h_1 - h_4}$ and the second law efficiency can be expressed as: $\psi = \eta_{th}\left(\dfrac{T_H}{T_H - T_O}\right)$

where, T_H is the highest temperature in the entire system and T_O is the condensation temperature.

[CSE (Mains) 2008 : 15 Marks]

Solution:



For boiler ($W = 0$):

$$Q_{in} = h_1 - h_4$$

For condenser ($W = 0$):

$$Q_{out} = h_2 - h_3$$

For turbine ($q = 0$):

$$W_{turb} = h_1 - h_2$$

Thermal efficiency of the Ranking cycle is determined from

$$\eta_{th} = \frac{W_{net}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{h_2 - h_3}{h_1 - h_4}$$

First expression derived

The second law efficiency is defined as:

$$\eta_{II} = \frac{\text{Energy recovered}}{\text{Energy expended}} = \frac{x_{recovered}}{x_{expended}}$$

$$x_{expended} = x_{heat,\,in} + x_{pump,\,in}$$

Assuming zero energy expended in pump work,

$$x_{expended} = x_{heat,\,in} = \left(1 - \frac{T_O}{T_H}\right) Q_{in}$$

$$x_{recovered} = W_{turbine} = (h_1 - h_2)$$

$$\eta_{II} = \frac{W_{turbine}}{\left(1 - \frac{T_O}{T_H}\right) Q_{in}} = \frac{\eta_{th}}{\left(1 - \frac{T_O}{T_H}\right)} = \eta_{th}\left(\frac{T_H}{T_H - T_O}\right)$$

Second expression derived

Q.18 For compressible flows, explain with the help of diagrams, the shapes of nozzles and diffusers in subsonic and supersonic regimes.

[CSE (Mains) 2008 : 10 Marks]

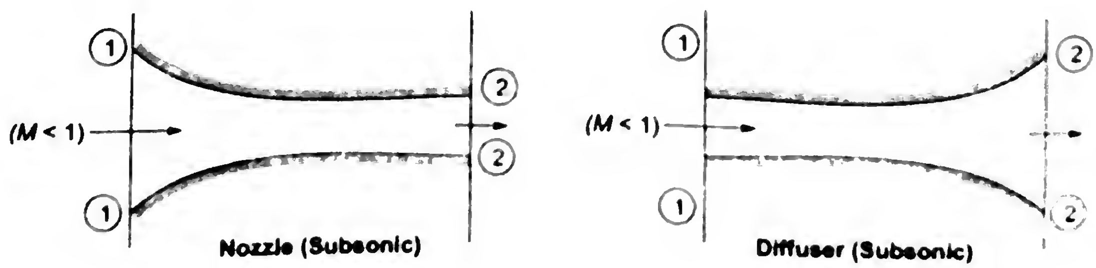
Solution:

Nozzles and Diffusers: For compressible flows,

We have, $$\frac{dA}{A} = (M^2 - 1)\frac{dv}{v}$$

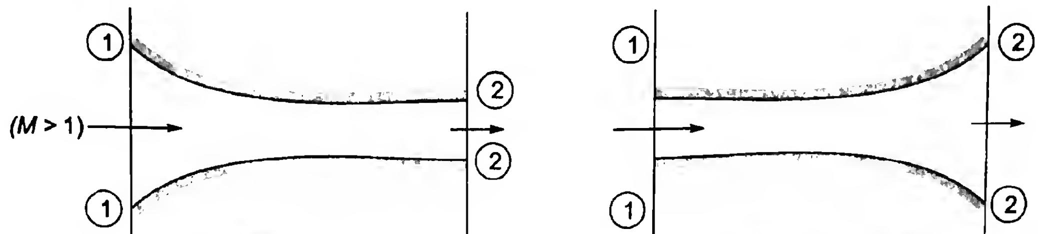
Subsonic regimes:

When $M < 1$, i.e. the inlet velocity is subsonic, as flow area A decreases, the pressure decreases and the velocity increases. so for subsonic flows, a convergent passage becomes a nozzle, and a divergent passage becomes a diffuser.



Supersonic regimes:

When the $M > 1$, i.e. when the inlet velocity is supersonic, as flow area A decreases, pressure increases and velocity decreases, and as flow area A increases, pressure decreases and velocity increases. So, for supersonic flow, α convergent is diffuser and a divergent passage is a nozzle.



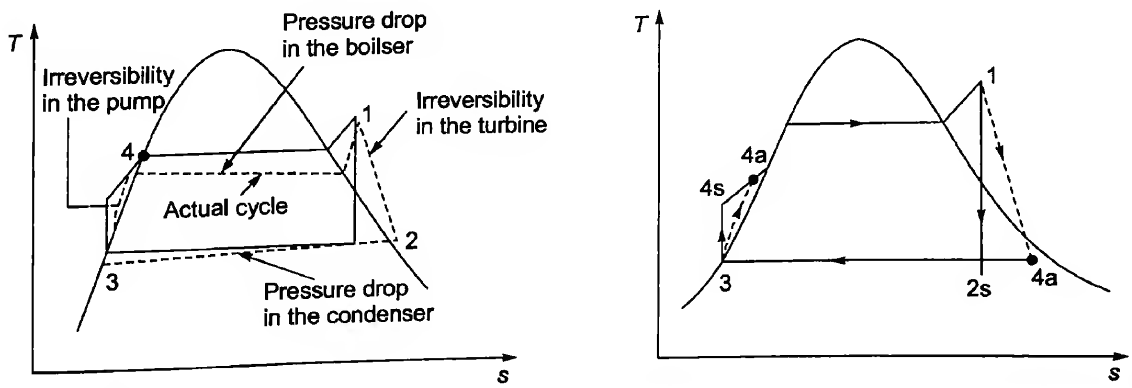
(Supersonic Diffuser) (Supersonic Nozzle)

Q.19 "The boiler efficiency is very high as against the thermal efficiency of Rankine cycle." – Comment on this statement with the help of laws of thermodynamics and schematic diagrams.

[CSE (Mains) 2008 : 10 Marks]

Solution:

The way a boiler works is by heating water which is conducted through radiators, radiant floor systems or a coil. With a standard boiler, some of the energy that is used to heat the boiler, whether it is a fossil fuel or natural gas, is lost in the process of conducting. Boiler efficiency is the ratio of heat actually utilized in generation of steam to the heat supplied by the fuel in the same period. It's value is of the order of 75 to 80%. The Rankine cycle closely describes the process by which steam-operated heat engines commonly found in thermal power generation plants generate power. The heat sources used in these power plants are usually nuclear fission or the combustion of fossil fuels such as coal, natural gas, and oil. The efficiency of the Rankine cycle is limited by the high heat of vaporization of the working fluid. Also, unless the pressure and temperature reach super critical levels in the steam boiler, the temperature range the cycle can operate over is quite small: steam turbine entry temperatures are typically around 565 °C and steam condenser temperatures are around 30 °C. This gives a theoretical maximum Carnot efficiency for the steam turbine alone of about 63.8% compared with an actual overall thermal efficiency of up to 42% for a modern coal-fired power station.



(a)
Comparison of Actual and Ideal vapour Power cycles

(b)
Effect of Pump and Turbine Irreversibilities on the Ideal Rankine cycle

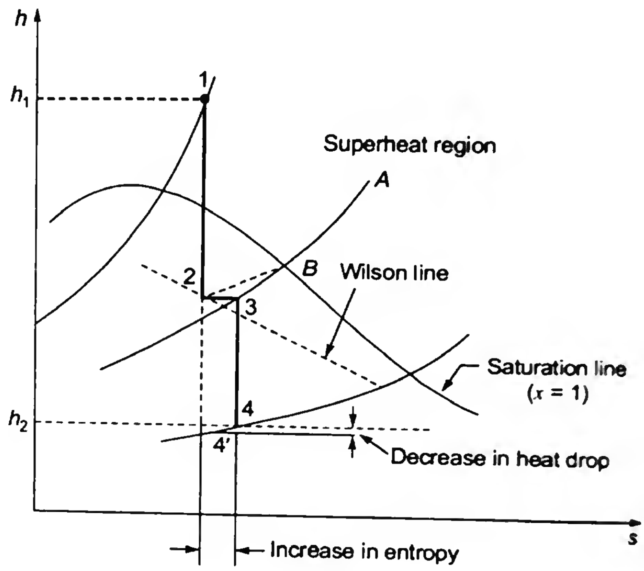
Q.20 Explain supersaturated flow in steam nozzles with the help of skeleton Mollier diagram inserting nomenclatures like dry saturated line, Wilson line and supersaturated zone. List also the five effects of supersaturation in the steam nozzles.

[CSE (Mains) 2009 : 20 Marks]

Solution:

When steam flows through a nozzle, it would normally be expected that the discharge of steam through the nozzle would be slightly less than the theoretical value. But it has been observed during experiments on flow of wet steam that the discharge is slightly greater than that calculated by the formula. This phenomenon is explained as follows: The converging part of the nozzle is so short and the steam velocity is so high that the molecules of steam have insufficient time to collect and form droplets so that normal condensation does not take place. Such rapid expansion is said to be metastable and produces a supersaturated state. In this state of super saturation, the steam is undercooled to a temperature less than that corresponding to its pressure; consequently the density of steam increases and hence the weight of discharge. Prof. Wilson through experiments showed that dry saturated steam, when suddenly expanded in the absence of dust, does not condense until its density is about 8 times that of the saturated vapour of the same pressure. This effect is discussed below:

Point 1 represents initial state of the steam. The steam expands isentropically without any condensation to point 2, 2 being on the superheat constant pressure curve A-3 produced. At point 2 the limit of super saturation is reached and steam reverts to its normal condition at 3 at the same enthalpy value as 2, and at the same pressure. The steam continues expanding isentropically to a lower pressure to point 4 instead of 4' which would have been reached if thermal equilibrium had been maintained. Consequently, enthalpy drop is reduced and the condition of the final steam is improved. The limiting condition of under-cooling at which condensation commences and is assumed to restore conditions of normal thermal equilibrium is called the "Wilson Line".



Supersaturated flow of steam

It may be noted that when metastable conditions prevail the h-s chart-I diagram should not be used and the expansion must be considered to follow the law $pv^{1.3} = C$, i.e., with the index of expansion for superheated steam. Thus,

$$\text{Enthalpy drop} = \frac{n}{(n-1)} p_1 v_1 \left[1 - \left(\frac{p_2}{p_2} \right)^{\frac{n-1}{n}} \right]$$

The relationship, $T_2/T_1 = (p_2 - p_1)^{\frac{n-1}{n}}$ may be used to calculate supercooled temperature.

The 'degree of undercooling' is then the difference between the saturation temperature and the supercooled temperature.

Effects of supersaturation: In a nozzle in which supersaturation occurs the effects may be summarised as follows:

(i) There is an increase in the entropy and specific volume of steam.

(ii) The heat drop is reduced below that for thermal equilibrium as a consequence the exit velocity of steam is reduced.

(iii) Since the condensation does not take place during supersaturated expansion, so the temperature at which the supersaturation occurs will be less than the saturation temperature corresponding to the pressure. Therefore, the density of supersaturated steam will be more than that for the equilibrium conditions which gives the increase in the mass of steam discharged.

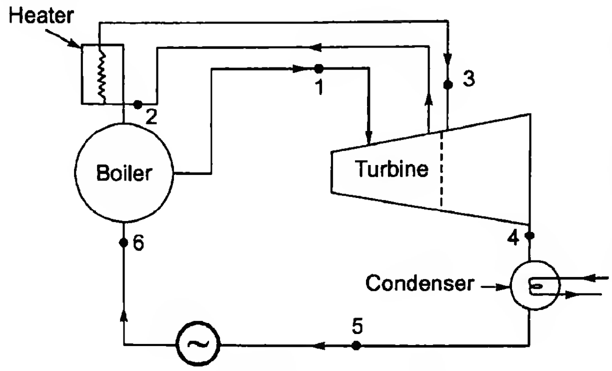
(iv) The dryness fraction of steam is improved

Q.21 **With the help of schematic and T - s diagrams explain a reheating Rankine cycle. State its advantages over ordinary Rankine cycle and define the reheat factor.**

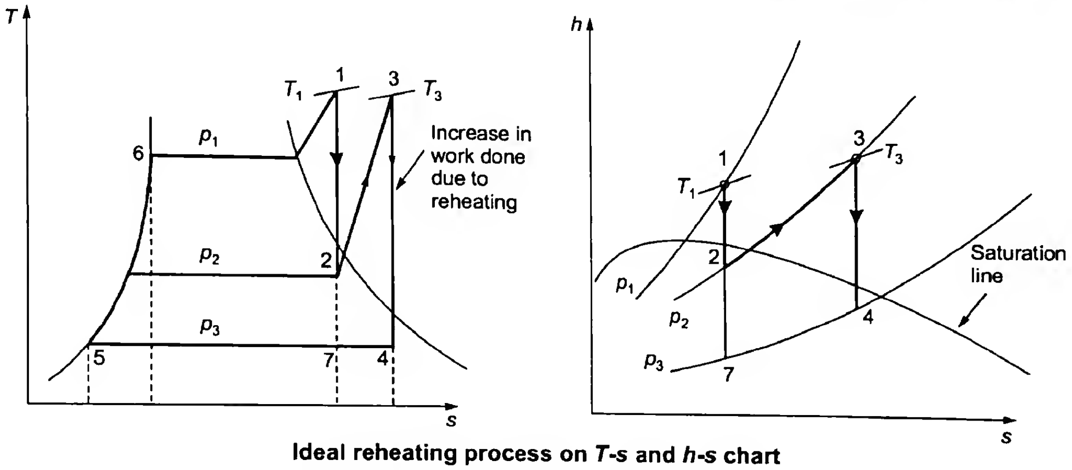
[CSE (Mains) 2009 : 20 Marks]

Solution:

Reheating in Ranking Cycle: Increasing the boiler pressure can increase the thermal efficiency of the Rankine cycle, but it also increases the moisture content at the exit of the turbine to an unacceptable level. To correct this side effect, the simple Rankine cycle is modified with a reheat process. In the reheating Rankine cycle, steam is expanded isentropically to an intermediate pressure in a high-pressure turbine (stage I) and sent back to the boiler, where it is reheated at constant pressure to the inlet temperature of the high-pressure turbine. Then the steam is sent to a low-pressure turbine and expands to the condenser pressure (stage II). Refer schematic and T-s cycle for reheat Rankine cycle as below.



Schematic of reheating Ranking cycle



Ideal reheating process on *T-s* and *h-s* chart

5–1 shows the formation of steam in the boiler. The steam as at state point 1 (i.e., pressure p_1 and temperature T_1) enters the turbine and expands isentropically to a certain pressure p_2 and temperature T_2. From this state point 2 the whole of steam is drawn out of the turbine and is reheated in a reheater to a temperature T_3. (Although there is an optimum pressure at which the steam should be removed for reheating, if the highest return is to be obtained, yet, for simplicity, the whole steam is removed from the high pressure exhaust, where the pressure is about one-fifth of boiler pressure, and after undergoing a 10% pressure drop, in circulating through the heater, it is returned to intermediate pressure or low pressure turbine). This reheated steam is then readmitted to the turbine where it is expanded to condenser pressure isentropically.

Advantages of 'Reheating:

1. There is an increased output of the turbine.
2. Erosion and corrosion problems in the steam turbine are eliminated/avoided.
3. There is an improvement in the thermal efficiency of the turbines.
4. Final dryness fraction of steam is improved.
5. There is an increase in the nozzle and blade efficiencies.

Reheat factor: It is defined as the ratio of cumulative heat drop to the adiabatic heat drop in all the stages of the turbine. The value of reheat factor depends on the type and efficiency of the turbine, the average value being 1.05.

$$\text{Reheat factor} = \frac{\text{Cumulative heat drop}}{\text{Adiabatic heat drop}}$$

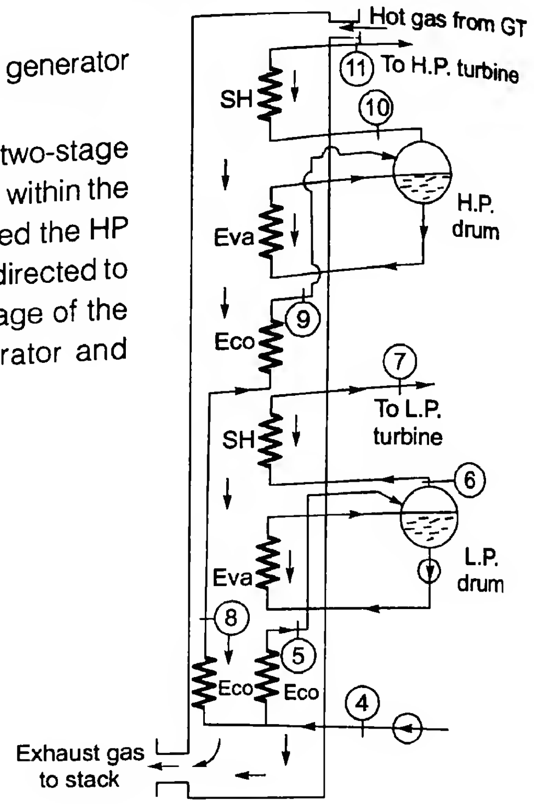
Q.22 With the help of a neat sketch show a steam/gas combined cycle with two pressures heat recovery steam generator (HRSG).

[CSE (Mains) 2010 : 10 Marks]

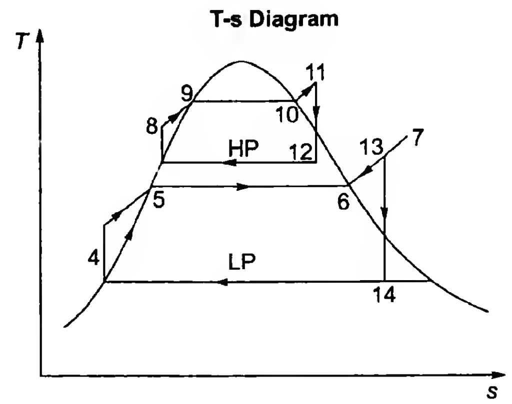
Solution:

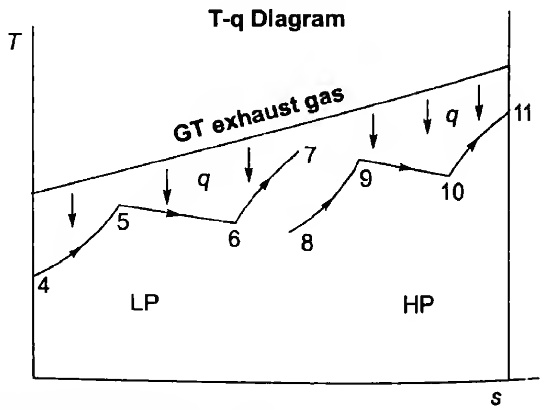
Steam/Gas combined cycle with two pressures heat recovery steam generator (HRSG).

(i) The dual pressure steam generator which is used to supply a two-stage steam turbine (HP and LP stages). There are two separate boilers, within the boiler casing, namely, the high pressure (HP) boiler which supplied the HP turbine, the exhaust steam from the HP stage of the turbine is then directed to the LP turbine. Steam from the LP boiler is directed to the LP stage of the turbine. Each boiler comprises the usual economiser, evaporator and superheater sections.



(ii)





(iii) Advantages of combined cycle:

a. It produces high power outputs at high efficiencies (upto 55%) and with low emissions.
b. Fewer moving parts and less vibrations than a reciprocating engine.
c. High operating speeds.
d. It can run on a wide variety of fuels.
e. These are more suitable for rapid start and shutdown than steam power plant.
f. The cooling water requirement of combined cycle plant is much lower than the normal steam turbine plant having same capacity output.
g. It has advantage of low operational personnel.
h. It has the advantage of high ratio of power output to the area occupied.
i. Maintenance duration (down time) for such combined cycle plant is significantly low.

Q.23 Air at the rate of 35 kg/s flows through a nozzle in which a normal shock occurs in the diverging section down-stream of the throat. The nozzle has an area of cross section equal to 40 cm^2 at the section of shock. The pressure and velocity of fluid just before the shock are 2.5 bar and 480 m/s respectively. Find the Mach number, pressure and temperature after the shock. Comment on the results.

Normal shock table:

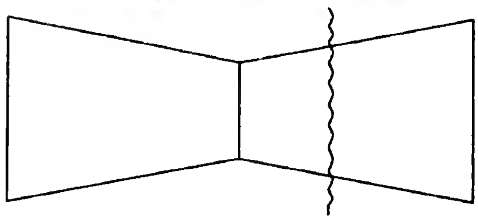
M_1	M_2	P_2/P_1	T_2/T_1	ρ_2/ρ_1	P_{02}/P_{01}
1.42	0.7314	2.1858	1.2676	1.7243	0.9531
1.43	0.7274	2.2190	1.2742	1.7416	0.9503

[CSE (Mains) 2010 : 20 Marks]

Solution:

Given:

Converging - Diverging Nozzle



$\dot{m}$ = 35 kg/s, A_s = Cross-sectional area at the section of shock = 40 cm^2

Before shock: P_x = 2.5 bar, V_x = 480 m/sec,

M_x = Mach number

After shock: P_y, T_y, V_y and M_y are pressure, temperature, velocity and mach number respectively.

As,
$$\dot{m} = \rho AV$$
$$35 = \rho_x A_x V_x \qquad [\because A_x = A_y = A_s = 40 \text{ cm}^2]$$
$$35 = \rho_x(40 \times 10^{-4}) \times 480$$
$$\rho_x = 18.23 \text{ kg/m}^3 \qquad \text{(This is absorb value)}$$

Note: Density of air cannot be of such value. Therefore mass flow rate (assumed) as 3.5 kg/sec

$$3.5 = \rho_x(40 \times 10^{-4}) \times 480$$
$$\rho_x = 1.858 \text{ kg/m}^3$$

Also,
$$P_x = \rho_x RT_x$$
$$T_x = \left(\frac{2.5 \times 10^5}{1.858 \times 287}\right) = 468.82 \text{ K}$$
$$M_x = \frac{V_x}{\sqrt{\gamma RT_x}} = \frac{480}{\sqrt{1.4 \times 287 \times 468.82}} = 1.106$$

Also,
$$M_y^2 = \frac{M_x^2 + \frac{2}{\gamma - 1}}{\frac{2\gamma}{\gamma-1}M_x^2 - 1} = \frac{(1.106)^2 + \frac{2}{0.4}}{\frac{2\times1.4}{0.4}(1.106)^2 - 1} = 0.907$$

Also,
$$\frac{P_y}{P_x} = \frac{1+\gamma M_x^2}{1+\gamma M_y^2} = \frac{1+1.4(1.106)^2}{1+1.4(0.907)^2} = \frac{2.7125}{2.1517} = 1.26$$
$$P_y = 1.26 \times 2.5 = 3.151 \text{ bar}$$

And,
$$\frac{T_y}{T_x} = \left(\frac{P_y}{P_x}\right)^2\left(\frac{M_y}{M_x}\right)^2 = \left(\frac{3.151}{2.5}\right)^2\left(\frac{0.907}{1.106}\right)^2 = 1.0687$$
$$T_y = 1.0687 \times 468.82 = 501.052 \text{ K}$$

After the normal shock, mach number becomes less than unity (subsonic), pressure and temperature increases.

Q.24 Steam enters a nozzle operating at steady state at 40 bars, and 400°C and 10 m/s. Steam flows through this nozzle with negligible heat transfer and change in potential energy. Steam exits at 15 bars and 665 m/s. Mass flow rate is 2 kg/s. Compute exit area of the nozzle.

Given data for steam at 15 bar:

T(°C)	*h*(kJ/kg)	V(m^3/kg)
250	2923.9	0.15201
300	3038.2	0.16971

and Enthalpy of steam at inlet 3214.5 kJ/kg.

[CSE (Mains) 2011 : 20 Marks]

Solution:

Given: Inlet conditions : P_1 = 40 bars, T_1 = 400°C, V_1 = 10 m/sec, $\dot{m}$ = 2 kg/sec, h_1 = 3214.5 kJ/kg

Outlet Conditions: P_2 = 15 bars, V_2 = 665 m/sec

As the steam flows through nozzle with negligible heat transfer and change in potential energy, so from steady flow energy equation:

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

$$h_2 = 3214.5 + \left[\frac{10^2 - 665^2}{2000}\right] = 2993.44 \text{ kJ/kg)}$$

Using Table,

$$(h_2 - 2923.9) = \left(\frac{3038.2 - 2923.9}{0.16971 - 0.15201}\right)(V_2 - 0.15201)$$

$$(V_2 = 0.16277 \text{ m}^3\text{/kg)}$$

Let A_2 be the exit area,

$$A_2V_2 = (\dot{m}v_2)$$

$$A_2V_2 = (2 \times 0.16227) \text{ m}^3\text{/sec}$$

$$A_2 = \frac{(2 \times 0.16227)}{665} = 4.895 \times 10^{-4} \text{ m}^2 = 489.55 \text{ mm}^2$$

Q.25 Air is supplied to a convergent-divergent nozzle with a static temperature of 300 K, static pressure of 5 bar and a velocity of 150 m/s. The inlet area of the nozzle is 10 cm^2. A normal shock occurs at a section of the nozzle where flow Mach number is 2. The flow Mach number downstream of shock corresponding to Mach number 2 is 0.577. The flow Mach number at the exit of the nozzle is 0.4. Considering isentropic flow before and after shock and using isentropic flow relation for A/A* and Mach number provided below, find:

(i) throat area of the nozzle

(ii) area of the nozzle where shock occurs

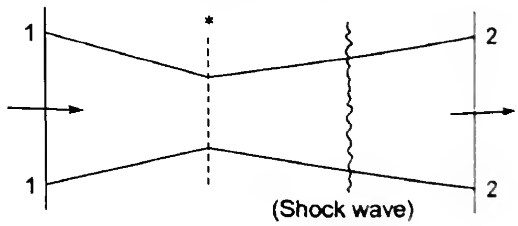
(iii) exit area of the nozzle

(iv) loss of stagnation pressure across shock.

Sketch the variation of pressure and Mach number along the length of the nozzle:

[CSE (Mains) 2011 : 30 Marks]

Solution:



Given : A convergent -Divergent Nozzle.

Inlet Conditions :

T_1 = 300 K, P_1 = 5 bar, V_1 = 150 m/sec, A_1 = 10 cm^2

Considering isentropic flow before and after shock,

$$M_1 = \frac{V_1}{\sqrt{\gamma R T_1}} = \frac{150}{\sqrt{(1.4 \times 287 \times 300)}}$$

$$M_1 = 0.432$$

$$\frac{T_{0_1}}{T_1} = 1 + \left(\frac{\gamma - 1}{2}\right)m_1^2 = 1 + \left(\frac{1.4 - 1}{2}\right)(0.432)^2$$

$$T_{01} = 311.2 \text{ K}$$

$$T_{01} = T_{02} = 311.2 \text{ k}$$

Also,

$$\frac{T_0^*}{T^*} = 1 + \left(\frac{\gamma - 1}{2}\right) \Rightarrow \left(\frac{\gamma + 1}{2}\right)$$

$$T^* = \left(\frac{2}{\gamma+1}\right)T_0 = \left(\frac{2}{1.4+1}\right)\times 311.2 = 259.33\text{ K}$$

$$\frac{P_{01}}{P_1} = \left(\frac{T_{01}}{T_1}\right)^{\gamma/\gamma-1} = \left(\frac{311.2}{300}\right)^{(1.4/0.4)}$$

$$P_{01} = 5.684\text{ bar}$$

$$\frac{P^*}{P_0} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \Rightarrow \left(\frac{2}{2.4}\right)^{\frac{1.4}{0.4}} \Rightarrow 0.528$$

$$P^* = 0.528\times 5.684 = 3\text{ bar}$$

Also,

$$m = m^*$$

$$\rho_1 A_1 V_1 = \rho^* A^* V^*$$

$$\left(\frac{P_1}{RT_1}\right)A_1V_1 = \left(\frac{P^*}{RT^*}\right)A^*V^*$$

$$\left(\frac{5}{300}\right)\times 10\times 150 = \left(\frac{3}{259.33}\right)A^*.\sqrt{(1.4)\times 287\times 259.33}$$

$$A^* = 6.7\text{ cm}^2$$

(i) Throat Area of nozzle, $A^* = 6.7\text{ cm}^2$

A normal shock occurs where Mach number is 2.

From Table, When $M = 2$ $\left(\frac{A}{A^*} = 1.688\right)$

So, Area of nozzle where shock occurs

$$A = (1.688\times 6.7)\text{ cm}^2$$

(ii)

$$A = 11.3096\text{ cm}^2$$

At exit, $(M = 0.4)$

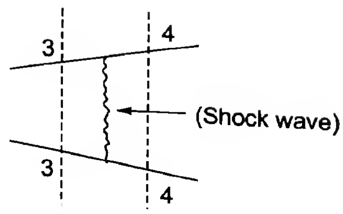
and

$$\frac{A}{A^*} = 1.59$$

(iii)

$$A = 1.59\times 6.7 = 10.653\text{ cm}^2$$

Let conditions upstream and downstream of shock are represented as



$$\frac{T_{0_3}}{T_3} = 1+\left(\frac{\gamma-1}{2}\right)M_3^2$$

$$\Rightarrow \quad 1+\left(\frac{1.4-1}{2}\right)(2)^2 \Rightarrow 1.8$$

$$T_3 = \frac{T_{0_3}}{1.8} \Rightarrow \frac{311.2}{1.8} = 172.88\text{ K}$$

and

$$V_3 = M_3\sqrt{\gamma RT_3} = 2\sqrt{1.4\times 287\times 172.88} = 527.13\text{ m/sec}$$

Also,

$$\dot{m}_1 = \dot{m}_3 \qquad \text{(mass flow rate is constant)}$$

$$\rho_1 A_1 V_1 = \rho_3 A_3 V_3$$

$$\left(\frac{P_1}{RT_1}\right)A_1V_1 = \left(\frac{P_3}{RT_3}\right)A_3V_3$$

$$\left(\frac{5}{300}\right)\times 10\times 150 = \left(\frac{P_3}{172.88}\right)\times 11.3096\times 527.13$$

$$P_3 = 0.725 \text{ bar}$$

and,
$$\frac{P_{03}}{P_3} \Rightarrow \left(\frac{T_0}{T_3}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{311.2}{172.88}\right)^{\frac{1.4}{0.4}}$$

$$P_{03} = 5.673 \text{ bar}$$

and
$$V_4 = M_4\sqrt{\gamma R T_4}$$

$$\frac{T_{04}}{T_4} = 1+\left(\frac{\gamma-1}{2}\right)M_4^2 = 1+\left(\frac{1.4-1}{2}\right)(0.577)^2$$

$$T_4 = 291.77 \text{ k}$$

$$V_4 = 0.577\times\sqrt{1.4\times 287\times 291.77} = 197.56 \text{ m/sec}$$

$$\dot{m}_3 = \dot{m}_4$$

$$\frac{P_4}{P_3} = \frac{1+\gamma M_3^2}{1+\gamma M_4^2} = \frac{1+(1.4\times 2^2)}{1+(1.4\times 0.577^2)} = \frac{6.6}{1.466} = 4.5$$

$$(P_4 = 4.5\times 0.725 = 3.263 \text{ bar})$$

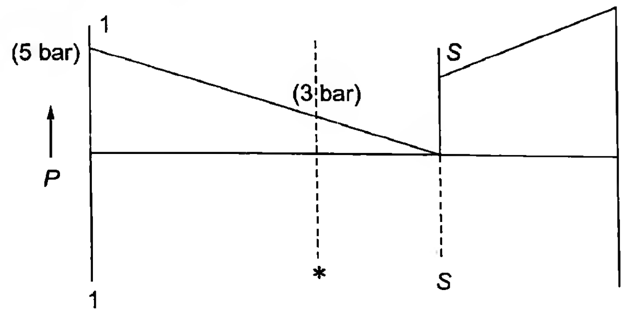
and
$$\frac{P_{04}}{P_4} \Rightarrow \left(\frac{311.2}{291.77}\right)^{\frac{1.4}{0.4}}$$

$$P_{04} = 4.088 \text{ bar}$$

(iv) Loss os stagnation pressure across shock,

$$\Delta P_0 = P_{03} - P_{04} = 5.673 - 4.088 = 1.584 \text{ bar}$$

Variation of pressure and Mach number along the length of nozzle

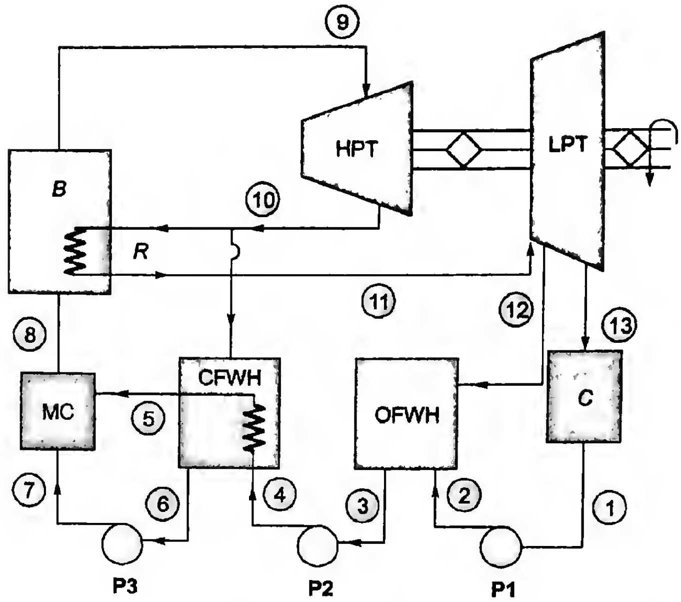


Q.26 A steam power plant runs on an ideal reheat-regenerative Rankine cycle. Details are given in Figure. Steam at turbine inlet is at, 150 bars and 600°C. Condenser pressure is 0.10 bar. Steam exiting the HPT is at 40 bars. This steam is split in two parts, y and x. Part x is reheated at the same pressure to 600°C and sent to LPT. Part y is condensed completely in CFWH and it is pumped to 150 bars before it mixes with the flow at same pressure. A fraction of steam, z, is extracted from LPT at 5 bars. Assume steam output of boiler as 1 kg/s for ease of computations. Determine:

(i) fraction y (ii) fraction z (iii) thermal efficiency of the cycle.

Given data is

h_1 = 191.81 kJ/kg	h_2 = 192.30 kJ/kg	h_9 = 3155.0 kJ/kg	h_{10} = 3155.0 kJ/kg
h_3 = 640.09 kJ/kg	h_4 = 643.92 kJ/kg	h_{11} = 3674.9 kJ/kg	h_{12} = 3014.8 kJ/kg
h_5 = 1087.4 kJ/kg	h_6 = 1087.4 kJ/kg	h_{13} = 2335.7 kJ/kg	w_{pump1} = 0.49 kJ/kg
h_7 = 1101.2 kJ/kg	h_8 = 1089.8 kJ/kg	w_{pump2} = 3.83 kJ/kg	w_{pump3} = 13.77 kJ/kg



R : Reheater
B : Boiler
HPT : High pressure turbine
LPT : Low pressure turbine
C : Condenser
OFWH : Open feedwater heater
CFWH : Closed feedwater heater
MC : Mixing chamber
P1 : Pump 1
P2 : Pump 2
P3 : Pump 3

Fig. : Schematic of steam power plant

[CSE (Mains) 2011 : 30 Marks]

Solution:

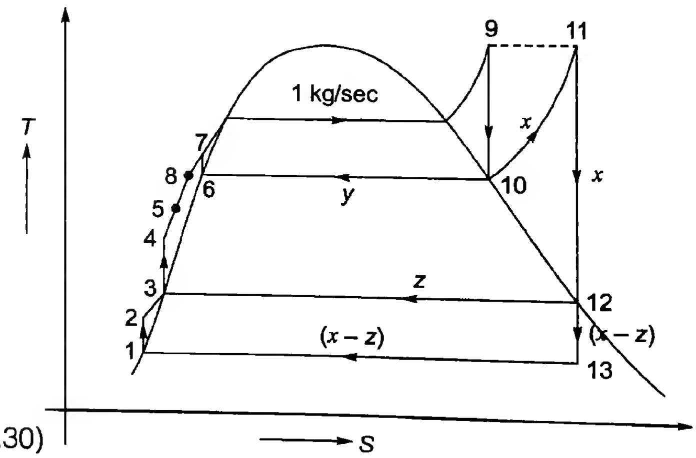
Steam output of boiler = 1 kg/sec

So, $x + y = 1$

Heat balance for *CFWH*

$$y(h_{10} - h_6) = x(h_5 - h_4)$$
$$y(h_{10} - h_6) = (1 - y)(h_5 - h_4)$$
$$y(3155 - 1087.4) = (1 - y)(1087.4 - 643.92)$$
$$2067.6y = 443.48 - 443.48\,y$$
$$y = 0.176 \text{ kg/s}$$

and
$$x = 1 - y = 0.823 \text{ kg/s}$$



Heat Balance for *OFWH*

$$Z(h_{12} - h_3) = (x - 2)(h_3 - h_2)$$
$$Z(3014.8 - 640.09) = (0.823 - Z)(640.09 - 192.30)$$
$$2374.71\,Z = 368.53 - 447.79Z$$
$$Z = 0.13 \text{ kg/s}$$

Thermal Efficiency of the cycle,

$$\eta_T = \frac{W_{net}}{Q_1}$$
$$W_T = 1(h_9 - h_{10}) + x(h_{11} - h_{12}) + (x - z)(h_{12} - h_{13})$$
$$W_T = 1(3155 - 3100) + 0.823(3674.9 - 3014.8) + (0.823 - 0.13)(3014.8 - 2335.7)$$

Note : In question value of h_{10} is given same as value of h_9, it is impossible. So, assuming value of (h_{10} = 3100 kJ/kg)

$$W_T = 55 + 543.26 + 470.6163$$
$$W_T = 1068.876 \text{ kJ}$$
$$W_P = W_{P1} + W_{P2} + W_{P3}$$
$$W_P = (0.823 - 0.13)(0.49) + (0.823)(3.83) + 0.176(13.77)$$
$$W_P = 5.915 \text{ kJ}$$
$$W_{net} = W_T - W_P = 1068.876 - 5.915 = 1062.96 \text{ kJ}$$

Heat added, $Q_1 = 1(h_9 - h_8) + x(h_{11} - h_{10})$

$$Q_1 = (3155 - 1089.8) + 0.823(3674.9 - 3100) = 2538.34 \text{ kJ}$$
$$\eta_{cycle} = \frac{W_{net}}{Q_1} = \frac{1062.96}{2538.34} = 0.4187 \text{ or } 41.87\%$$

Q.27 A small power plant produces 25 kg/s steam at 3 MPa, 600°C in the boiler. It cools the condenser with ocean water coming in at 12°C and return at 15°C. Condenser exit is 45°C. Find

(i) net power output.

(ii) required mass of ocean flow water.

Given: $h_{x=0}^{45°C} = 188.4$

$V_1 = 0.001\ m^3/kg$

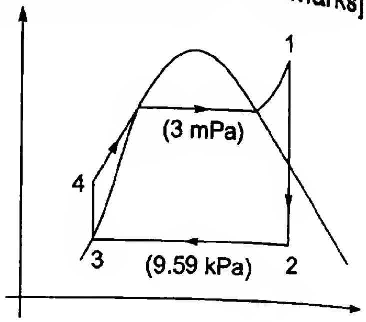
$P_{SQt}^{45°C} = 9.59\,kPa$

$h_{3\ MPa}^{600°C} = 3682$

$S_{3\ MPa}^{600°C} = 7.50\ kJ/kg\text{-}K$

[CSE (Mains) 2012 : 20 Marks]

Solution:



Given : $\dot{m}_s = 25$ kg/sec, $T_1 = 600°C$, $h_1 = 3682$ kJ/kg,

$S_1 = 7.50$ kJ/kgK $= S_2$, $h_3 = h_{x=0}^{45°C} = 188.4$ kJ/kg-K, $v_3 = 0.001$ m³/kg

$$W_P = \text{Pump work} = vdp$$

$$W_p = 0.001 \times [3 \times 10^6 - (9.59 \times 10^3)] \times 10^{-3} = 3\ kJ/kg$$

$$h_4 - h_3 = W_P = 3\ kJ/kg$$

$$h_4 = h_3 + 3 \Rightarrow 188.4 + 3 = 191.4\ kJ/kg$$

$$\text{Heat added, } Q_1 = h_1 - h_4 = 3682 - 191.4 = 3490.6\ kJ/kg$$

$$S_1 = S_2 = 7.50 = S_F + x_2 S_{fg}$$

$$7.50 = 0.6386 + x_2(7.5261)$$

$$x_2 = 0.91$$

$$h_2 = h_f + x_2 h_{fg}$$

$$h_2 = 188.42 + 0.91\,(2394.8) = 2367.68\ kJ/kg$$

$$W_T = (h_1 - h_2) = 3682 - 2367.68 = 1314.32\ kJ/kg$$

$$W_{net} = W_T - W_P = (1314.32 - 3)\ kJ/kg = 1311.32\ kJ/kg$$

(i) Power output, $W_{net} = 25 \times 1311.32 = 32.78$ MW

(ii) Let the required mass of ocean water be $\dot{m}_w$.

$$Q_R \Rightarrow 25 \times (2367.68 - 188.4) = 54.482\ MW$$

Also,

$$Q_R = \dot{m}_w C_P (\Delta T)$$

$$54.482 \times 10^6 = \dot{m}_w \times 4.18 \times 10^3 \times (15 - 12)$$

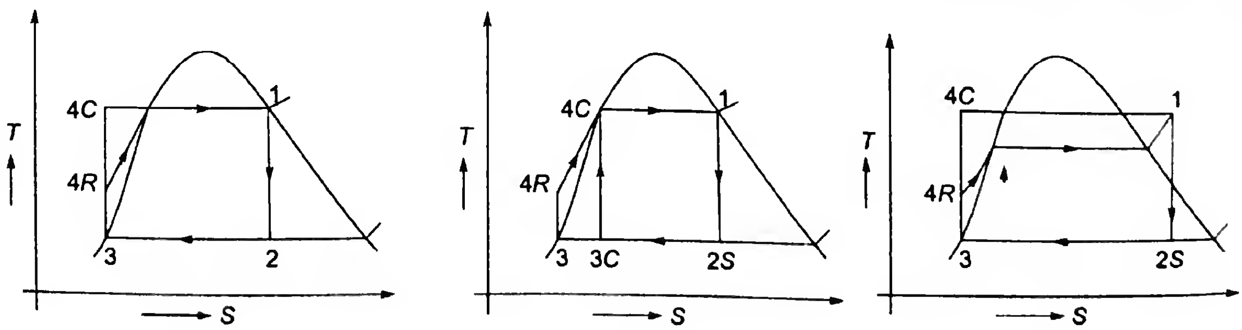
$$\dot{m}_w = 4344.6\ kg/s$$

Q.28 With the help of T-s diagrams, differentiate between Carnot and Rankine vapour cycles. State the advantages of Rankine cycle and derive the expression for its thermal efficiency.

[CSE (Mains) 2013 : 15 Marks]

Solution:

Difference between Carnot and Ranking Vapour Cycles: Although the carnot cycle has the maximum possible efficiency for the given temperature limits, but it is not suitable in steam power plants. Rather, Rankine cycle is preferred in steam power plants. Consider Rankine and Carnot cycles on the T-S diagram.



(a) (b) (c)

The reversible adiabatic expansion in the turbine, the constant temperature heat rejection in condenser, and the reversible adiabatic compression in the pump, are similar characteristic of features of both the rankine and carnot cycles. But whereas heat addition process in the Rankine cycle is reversible and at constant pressure, in the carnot cycle it is reversible and isothermal. The thermal efficiency (η) of carnot is greater than Rankine thermal efficiency. But the carnot cycle cannot be realised in practise because the pump work is very large. Whereas in diagram (a) and (c) it is impossible to add heat at infinite pressures and at constant temperature from state 4C to state 1. In figure (b) it is difficult to control the quality at (3C), so that the isentropic compression leads to saturated liquid state. Advantages of Rankine cycle are

1. It minimizes the pump work. Thus, the net output of cycle increase.
2. It increases the efficiency by increasing the pressure inside the boiler by increasing temperature, in Carnot, it is showing at constant temperature which is not possible.

Rankine cycle

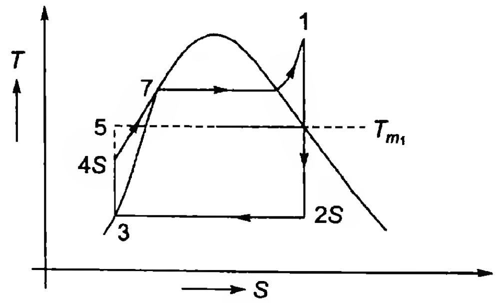
Let the mean temperature of heat addition be T_{M1}.

$$\text{Heat added, } Q_1 = h_1 - h_{4s} = T_{m1}(S_1 - S_{4s})$$

$$T_{m1} = \frac{h_1 - h_{4s}}{S_1 - S_{4s}}$$

$$Q_2 = \text{Heat rejected} = h_{2s} - h_3$$

$$Q_2 = T_2(S_1 - S_{4s})$$



$$\eta_{\text{Rankine}} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2(S_1 - S_{4s})}{T_{m_1}(S_1 - S_{4s})} = 1 - \frac{T_2}{T_{m_1}}$$

where T_2 is temperature of heat rejection.

The lower is the T_2 for a given T_{m1}, the higher will be the efficiency of Rankine cycle.

$$\eta_{\text{rankine}} = f(T_{m1})$$

The higher the mean temperature of heat addition, the higher will be the cycle efficiency.

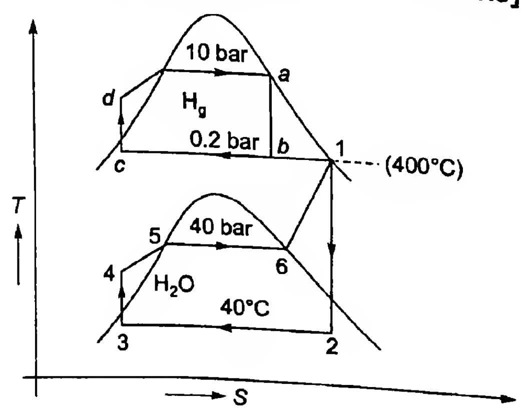
Q.29 A combined cycle power plant operates with mercury and steam cycles. Mercury cycle is superimposed over the steam cycle operating between boiler outlet condition of 40 bar and 400°C, (h = 3215.7 kJ/kg and s_g = 6.713 kJ/kg-K) and condenser temperature of 40°C. The heat released by mercury condensing at 0.2 bar is used to impart the latent heat of vaporization to the water in steam cycle. Mercury turbine receives mercury as saturated vapour at 10 bar. Calculate the mass of mercury circulated per unit mass of water and the efficiency of this binary cycle.

Properties of saturated mercury and steam are:

	(P) bar	T (°C)	Enthalpy (kJ/kg)		Entropy (kJ/kg-K)		Sp. volume (m³/kg)	
			h_f	h_g	s_f	s_g	v_f	v_g
Mercury	10	515.5	72.33	363	0.1478	0.5167	80.5×10^{-6}	0.0333
	0.2	277.3	38.35	336.55	0.0967	0.6385	77.4×10^{-6}	1.163
Steam	0.074	40	167.5	2574.4	0.572	8.258	0.001	19.546

[CSE (Mains) 2013 : 35 Marks]

Solution:



Given:

$$h_1 = 3215.7 \text{ kJ/kg}$$

$$S_1 = 6.713 \text{ kJ/kg-K}$$

As,

$$S_1 = S_2 = 6.713$$

$$6.713 = S_F + x_2 S_{fg}$$

$$6.713 = 0.572 + x_2(8.258 - 0.572)$$

$$x_2 = 0.8$$

$$h_2 = h_f + xh_{fg} = 167.5 + 0.8(2574.4 - 167.5)$$

$$h_2 = 2093.02 \text{ kJ/kg}$$

$$h_3 = 167.5 \text{ kJ/kg}$$

and $(h_4 - h_3) = v_f(\Delta P)$

$$h_4 = [167.5 + 0.001\,(40 - 0.074) \times 100] \text{ kJ/kg}$$

$$h_4 = 171.5 \text{ k3/kg}$$

$$h_5 = 1087.31 \text{ kJ/kg} \qquad \text{[From steam table]}$$

$$h_6 = 2801.4 \text{ kJ/kg}$$

For mercury cycle:

$$h_a = 363 \text{ kJ/kg}$$

$$S_a = 0.5167 \text{ kJ/kg}$$

$$S_a = S_b = 0.5167 = 0.0967 + x_b(0.6385 - 0.0967) \Rightarrow (x_b = 0.7751)$$

$$h_b = (h_f)_b + x_b(h_{fg})_b = 38.35 + 0.7751(336.55 - 38.35) = 269.48 \text{ kJ/kg}$$

$$h_c = 38.35 \text{ kJ/kg} = h_d$$

(Neglecting small mercury cycle pump work)

Let mass of mercury circulated per kg of steam be m.

From energy balance,

$$m(h_b - h_c) = 1 \times (h_6 - h_5)$$

$$m = \left(\frac{h_6 - h_5}{h_b - h_c}\right) = \left(\frac{2801.4 - 1087.31}{269.48 - 38.35}\right) = 7.4159 \text{ kg Hg/kg } H_2O$$

$$Q_1 = m(h_a - h_d) + 1(h_1 - h_6) + 1(h_5 - h_4)$$

$$Q_1 = 7.4159(363 - 38.35) + (3215.7 - 2801.4) + (1087.31 - 171.5)$$

Heat added, $Q_1 = 3737.68$ kJ/kg

Heat rejected, $Q_2 = h_2 - h_3$

$$Q_2 = (2093.02 - 167.5)\text{kJ/kg} = 1925.52 \text{ kJ/kg}$$

$$\eta_{\text{combined cycle}} = 1 - \frac{Q_2}{Q_1} = \left(1 - \frac{1925.52}{3737.68}\right) = 0.4848 \text{ or } 48.5\%$$

Q.30 The vapour, at the saturation temperature of an oil flowing at the rate of 500 kg/min, enters a heat exchanger tube, at 355 K and condenses while it is cooled by water flowing at the rate of 3600 kg/min entering the concentric tube of a parallel-flow heat exchanger at 286 K. Assuming overall heat transfer coefficient of 475 W/m²K, latent heat of oil as 600 kJ/kg K, calculate the number of tubes required of 25 mm outer diameter and 2 mm thick with a length of 4.87 m. What will be the number of tube passes, if cooling water velocity should not exceed 2 m/s? Take C_p for water as 4.18 kJ/kg K and density of water as 1000 kg/m³.

[CSE (Mains) 2014 : 20 Marks]

Solution:

Given : $P_1 = 1$ bar, $T_1 = 288$ k, $P_2 = 2$ bar, $T_4 = 1700$ K, $\eta_C = 0.87$, $\eta_T = 0.88$, $\eta_{comb.} = \eta_{HE} = 0.97$

$$(\Delta P)_{\text{combuster}} = 0.4 \text{ bar}$$

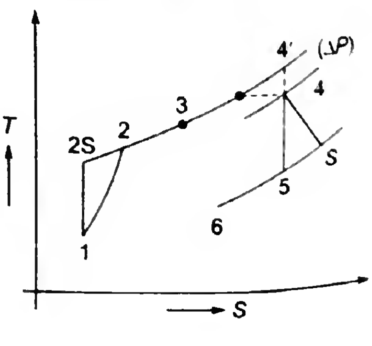
$$P_4 = 2 - 0.4 = 1.6 \text{ bar}$$

$$P_5 = 1 \text{ bar}$$

$$(W_T) = 350 \text{ mw}$$

$$CV = \text{calorific value} = 42 \text{ MJ/kg}$$

$$(C_P)_{air} = (C_P)_{gas} = 1.005 \text{ kJ/kgk}, \gamma = 1.4$$



$$\frac{T_{2S}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{2}{1}\right)^{\frac{1.4-1}{1.4}} = 1.219$$

$$\Rightarrow \quad T_{2s} = 351 \text{ k } [\text{As } T_1 = 288 \text{ k}]$$

$$\eta_C = 0.87 = \frac{T_{2S} - T_1}{T_2 - T_1}$$

$$(T_2 - T_1) = \left(\frac{351 - 288}{0.87}\right) = 72.5$$

$$T_2 = 360.5 \text{ k}$$

$$\frac{T_4}{T_{5S}} = \left(\frac{P_4}{P_5}\right)^{\frac{\gamma_g - 1}{\gamma_g}} = \left(\frac{1.6}{1}\right)^{\frac{1.4-1}{1.4}} \left(\gamma_g = \gamma = 1.4\right) = 1.143$$

$$T_{5S} = \frac{T_4}{1.143} = \frac{1700}{1.143} = 1486.38 \text{ k}$$

$$\eta_T = 0.88 = \frac{T_4 - T_5}{T_4 - T_{5S}}$$

$$(T_4 - T_5) = 0.88\,(1700 - 1486.38)$$

$$T_5 = 1512 \text{ k}$$

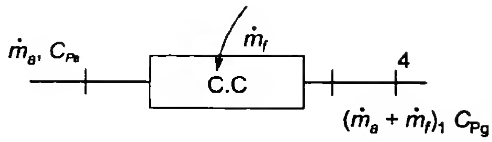
$$\eta_{H.E.} = 0.97 = \frac{T_3 - T_2}{T_5 - T_2}$$

$$T_3 - T_2 = 0.97(1512 - 360.5)$$

$$\Rightarrow \quad T_3 = 1477.5\text{k}$$

$$\text{Power developed by turbine} = (\dot{m}_a + \dot{m}_f) C_{pg} (T_4 - T_5)$$

$$(\dot{m}_a + \dot{m}_f) \times 1.005 \times (1700 - 1512) = 350 \times 10^3$$



$$\Rightarrow \quad \dot{m}_a + \dot{m}_f = 1852.45 \text{ kg/s}$$

$$\text{Heat released} = \text{Enthalpy rise of gas}$$

$$\dot{m}_f \times C.V. \times \eta_{comb} = (\dot{m}_a + \dot{m}_f) C_{pg} T_4 - \dot{m}_a C_{pa} T_3$$

$$\dot{m}_f (C.V. \times \eta_{comb} - C_{pa} T_4) = \dot{m}_a C_{pa} (T_4 - T_3)$$

$$\dot{m}_f \left[42 \times 10^3 \times 0.97 - 1.005 \times 1700\right] = \dot{m}_a \times 1.005 \times (1700 - 1477.5)$$

$$\Rightarrow \quad \frac{\dot{m}_a}{\dot{m}_f} = 174.5 \text{ kg/s}$$

As,
$$\begin{pmatrix} \dot{m}_f = 10.55 \text{ kg/sec} \\ \dot{m}_a = 1842.25 \text{ kg/sec} \end{pmatrix}$$

$$\text{Compressor Power Input} = \dot{m}_a C_{pa} (T_2 - T_1)$$

$$= 1842.25 \times 1.005 \times (360.5 - 288) = 134.24 \text{ MW}$$

$$\dot{W}_{net} = W_T - W_C = 350 - 134.24 = 215.76 \text{ MW}$$

$$\text{Work ratio} = \frac{W_{net}}{W_T} = \frac{215.76}{350} = 0.616$$

$$Q_1 \text{ (Heat added)} = \dot{m}_f \times C.V. \times \eta_{comb.}$$

$$Q_1 = 10.55 \times 42 \times 0.97 = Q_1 = 429.8 \text{ MW)}$$

$$\eta_{thermal} = \frac{W_{net}}{Q_1} = \frac{215.76}{429.8} = 0.5019 \text{ or } 50.19\%$$

$$\text{Specific fuel consumption} = \frac{\dot{m}_f}{W_{net}} = \frac{10.55}{215.76 \times 10^3} \times 3600 \text{ kg/kw.h} = 0.176 \text{ kg/kw-hr}$$

Q.31 Derive an expression for critical pressure ratio of a nozzle. Explain the phenomenon of choking in the nozzle.

[CSE (Mains) 2014 : 10 Marks]

Solution:

Expression for critical pressure ratio of a nozzle:

(a) Consider the isentropic flow of an ideal gas through a convergent-divergent nozzle.

Let
$$h_0 = \text{stagnation enthalpy}$$
$$h_0 = h + V^2/2$$
$$C_p(T_0 - T) = \frac{V^2}{2}$$

As
$$C_p = \frac{\gamma R}{\gamma - 1}$$
$$(T_0 - T) = \frac{V^2}{2\gamma R}(\gamma - 1)$$
$$\frac{T_0}{T} - 1 = \frac{V^2}{2\gamma RT}(\gamma - 1)$$
$$\text{Velocity of sound (c)} = \sqrt{\gamma RT}$$

and
$$M = \frac{V}{C}$$
$$M^2 = \frac{V^2}{\gamma RT} \quad \text{...(i)}$$

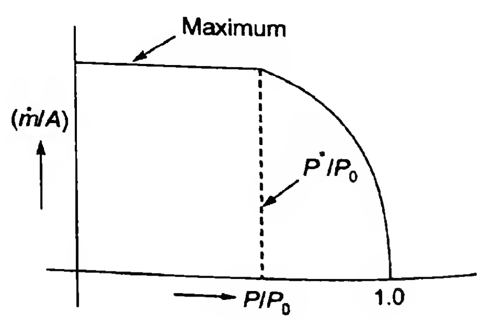
From equation (1)
$$\frac{T_0}{T} - 1 = \left(\frac{\gamma-1}{2}\right)M^2 = 1+\left(\frac{\gamma-1}{2}\right)M^2 \quad \text{...(ii)}$$

and
$$\frac{P_0}{P} = \left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma-1}} = \left[1+\left(\frac{\gamma-1}{2}\right)M^2\right]^{\frac{\gamma}{\gamma-1}} \quad \text{...(iii)}$$

When $M = 1$ occurs at the throat, the discharge is maximum and the nozzle is said to be choked. It is incapable of allowing more discharge even with further decrease in exhaust pressure. This discharge is known as critical discharge and the properties at the throat under this condition are called critical properties designated by superscript (*)

For critical pressure ratio of a nozzle, putting ($M = 1$) in equation (ii)

$$\frac{T_0}{T^*} = 1+\left(\frac{\gamma-1}{2}\right)(1)^2 = \left(\frac{\gamma+1}{2}\right)$$
$$\frac{T^*}{T_0} = \left(\frac{2}{\gamma+1}\right)$$



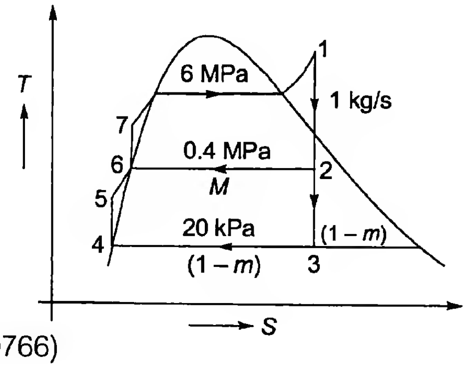
Also,
$$\frac{P^*}{P_0} = \left(\frac{T^*}{T_0}\right)^{\frac{\gamma}{\gamma-1}}$$
$$\frac{P^*}{P_0} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$$

Note: Thus, critical pressure ratio for an ideal gas depends only on the specific heat ratio (γ).

Q.32 A steam power plant operates on ideal regenerative Rankine cycle. Steam enters the turbine at 6 MPa, 450°C (h = 3301.8 kJ/kg, s = 6.7193 kJ/kg K) and is condensed in the condenser at 20 kPa (h_f = 251.4 kJ/kg, h_{fg} = 2358.3 kJ/kg, v_f = 0.001 m³/kg, s_f = 0.832 kJ/kg K, s_{fg} = 7.0766 kJ/kg K). Steam is extracted from the turbine at 0.4 MPa (h_f = 604.74 kJ/kg, v_f = 0.001 m³/kg, h_{fg} = 2133.8 kJ/kg, s_f = 1.7766 kJ/kg K, s_{fg} = 5.1193 kJ/kg K) to heat feedwater heater. Water leaves feedwater heater as saturated liquid. Show the cycle on T-s diagram and find net work output/kg of steam, the boiler and thermal efficiencies of the cycle. [CSE (Mains) 2014 : 20 Marks]

Solution:

Given: h_1 = 3301.8 kJ/kg, S_1 = 6.7193 kJ/kg-k = $S_2 = S_3$



$$S_1 = S_2 = 6.7193 = S_f + x_2 S_{fg}$$

$$6.7193 = 1.7766 + x_2(5.1193)$$

⇒ $$x_2 = 0.965$$

As, $$h_2 = h_f + x_2 h_{fg}$$

⇒ $$h_2 = 604.76 + 0.965\,(2133.8)$$

or $$h_2 = 2664.9 \text{ kJ/kg}$$

Also, $$S_1 = S_3 = 6.7193 = 0.832 + x_3\,(7.0766)$$

$$x_3 = 0.83$$

$$h_3 = h_f + x_3 h_{fg} = 251.4 + (0.83 \times 2358.3) = 2208.78 \text{ kJ/kg}$$

Neglecting pump work (ω_p is very small)

Assuming, $$h_4 = h_5 = (h_f)_{20\text{ kpa}} = 251.4 \text{ kJ/kg}$$

$$h_6 = (h_f)_{0.4\text{ mPa}} = 604.74 \text{ kJ/kg}$$

Energy balance for the heater gives,

$$m(h_2 - h_6) = (1-m)(h_6 - h_5)$$

$$m(2664.9 - 604.74) = (1-m)(604.74 - 251.4)$$

$$(2060.16)m = (1-m)(353.34)$$

$$2413.5\,m = 353.34 = 0.15 \text{ kg}$$

$$w_T = (h_1 - h_2) + (1-m)(h_2 - h_3)$$

$$w_T = (3301.8 - 2664.9) + (1 - 0.15)(2664.9 - 2208.78) = 1024.602 \text{ kJ/kg}$$

Let pump work be w_{P_1} and w_{P_2}.

$$w_{P_1} = (v\Delta P) \quad \text{[Process 4-5]}$$

$$= (0.001) \times \frac{(0.4 \times 10^6 - 20 \times 10^3)}{10^3} = 0.38 \text{ kJ/kg}$$

$$w_{P_1} = (1 - 0.15) \times 0.38 \text{ kJ} = 0.323 \text{ kJ}$$

Similarly $$w_{P_2} = V(\Delta P) \quad \text{[Process 6-7]}$$

$$= 0.001 \frac{(6 \times 10^6 - 0.4 \times 10^6)}{10^3} \text{kJ} = 5.6 \text{ kJ}$$

$$w_P = w_{P_1} + w_{P_2} = (0.323 + 5.6) = 5.923 \text{ kJ}$$

$$w_P/\text{kg} = 5.923 \text{ kJ/kg}$$

$$w_{net} = w_T - w_P = 1024.602 - 5.923 = 1018.68 \text{ kJ/kg}$$

$$Q_1 = (h_1 - h_7)$$

$$h_7 = h_6 + W_{P_2} = 604.74 + 5.6 = 610.34 \text{ kJ/kg}$$

$$Q_1 = (3301.8 - 610.34) \text{ kJ/kg} = 2691.46 \text{ kJ/kg}$$

$$\eta_{cycle} = \frac{w_{net}}{\theta_1} = \frac{1018.68}{2691.46} = 0.3784 \text{ or } 37.84\%$$

Q.33 Consider one-dimensional isentropic flow of a perfect gas. Derive an appropriate expression to show that the shape of the nozzle for supersonic flow is divergent in cross-section.

[CSE (Mains) 2015 : 10 Marks]

Solution:

Consider one-dimensional isentropic flow of a perfect gas.
Stagnation enthalpy (h_0) remains constant.

$$h_0 = h + \frac{v^2}{2} = k \text{ (constant)}$$

on differentiating,

$$0 = dh + \frac{2vdv}{2}$$

$$dh = -vdv \quad \text{...(i)}$$

Also,

$$Tds = dh - vdP = dh - \frac{dP}{\rho}$$

For isentropic process, ($ds = 0$)

$$dh = \frac{dP}{\rho} \quad \text{...(ii)}$$

From (i) and (ii)

$$\frac{dP}{\rho} = -vdv = -v^2\frac{dv}{v}$$

So,

$$\frac{dv}{v} = -\frac{dP}{\rho v^2} \quad \text{...(iii)}$$

As mass flow rate remains constant

$$\dot{m} = \rho A v$$

So,

$$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dv}{v} = 0$$

$$\frac{dA}{A} = -\frac{dv}{v} - \frac{d\rho}{\rho} = \frac{dA}{A} = \frac{dP}{\rho v^2} - \frac{d\rho}{\rho} = \frac{dP}{\rho v^2}\left[1 - \frac{d\rho}{dP}\cdot v^2\right]$$

As,

$$C = \sqrt{\frac{dP}{d\rho}} \Rightarrow C^2 = \frac{dP}{d\rho}$$

$$\frac{dA}{A} = \frac{dP}{\rho v^2}\left[1 - \frac{v^2}{c^2}\right]$$

Also,

$$\frac{dP}{\rho v^2} = -\frac{dv}{v} \quad \text{and} \quad M = \frac{v}{c}$$

$$\frac{dA}{A} = \frac{-dv}{v}(1 - M^2) = (M^2 - 1)\frac{dv}{v} \quad \text{...(iv)}$$

For supersonic flow, $M > 1$
From equation (iv),

For nozzle,

$$\frac{dv}{v} > 0, (M^2 - 1) > 0$$

So,

$$\frac{dA}{A} > 0$$

∴ Shape of nozzle for supersonic flow is divergent in cross-section.

Q.34 Which are the factors effecting Nozzle efficiency?

[CSE (Mains) 2015 : 4 Marks]

Solution:

Factors affecting nozzle efficiency are:

1. Friction between the fluid and walls of the nozzle, due to which the process becomes irreversible.
2. Semi-divergence angel (α) of nozzle, when this angle (α) is large, there will be flow separation from the wall with the formation of eddies, which entails energy loss.
3. The length of the nozzle.
4. The shape of the entrance and exit sections. As the foil nozzle is formed by curved aerofoil sections. This nozzle has well-rounded entrance edges and sharp exit edges and possess high efficiency.

Q.35 In a cogeneration plant, the power load is 5.6 MW and the heating load is 1.163 MW. Steam is generated at 40 bar and 500°C and is expanded isentropically through a turbine to a condenser at 0.06 bar. The heating load is supplied by extracting steam from the turbine at 2.0 bar, which is condensed in the processor device to saturated liquid at 2.0 bar and then pumped back to the boiler. Compute:

(i) the steam generation capacity of the boiler in kg/hr,

(ii) the heat input to the boiler in kW,

(iii) the fuel burning rate of the boiler in *t/h*, if a coal of calorific value 25 MJ/kg is burned and the boiler efficiency is 88,

(iv) the heat rejected to the condenser

(v) the rate of flow of cooling water in the condenser if the temperature rise of water is 6°C. Neglect pump works.

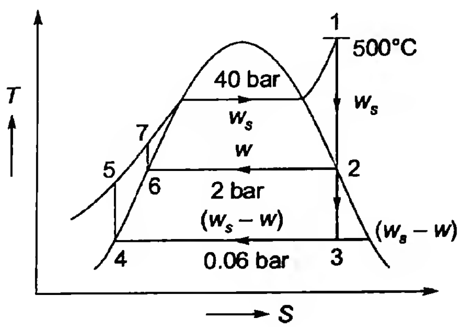
Draw the T-s diagram. Properties of steam: At 40 bar 500°C.

$v = 0.08643$ m³/kg, $h = 3445.3$ kJ/kg, $s = 7.0901$ kJ/kg K.

P_{sat} (bar)	Volume (m³/kg)		Enthalpy (kJ/kg		Entropy (kJ/kg K)	
	v_f	v_g	h_f	h_g	s_f	s_g
2.0	0.00106	0.8857	520.72	2712.1	1.5706	7.0878
0.06	0.00101	25.22	149.79	2565.79	0.520	8.335

[CSE (Mains) 2015 : 20 Marks]

Solution:



Given: $h_1 = 3445.3$ kJ/kg, $S_1 = 7.0901 = S_2 = S_3$

$$S_1 = 7.0901 = S_2 = 1.5706 + x_2(7.0878 - 1.5706)$$
$$x_2 = 1$$
$$h_2 = h_g = 2712.1 \text{ kJ/kg}$$
$$h_2 - h_6 = h_{fg} = h_g - h_f = 2712.1 - 520.72$$
$$h_2 - h_6 = 2191.38 \text{ kJ/kg}$$

Let the rate of steam extraction for process heating be w.

$$w(h_2 - h_6) = 1.163 \times 10^3$$
$$w = \frac{1.163 \times 10^3}{2191.38} = 0.53 \text{ kg/sec or } 1901.5 \text{ kg/hr}$$

As,
$$S_1 = 7.0901 = S_f + x_3 S_{fg} = 0.520 + x_3(8.335 - 0.520)$$
$$x_3 = 0.84$$
$$h_3 = 149.79 + 0.84(2565.79 - 149.79) = 2179.23 \text{ kJ/kg}$$

Total work output $w_T = w_s(h_1 - h_2) + (w_s - w)(h_2 - h_3)$

$$w_T = 5.6 \times 10^3 = w_s(3445.53 - 2712.1) + \{(w_s - 0.53) \times (2712.1 - 2179.23)\}$$
$$5.6 \times 10^3 = 733.2\, w_s + (w_s - 0.53)(532.87) = 1266.07\, w_s - 282.42$$

$\Rightarrow \quad w_s = 4.646$ kg/sec or 16726.3 kg/hr

(i) $\quad h_7 = 520.72 + 0.00106(40-2) \times 10^2$

$h_7 = 524.748$ kJ/kg

Similarly, $\quad h_s = 149.79 + 0.00101 \times (40 - 0.06) \times 10^2 = 153.8$ kJ/kg

$Q_1 = (w_s - w)(h_1 - h_5) + w(h_1 - h_7)$

$= (4.646 - 0.53)(3445.3 - 153.8) + 0.53(3445.3 - 524.748) = 15095.706$ kJ/s

(ii) $\quad Q_1 = 15.1$ mW

$$\eta_{boiler} = 0.88 = \frac{\theta_1}{\dot{m}_f \times CV} = \frac{15.1}{\dot{m}_f \times 25}$$

$\dot{m}_f = 0.687$ kg/sec $= 2473.2$ kg/h

(iii) $\quad \dot{m}_f = 2473.2$ kg/hr $= 2.473$ t/h

$Q_2 = (w_s - w)(h_3 - h_4) = (4.646 - 0.53)(2179.23 - 149.79)$

(iv) $\quad Q_2 = 8.353$ MW

Let the water flow rate in condenser be w_c,

$Q_2 = w_c C_p(T_2 - T_1)$

(v) $$w_c = \frac{8353}{4.187 \times 6} = 332.5 \text{ kg/sec}$$

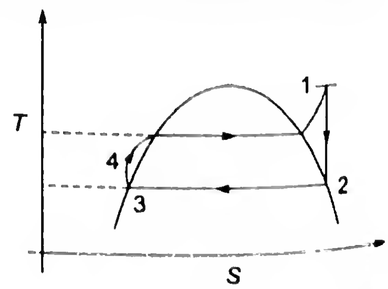
Q.36 What are the desirable properties of a fluid to be used as a working substance in Rankine cycle based heat engine plant? Discuss with the help of T-s diagram.

[CSE (Mains) 2016 : 10 Marks]

Solution:

The desirable properties of a fluid to be used as a working substance in Rankine cycle based plants are:

1. The fluid should have a high critical temperature so that the saturation pressure at the maximum permissible temperature (metallurgical limit) is relatively low. It should have a large enthalpy of vaporization at that pressure.
2. The saturation pressure at the temperature of heat rejection should be above the atmospheric pressure so as to avoid the necessity of maintaining vacuum in condenser.
3. The specific heat of liquid should be small so that little heat transfer is required to raise the liquid to boiling point.
4. The saturated vapour line of T-s diagram should be steep, very close to the turbine expansion process so that excessive moisture does not appear during expansion.
5. The freezing point of the fluid should be below the room temperature, so that it does not get solidified while flowing through the pipelines.
6. The fluid should be chemically stable and should not contaminate the materials of construction at any temperature.
7. The fluid should be non-toxic, non-corrosive, not excessively viscous, and low in cost.
   T-s diagram of an ideal working fluid for a vapour power cycle.
   Some superheat is desired to reduce piping losses and improve the turbine efficiency. The bounded are of the cycle is almost a rectangle, and its thermal efficiency is very close to the Carnot efficiency.



Q.37 A convergent-divergent nozzle receives steam at 5.0 bar, 200°C and expands isentropically into a space at 2.0 bar. Neglecting the inlet velocity, calculate the exit area required for a mass flow of 0.3 kg/s in the following cases:

(i) When the flow is in equilibrium throughout

(ii) When the flow is supersaturated with $PV^{1.3}$ = constant

Calculate also for this supersaturated flow, the degree of supercooling and the degree of supersaturation.

Properties of steam:

At 5.0 bar and 200°C, h = 2855.4 kJ/kg, s = 7.0592 kJ/kg K, V = 0.4249 m³/kg

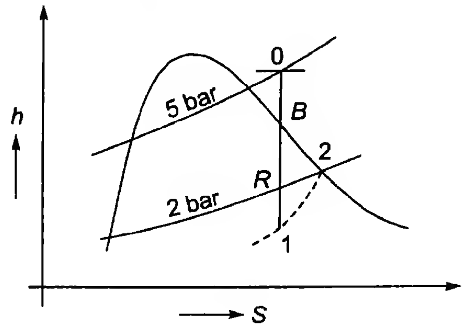
At 2.0 bar and T_{sat} = 120.23°C, h_f = 504.7 kJ/kg, h_g = 2706.7 kJ/kg,

s_f = 1.5301 kJ/kg K, s_g = 7.1271 kJ/kg K, V_f = 0.001061 m³/kg, V_g = 0.8857 m³/kg

[CSE (Mains) 2016 : 20 Marks]

Solution:

(i) When the flow is in equilibrium throughout;



$$h_0 = 2855.4 \text{ kJ/kg}$$

$$s_0 = 7.0592 \text{ kJ/kg} = s_1$$

$$s_0 = s_1 = 7.0592$$

$$= 1.5301 + x_1(7.1271 - 1.5301)$$

$$x_1 = 0.98$$

$$h_1 = 504.7 + 0.98\,(2706.7 - 504.7) = 2662.66 \text{ kJ/kg}$$

$$v_1 = 0.98 \times 0.8857 = 0.86 \text{ m}^3\text{/kg}$$

Velocity at exit $(V_1) = 44.72\,(h_0 - h_1)^{1/2}$

$$V_1 = 44.72(2855.4 - 2662.66)^{1/2} = 620.85 \text{ m/sec}$$

and $$\dot{m}_s = \frac{A_1 V_1}{v_1}$$

$$0.3 = \frac{A_1 \times 620.85}{0.86}$$

$$A_1 = 415.5 \text{ mm}^2$$

Exit area when the flow is in equilibrium,

$$A_1 = 415.5 \text{ mm}^2$$

(ii) When the flow is supersaturated with

$$Pv^{1.3} = \text{Constant}$$

$$dh = Tds + vdp = vdp \; [ds = 0]$$

and $$h_0 = h + \frac{V^2}{2} = \text{constant}$$

$$dh = Vdv$$

$$Vdv = -vdp$$

As $$PV^{1.3} = \text{Constant}$$

$$PV^n = C \qquad \text{[where } n = 1.3]$$

$$\int_{V_0}^{V_R} VdV = \int_{P_0}^{P_1} -\left(\frac{C}{P}\right)^{1/n} dp = \int_{P_0}^{P_1} -(C)^{1/n}(P)^{-1/n} dp$$

$$\left(\frac{V_R^2 - V_0^2}{2}\right) = -(C)^{1/n}\left(\frac{n}{n-1}\right)\left(P_1^{1-1/n} - P_0^{1-1/n}\right)$$

Since $V_0 = 0$

$$\frac{V_R^2}{2} = \left(\frac{n}{n-1}\right)(P_0V_0 - P_1V_R)$$

Now, $$\frac{V_R}{V_0} = \left(\frac{P_0}{P_1}\right)^{1/n} = \left(\frac{5}{2}\right)^{1/1.3} = 2.023$$

$$V_R = 2.023 \times 0.4249 = 0.86\ \text{m}^3/\text{kg}$$

So,
$$\frac{V_R^2}{2} = \left(\frac{1.3}{0.3}\right)[500 \times 0.4249 - 200 \times 0.86] = 175.28 \times 10^3$$

$$V_R = 592\ \text{m/sec}$$

$$A_1 = \text{Exit area} = \frac{\dot{m}V_R}{592} = \frac{\dot{m}V_R}{V_R} = \frac{0.3 \times 0.86}{592} = 435.8\ \text{mm}^2$$

$$\frac{T_0}{T_R} = \left(\frac{P_0}{P_1}\right)^{\left(\frac{n-1}{n}\right)} = \left(\frac{5}{2}\right)^{0.3/1.3} = (2.5)^{0.23} = 1.2346$$

$$T_R = \frac{473}{1.2346} = 383.12\ \text{k} = 110.12°\text{C}$$

$$(t_{sat})_{P1} = 120.23°\text{C}$$

$$\text{Degree of subcooling} = 120.23 - 110.12 = 10.11°\text{C}$$

$$(P_{sat})_{t_R=110°C} = 1.4327\ \text{bar}$$

$$\text{Degree of supersaturation} = \frac{2\ \text{bar}}{1.4327\ \text{bar}} = 1.396$$

Q.38 Write a note on the off-design performance characteristics of a convergent-divergent nozzle. Plot the pressure distribution along the axis of the nozzle for different back pressures.

[CSE (Mains) 2016 : 20 Marks]

or

Show the effect of variation of back pressure on distribution of pressure and velocity all along in a convergent-divergent nozzle.

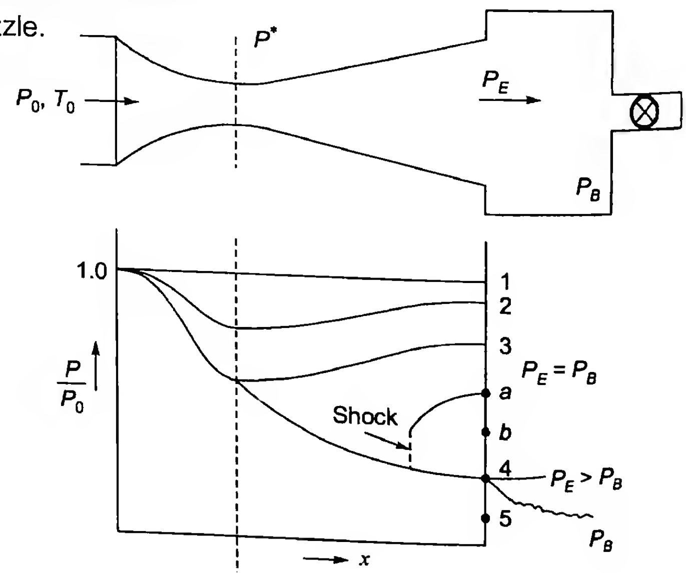
[CSE (Mains) 2010 : 20 Marks]

Solution:

Performance characteristics of a convergent-divergent nozzle:

Consider a flow through a convergent divergent nozzle.

In the diagram, point 1 designates the condition when ($P_B = P_0$) and there is no flow. When P_B is lowered to the pressure denoted by 2, so that $\frac{P_B}{P_0} < 1$, but $\left(\frac{P_B}{P_0} > \frac{P}{P_0}\right)$, the velocity increases in the convergent section but $M < 1$ occurs at the throat. The diverging section acts as the subsonic diffuser in which the pressure increases and velocity decreases. Point 3 indicates the back pressure when M = 1 occurs at the throat but the diverging section acts as a subsonic diffuser.



Point 4 indicates one another back pressure for which the flow is isentropic throughout and the diverging section acts as a supersonic nozzle with a continuous decrease in pressure and a continuous increase in velocity and ($P_{E4} = P_{B4}$). This condition of supersonic flow indicates the design pressure ratio of the nozzle. If the pressure ratio is lowered to 5, no further decrease in exit pressure occurs and the drop of pressure from P_E to P_B occurs outside the nozzle.

Between the back pressure between 3 and 4, flow is not isentropic in the diverging part, and it is accompanied by a irreversible phenomenon known as shocks. Shocks occur only when the flow is supersonic, and after the shock the flow becomes subsonic and the rest of the duct acts as a diffuser properties vary continuously across the shock. When the back pressure is as indicated by point b, the flow throughout the nozzle is isentropic, with pressure continuously decreasing and the velocity increasing. A shock appears just at the exit of the nozzle $P_b < P_E$. When the back pressure is increased from b to a, the shock moves upstream as indicated. When the back pressure is further increased, the shock-moves upstream and disappears at the nozzle throat where the back pressure corresponds to 3. Since the flow throughout is subsonic, no shock is possible.

Q.39 A cyclic steam power plant is to be designed for a steam temperature at turbine inlet of 360°C and an exhaust pressure of 0.08 bar. After isentropic expansion of steam in the turbine, the moisture content at the turbine exhaust is not to exceed 15%. Determine the greatest allowable steam pressure at the turbine inlet, and calculate the Rankine cycle efficiency for these steam conditions. Estimate also the mean temperature of heat addition. Compare the Rankine cycle efficiency with the Carnot cycle efficiency operating between the mean temperature of heat addition and the condenser temperature. Neglect the pump work input.

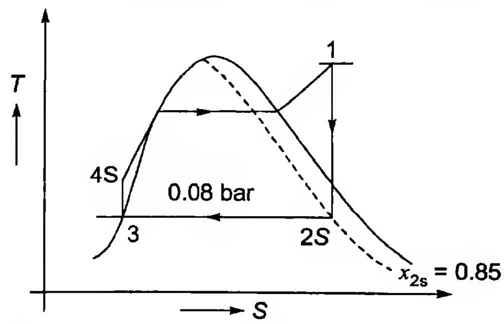
Properties of steam:

At P = 0.08 bar and T_{sat} = 41°C, h_f = 173.88 kJ/kg,

s_f = 0.5926 kJ/kg K, h_{fg} = 2403.1 kJ/kg, s_{fg} = 7.6361 kJ/kg K

[CSE (Mains) 2016 : 20 Marks]

Solution:



As the quality and pressure are known at state 25.

$$S_{2s} = S_F + x_{2s} S_{fg}$$
$$= 0.5926 + 0.85(7.6361)$$
$$= 7.0833 \text{ kJ/kg-K}$$

Also, $S_1 = S_{2s} = 7.0833$ kJ/kg-K

At state 1, the temperature and entropy are thus known. At 360°C,

S_g = 5.0526 kJ/kg.K, which is less than S_1, so it is in superheated state.

From the table of superheated steam, at t_1 = 360°C and S_1 = 7.0833 kJ/kg-K, the pressure is found to be 16.832 bar (by interpolations)

∴ The greatest allowable steam pressure is

$$P_1 = 16.832 \text{ bar}$$
$$h_1 = 3165.54 \text{ kJ/kg}$$
$$h_{2s} = h_f + x_{2s} h_{fg} = 173.88 + 0.85(2403.1) = 2216.52 \text{ kJ/kg}$$
$$h_3 = h_f = 173.88 \text{ kJ/kg}$$

Neglecting the pump work input

$$h_3 = h_{4s} = 173.88 \text{ kJ/kg}$$
$$Q_1 = \text{Heat added} = h_1 - h_{45} = 3165.54 - 173.88 = 2991.66 \text{ kJ/kg}$$
$$W_T = h_1 - h_{4s} = 3165.54 - 2216.52 = 949 \text{ kJ/kg}$$
$$W_{net} = W_t - W_P = W_T = 949 \text{ kJ/kg}$$
$$\eta_{cycle} = \frac{W_{net}}{\theta_1} = \frac{949}{2991.66} = 0.3172 \text{ or } 31.72\%$$

Let be the mean temperature of heat addition be T_{m1}.

$$T_{m1} = \frac{h_1 - h_{4s}}{S_1 - S_{4s}} = \frac{3165.4 - 173.88}{7.0833 - 0.592.6} = 460.66 \text{ k} = 187.51°\text{C}$$

Let be Carnot cycle efficiency operating between the mean temperature of heat addition and the condenser temperature be η_{carnot}.

$$\eta_{Carnot} = 1 - \frac{T_0}{T_{m_1}}$$

$$T_0 = T_{sat} = 41°C \Rightarrow 314 \text{ k}$$

$$T_{m1} = 460.66 \text{ k}$$

$$\eta_{Carnot} = 1 - \frac{314}{460.66} = 31.83\%$$

Note: So, the efficiency of Rankine cycle is almost similar to that of Carnot cycle.

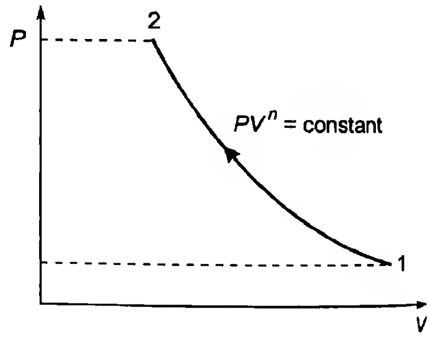
4. Compressors

Q.40 The ratio of heat transfer to work transfer in the process of an air compressor reciprocating type is 1/4. If the compression follows pv^n = constant, what is the value of n? Derive the equation that you use. In such a compression process the work required is 200 kJ/kg and the specific heat at constant volume is 0.75 kJ/kg K. What rise of temperature is expected at the end of compression process?

[CSE (Mains) 2001 : 20 Marks]

Solution:

Work done during polytropic compression is given by,



$$W = \int_1^2 pdV$$

We have,

$$PV^n = C$$

$$P = \frac{C}{V^n} = CV^{-n}$$

$$W = \int_1^2 V^{-n} dV = C\left(\frac{V^{-n+1}}{-n+1}\right)\Bigg|_1^2$$

$$W = \left|PV^n \frac{V^{1-n}}{1-n}\right|_1^2 = \frac{(PV)}{n-1}\Bigg|_1^2 = \frac{P_2V_2 - P_1V_1}{1-n} = \frac{R(T_2 - T_1)}{1-n} \quad \text{... (i)}$$

Heat transfer for a polytropic process is given by

$$dQ = du + PdV = C_V(T_2 - T_1) + \frac{R(T_1 - T_1)}{1-n} = C_V(T_2 - T_1) - \frac{C_V(\gamma - 1)(T_2 - T_1)}{1-n}$$

$$= C_V(T_2 - T_1)\left[1 - \frac{\gamma - 1}{n-1}\right]$$

$$Q = C_V \frac{n-\gamma}{n-1}(T_2 - T_1) \quad \text{... (ii)}$$

As given,

$$\frac{Q}{W} = \frac{1}{4}$$

From equation (i) and (ii),

$$C_V \frac{n-\gamma}{n-1}(T_2 - T_1) = \frac{R}{4} = \frac{(\gamma - 1)C_V}{4}$$

$$\gamma - n = \frac{\gamma - 1}{4}$$

$$4\gamma - 4n = \gamma - 1$$
$$4\gamma - \gamma + 1 = 4n$$
$$n = \frac{1+3\gamma}{4} = \frac{1+4.2}{4} = \frac{5.2}{4} = 1.3$$
$$\text{Work done, } W = \frac{P_1V_1 - P_2V_2}{n-1} = \frac{R(T_1 - T_2)}{n-1}$$

As work done during compression process is negative,

$$-W = 200 \text{ kJ/kg} = \frac{R\Delta T}{n-1}$$
$$\Delta T = \frac{200(n-1)}{(\gamma-1)C_V} = \frac{200(1.3-1)}{0.75(1.4-1)} = 200 \text{ K}$$

Q.41 Show that volumetric clearance efficiency of a reciprocating compressor is given by:

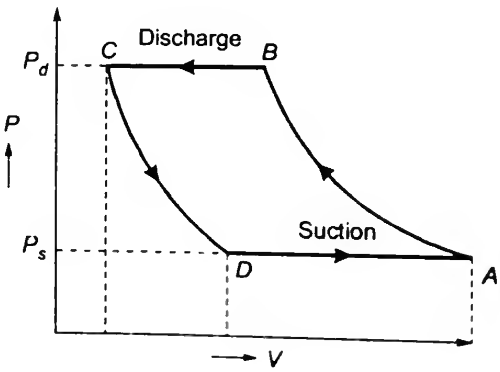
$$\eta_w = I + CI - CI\,(pd/ps)^{1/n}$$

where CI is clearance ratio and suffix d & s refer to discharge and suction pressures.

[CSE (Mains) 2001 : 20 Marks]

Solution:

Volumetric efficiency of the compressor with clearance is given by



$$\eta_{V,Cl} = \frac{\text{Actual volume of air compressed}}{\text{Swept volume of compressor}}$$
$$\eta_V = \frac{V_A - V_D}{V_A - V_C} = \frac{(V_A - V_C) + (V_C - V_D)}{(V_A - V_C)} = 1 + \frac{V_C - V_D}{V_A - V_C}$$

Since the clearance ratio,

$$CI = \frac{V_C}{\text{Swept volume}} = \frac{V_C}{V_A - V_C}$$
$$V_A - V_C = \frac{V_C}{CI}$$
$$\eta_V = 1 + \frac{CI(V_C - V_D)}{V_C} = 1 + CI - CI\left(\frac{V_D}{V_C}\right)$$

If we assume the re-expansion process as pV^n = Constant, then

$$\frac{V_0}{V_C} = \left(\frac{P_C}{P_D}\right)^{1/n} = \left(\frac{P_{\text{discharge}}}{P_{\text{suction}}}\right)^{1/n}$$

So, the volumetric clearance efficiency is given by

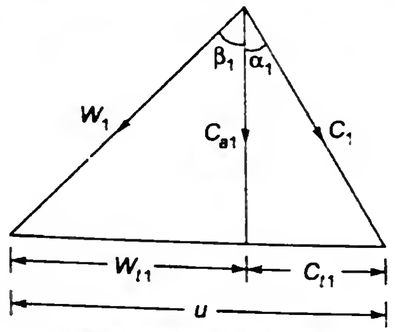
$$\eta_V = 1 + CI - CI\left(\frac{P_d}{P_s}\right)^{1/n}$$

Q.42 An axial flow compressor of 50% reaction blading has isentropic efficiency of 82%. It draws air at 17°C and compresses in the pressure ratio of 4 : 1. The mean blade speed and flow velocity are constant throughout the compressor. The inlet and outlet angles of blades are 15° and 45° respectively (angles measured from axial direction). Blade speed = 180 m/s and work input factor = 0.84. Calculate (i) flow velocity and (ii) number of stages.

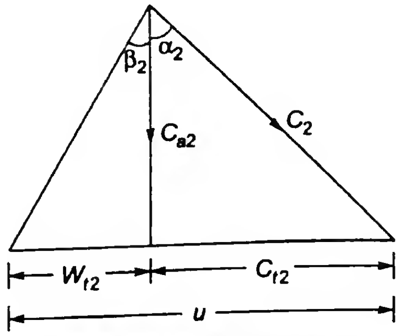
[CSE (Mains) 2002 : 30 Marks]

Solution:

Given: $r_p = 4$, $\eta_c = 82\%$, 50% reaction blading i.e. $R = 0.5$, work factor 'Ω' = 0.84, C_a = blade sped = 180 m/s



Inlet velocity triangle



Outlet velocity triangle

Since the stages are symmetrical, degree of reaction is 50% and also $\alpha_1 = \beta_1$ and $\alpha_2 = \beta_2$

$$\text{Degree of reaction, } R = \frac{C_a}{2u}(\tan\beta_1 + \tan\beta_2)$$

Since,

$$R = 0.5$$

$$C_a = \frac{u}{\tan\beta_1 + \tan\beta_2} = \frac{180}{\tan 15^\circ + \tan 45^\circ} = \mathbf{141.96\ m/s}$$

Temperature rise per stage is given by,

$$\Delta T_s = \frac{\Omega u C_a}{C_p}(\tan 45^\circ - \tan 15^\circ) = \frac{0.84 \times 180 \times 141.96}{1005}(1 - 0.268)$$

$$= 15.634\ \text{K}$$

$$\Delta T_{\text{overall}} = \frac{T_1}{\eta_c}\left(r_p^{\frac{\gamma-1}{\gamma}} - 1\right) = \frac{290}{0.82}(4^{0.286} - 1) \quad \left[\frac{\gamma - 1}{\gamma} = 0.286\right] = 172.084\ \text{K}$$

$$\text{Number of stages, } n = \frac{\Delta T_{\text{overall}}}{\Delta T_{\text{stage}}} = \frac{172.084}{15.634} = 11$$

Q.43 Discuss the effect of pre-whirl on the performance of centrifugal compressors.
A single sided centrifugal air compressor delivers 1800 kg of air per minute. The air enters the eye of the impeller axially at total pressure of 100 kPa and total temperature of 290 K. The overall diameter of the impeller is 700 mm and it rotates at 1600 RPM. The slip factor is 0.9 and work input factor is 1.1. Isentropic efficiency is 85%. Calculate
(i) power required by the compressor, (ii) pressure coefficient and
(iii) total pressure at delivery.

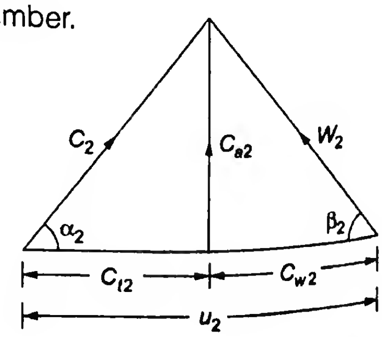
[CSE (Mains) 2003 : 30 Marks]

Solution:

Effect of pre-whirl on the performance of Centrifugal compressor:

Pre-whirl can be achieved by fixing inlet guide vanes to the compressor casing. This changes the inlet velocity triangles. The work capacity of the compressor decreases due to change in velocity triangle. Pre-whirl is a special type of motion provided to the air at the entry of the compressor.

(i) Pre-whirl avoids shock wave formation as it reduces the relative Mach number.
(ii) It reduces losses which were to be there due to shock wave formation.
(iii) Pre-whirl reduces the required output.
(iv) It also reduces the pressure ratio.



Numerical Part

Given: Power input factor, $P_{if} = 1.1$, Slip factor, $\mu = 0.9$, Isentropic efficiency, $\eta_c = 0.85$, Mass flor rate, $\dot{m} = 1800\ \text{kg/min} = \frac{1800}{60} = 30\ \text{kg/s}$

Pressure at impeller eye, P_{01} = 100 kPa

Temperature at impeller eye, T_{01} = 290 k

Blade velocity at outlet, $u_2 = \frac{\pi DN}{60} = \frac{\pi \times 0.7 \times 1600}{60} = 58.64$ m/s

(i) Power required by the compressor,

$$W = \dot{m} P_{if} \mu u_2^2 = \frac{30 \times 1.1 \times 0.9 \times (58.64)^2}{100} = \mathbf{102.13\ kW}$$

(ii) Pressure coefficient is given by,

$$\psi_p = \frac{C_p T_{01}\left[(r_p)^{\frac{\gamma-1}{\gamma}} - 1\right]}{u_2^2}$$

$$\frac{P_{03}}{P_{01}} = \left[1 + \frac{\eta_c (T_{03} - T_{01})}{T_{01}}\right]^{\frac{\gamma}{\gamma-1}} \quad \text{(03 refers to diffuser outlet)}$$

$$T_{03} - T_{01} = \frac{P_{if} \mu u_2^2}{C_p} = \frac{1.1 \times 0.9 \times (58.64)^2}{1005} = 3.387 \text{ K}$$

Now,
$$\frac{P_{03}}{P_{01}} = \left[1 + \frac{0.85(3.387)}{290}\right]^{\frac{1.4}{0.4}} \approx 1.035$$

Total pressure at delivery, P_{03} = 100 × 1.035 = 103.5 kPa

$$\psi_p = \frac{C_p T_{01}\left[(1.035)^{\frac{0.4}{1.4}} - 1\right]}{(58.64)^2} = 0.8372$$

(iii) Total pressure at delivery, P_{03} = **103.5 kPa**

Q.44 Air at a temperature of 300 K enters a ten-stage axial flow compression at the rate of 3.5 kg/s. The pressure ratio is 6.0 and the isentropic efficiency is 90%. The process is adiabatic and the compressor has symmetrical stages. The axial velocity of 120 m/s is uniform across the stages and the mean blade speed is 200 m/s. Assume that the temperature change is same in each stage.
Determine the direction of the air at entry to and exit from the rotor and stator blades. Also find the power given to the air.
For air, take C_p = 1.005 kJ/kg and γ = 1.4.

[CSE (Mains) 2004 : 30 Marks]

Solution:

Given: T_1 = 300 K, η_C = 90%, $\dot{m}$ = 3.5 Kg/s, u = mean blade speed = 200 m/s, $C_a = C_{a_1} = C_{a_2} = 120$ m/s

As the temperature change is same in each stage, the power input may be obtained by considering the overall conditions.

$$\frac{T_{2'}}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_{2'} = (6)^{0.286} \times 300 = 500.8 \text{ K}$$

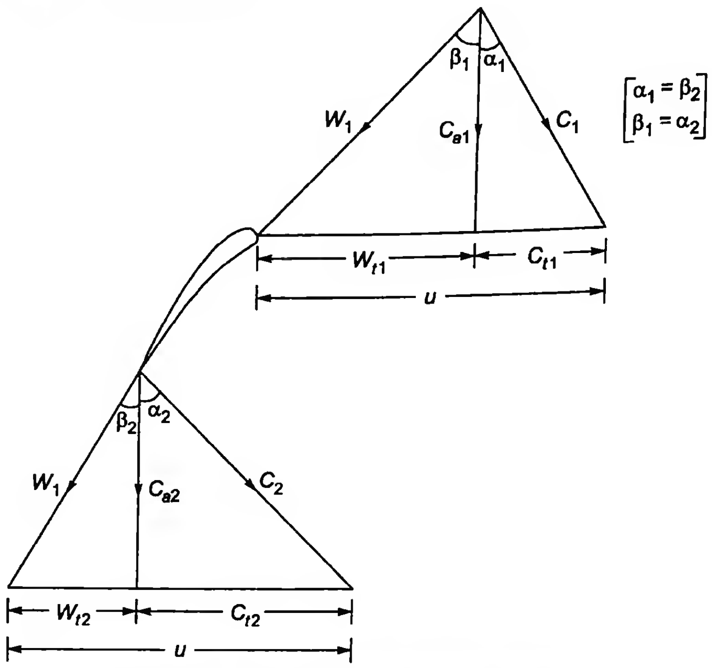
Isentropic efficiency,
$$\eta_c = \frac{T_{2'} - T_1}{T_2 - T_1}$$

$$T_2 = \frac{T_{2'} - T_1}{\eta_c} + T_1 = 523.12 \text{ K}$$

$$= \frac{500.8-300}{0.9} + 300$$

Power given to the air, $W = \dot{m}C_p\Delta T$

$= 3.5 \times 1.005(523.12-300) =$ **784.83 kW**



Entry to and exit from rotor : Velocity Diagram

$$\text{Temperature change per stage, } \Delta T_s = \frac{\Delta T}{10} = \frac{523.12-300}{10} = 22.312 \text{ K}$$

Work done/kg of air second $= u\Delta C_t = 200\,\Delta C_t$

Also work done/per kg of air per second $= C_p\Delta T_s = 200\,\Delta C_t$

$$\Delta C_t = \frac{1005 \times 22.312}{200} = 112.118 \text{ m/s}$$

For symmetrical stages, $\Delta C_t = C_a(\tan\beta_1 - \tan\beta_2)$

$$112.118 = 120(\tan\beta_1 - \tan\beta_2)$$

$$\tan\beta_1 - \tan\beta_2 = \frac{112.118}{120} = 0.9343 \qquad \text{... (i)}$$

$$\text{Degree of reaction, } R = \frac{C_a}{2u}(\tan\beta_1 - \tan\beta_2)$$

$$\tan\beta_1 + \tan\beta_2 = \frac{2Ru}{C_a} = \frac{2\times 0.5\times 200}{120} = 1.67 \qquad \text{... (ii)}$$

Solving (i) and (iii) $\tan\beta_2 = \dfrac{0.9343+1.67}{2} = 1.30215 =$ **52.47°**

$\Rightarrow$ $\tan\beta_1 = 1.30215 \Rightarrow \beta_1 = 52.47°$

Now from equation (ii)

As, $\tan\beta_1 + \tan\beta_2 = 1.67$

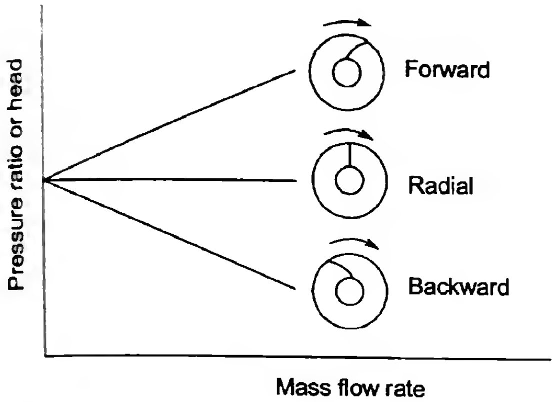
$\Rightarrow$ $\tan\beta_2 = 1.67 - \tan\beta_1 = 1.67 - 1.30215$

$\beta_2 = 20.19°$

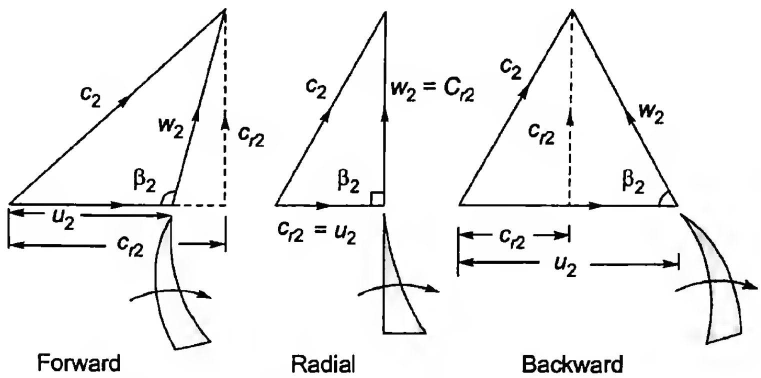
Q.45 Sketch and explain the curves showing the variation of pressure ratio (or head) versus volume flow rate for the three types of blades generally used in centrifugal compressor. Indicate on each curve the stable range of operation.

[CSE (Mains) 2004 : 30 Marks]

Solution:



The effect of impeller blade shape on performance



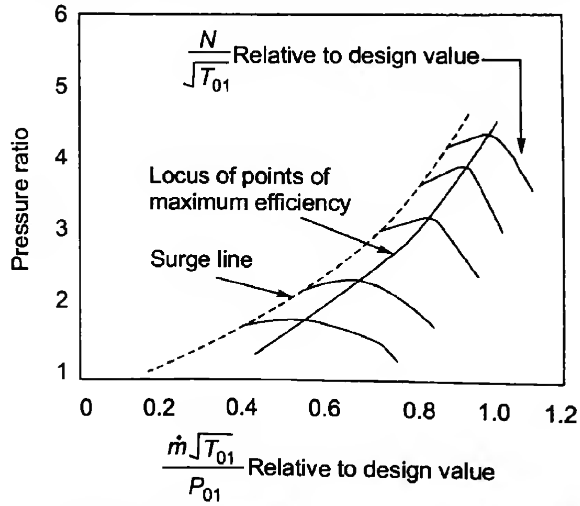
Exit velocity diagrams for different shapes of blades

The comparison of performance for the three types of vanes are made for the same volume flow rate, each blade having unit depth, and for the same vector value of c_{r2} Figure (ii) shows the exit velocity triangles for three types of impellers from which the relative performance of the blades can be evaluated.

Centrifugal effects of the curved blades create a bending moment and produce increased stresses which reduce the maximum speed at which the impeller can be run. Good performance can be obtained with radial impeller blades. Backward-curved blades are slightly better in efficiency and are stable over a wider range of flows than the radial or forward-curved blades.

The forward-curved impeller can produce the highest pressure ratio for a given blade tip speed; but is inherently less stable and has a narrow operating range. Its efficiencies are lower than that are possible with the backward-curved or radial-curved blades.

Although all the three types can be used in compressors, the radial blade is used almost exclusively in turbojet engine applications.



Actual characteristics of a centrifugal compressor

Q.46 An air compressor has eight stages of equal pressure ratio 1.35. The flow rate through the compressor and its overall efficiency are 50 kg/s and 82 percent respectively. If the condition of air at entry are 10 bar and 40°C, determine:

(i) the state of air at the compressor exit,
(ii) polytropic or small stage efficiency,
(iii) efficiency of each stage,
(iv) power required to drive the compressor assuming overall efficiency of the drive as 90%.

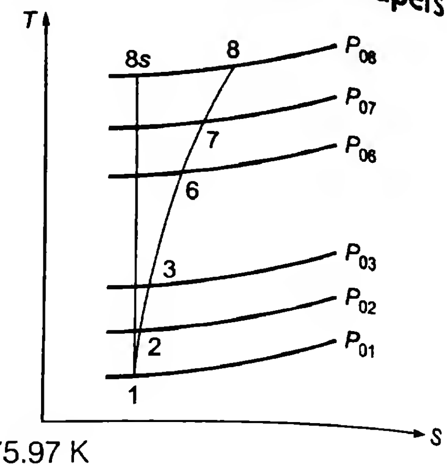
Take C_p = 1.005 kJ/kg K and γ = 14.

[CSE (Mains) 2005 : 20 Marks]

Solution:

Given: Stages = 8, r_p = 1.35/stage, η_c = 82%, $\dot{m}$ = 50 kg/s,

$T_{01} = 273 + 40 = 313$ K



$$\text{Overall pressure ratio} = (\gamma_s)^8 = (1.35)^8 = 11$$

$$\frac{T_{8s}}{T_{01}} = \left(\frac{p_{08}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} = (11)^{\frac{0.4}{1.4}} = 1.985$$

$$T_{8S} = 621.3 \text{ K}$$

$$\eta_c = \frac{T_{8s} - T_{01}}{T_8 - T_{01}}$$

$$T_{08} - T_{01} = \frac{T_{8s} - T_{01}}{\eta_c} = \frac{621.3 - 313}{0.82} = 375.97 \text{ K}$$

$$\Delta T_{\text{overall}} = T_{08} - T_{01} = 375.97 \text{ K}$$

$$\Delta T_{\text{stage}} = \frac{375.97}{8} \approx 47 \text{ K}$$

$$T_{08} = 375.97 + 313 = 388.97 \text{ K}$$

$$\text{Density of air at inlet, } \rho_1 = \frac{p_1}{RT_1} = \frac{1 \times 10^5}{287 \times 313} = 1.113 \text{ kg/m}^3$$

$$\text{Density of air at inlet, } \rho_8 = \frac{p_8}{RT_8} = \frac{11 \times 10^5}{287 \times 688.97} = 5.563 \text{ kg/m}^3$$

State of air at the compressor exit are **11 bar and 415.97°C.**

Relation between isentropic efficiency and polytropic efficiency is given by

$$\eta_c = \frac{r^{\frac{\gamma-1}{\gamma}} - 1}{r^{\frac{\gamma-1}{\gamma\eta_p}} - 1}$$

$$\Rightarrow \qquad 0.82 = \frac{11^{0.286} - 1}{11^{\frac{0.286}{\eta_p}} - 1}$$

$$\Rightarrow \qquad 11^{\frac{0.286}{\eta_p}} = 1 + \frac{(11^{0.286} - 1)}{0.882} = 2.2$$

$$\Rightarrow \qquad \frac{0.286}{\eta_p} \log_e 11 = \log_e 2.2$$

$$\eta_p = \frac{0.286 \times \ln(11)}{\ln(2.2)} = \mathbf{0.87 \text{ or } 87\%}$$

$$\Rightarrow \qquad \frac{T_{2s}}{T_1} = (r_s)^{\frac{\gamma-1}{\gamma}}$$

$$\Rightarrow \qquad T_{2s} = 313 \times (1.35)^{0.286} = 341 \text{ K}$$

Efficiency of each stage is given by

$$\eta_s = \frac{T_{2s} - T_1}{T_2 - T_1} = \frac{341 - 313}{\Delta T_{\text{stage}}} = \frac{28}{47} = \mathbf{0.5957 \text{ or } 59.57\%}$$

Power required to drive the compressor,

$$p = \frac{\dot{m}C_p\Delta T_{\text{overall}}}{\eta} = \frac{50 \times 1.005 \times 375.97}{0.9}$$

$$\text{Power} = \mathbf{20991.658\ kW\ or\ 20.99\ MW}$$

Q.47 A small compressor has the following data:

Air flow rate = 1.5778 kg/s **Pressure = 1.6**

Rotational speed = 54,000 rpm **Efficiency = 85%**

State of air at entry, P_{0_1} = 1.008 bar; T_{0_1} = 300 K C_p for air = 1.009 kJ/kg K.

(i) Calculate the power required to drive this compressor.

(ii) A geometrically similar compressor of three times the size is constructed. Determine, for this compressor

I. mass flow rate, **II. pressure ratio,**

III. speed and **IV. the power required.**

Assume same entry conditions and efficiency for the two compressors and also assume kinematic and dynamic similarities between the two machines. **[CSE (Mains) 2005 : 30 Marks]**

Solution:

Given: Air flow rate, $\dot{m}$ = 1.5778 kg/s, Pressure ratio, r = 1.6, Speed, N = 54000 rpm

Let us assume kinematic and dynamic similarities between the two machines. Let subscripts 1 and 2 represent the small and large compressors respectively.

(i) Actual enthalpy rise through the compressor

$$\Delta h_{01} = C_p(T_{02} - T_{01}) = C_p \frac{T_{01}}{\eta_c}\left\{\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right\} = 1.009 \times \frac{300}{0.85}(1.6^{0.286} - 1)$$

$$= 51.20 \text{ kJ/kg}$$

Power required to drive the compressor $= \dot{m}\Delta h_{01} = 1.5778 \times 51.20 = \mathbf{80.78\ kW}$

(ii)
$$\pi_2 = \frac{N_2D_2}{\sqrt{\gamma_2R_2T_{01}}} = \frac{N_1D_1}{\sqrt{\gamma_1R_1T_{01}}}$$

For the same working fluid, $\gamma_1 = \gamma_2$, $R_1 = R_2$

$\therefore$
$$N_2D_2 = N_1D_1$$

$$N_2 = \frac{D_1}{D_2}\cdot N_1 = \frac{N_1}{3} = \frac{54000}{3} = \mathbf{18000\ rpm}$$

$$\pi_2 = \frac{\dot{m}_2\sqrt{R_2T_{01}}}{p_{01}D_2^2} = \frac{\dot{m}_1\sqrt{R_1T_{01}}}{p_{01}D_1^2}$$

(p_{01} and T_{01} remains same for the same entry condition)

For the same working fluid, $R_2 = R_1$ (Same gas constant)

$$\frac{\dot{m}_2}{D_2^2} = \frac{\dot{m}_1}{D_1^2}\ ;\ \dot{m}_2 = \left(\frac{D_2}{D_1}\right)^2 \dot{m}_1 = 3^2 \times 1.5778 = \mathbf{14.20\ kg/s}$$

We also have for the two compressors

$$\frac{\Delta h_{02}}{(N_2D_2)^2} = \frac{\Delta h_{01}}{(N_1D_1)^2}$$

$$\Delta h_{02} = \left(\frac{N_2}{N_1}\times\frac{D_2}{D_1}\right)^2 \Delta h_{01} = \left(\frac{1800}{54000}\times 3\right)^2 \Delta h_{01} = \Delta h_{01}$$

For the same efficiency, $\eta_c = 0.85$

$$\Delta h_{02} = \Delta h_{01} = f(Pr)$$

Pressure ratio of the large compressor is also **1.6**

$$\frac{P_2}{N_2^3 D_2^5} = \frac{P_1}{N_1^3 D_1^5}$$

$$P_2 = \left(\frac{N_2}{N_1}\right)^3 \times \left(\frac{D_2}{D_1}\right)^5 \times P_1 = \left(\frac{1}{3}\right)^3 \times 3^5 \times 80.78 = 9\times 80.78 = 727.02\ \text{kW}$$

Q.48 For flow through a compressor cascade, show that lift and drag coefficients are given by the following expressions:

$$C_L = Z\left(\frac{s}{c}\right)\frac{\Delta C L_w}{C_m} - \xi\left(\frac{s}{c}\right)\frac{\cos^3\alpha_m}{\cos^2\alpha_1}\tan\alpha_m \quad \text{and} \quad C_D = \xi\left(\frac{s}{c}\right)\frac{\cos^3\alpha_m}{\cos^2\alpha_1}$$

where

s/c = Pitch-chord ratio

α_m = Mean flow angle

$\Delta C_w = C_{w_1} - C_{w_2}$

ξ = Total pressure loss coefficient

[CSE (Mains) 2006 : 20 Marks]

Solution:

Due to friction the total pressure loss in the cascade is given by the difference of stagnation pressures for up stream and far down stream of the cascade.

Thus, $\Delta p_0 = P_{01} - P_{02}$ and the total pressure loss coefficient is given by

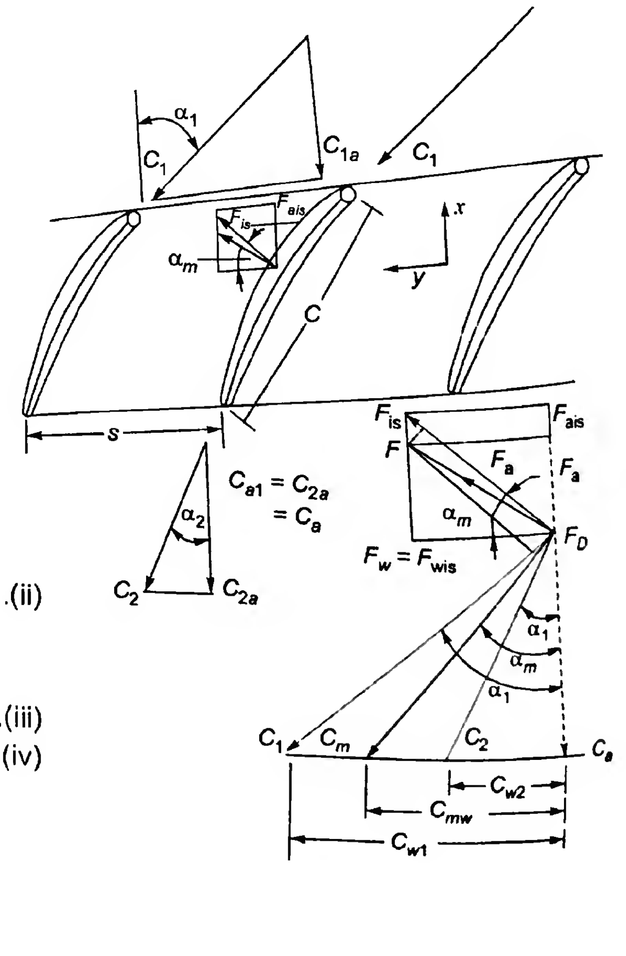
$$\xi = \frac{P_{01} - P_{02}}{\frac{1}{2}\rho C_1^2}$$

Actual pressure rise in the cascade is

$$\Delta p = P_2 - P_1 = \frac{\rho}{2}(C_1^2 - C_2^2) - \Delta P_0$$

$$= \rho C_a^2(\tan\alpha_1 - \tan\alpha_2)\tan\alpha_m - \Delta P_0$$

$$= \rho C_a^2(\tan\alpha_1 - \tan\alpha_2)\tan\alpha_m - \frac{1}{2}\xi P\frac{C_a^2}{\cos^2\alpha_1}$$



Peripheral and axial component of the force on a blade of unit height are given by,

$$F_w = S\rho C_a^2(\tan\alpha_1 - \tan\alpha_2) \quad \text{...(i)}$$

$$F_a = S\rho C_a^2(\tan\alpha_1 - \tan\alpha_2)\tan\alpha_m - \frac{1}{2}\xi S\rho\frac{C_a^2}{\cos^2\alpha_1} \quad \text{...(ii)}$$

The Lift (L) and drag force (D) are given by,

$$L = F_W\cos\alpha_m + F_a\sin\alpha_m \quad \text{...(iii)}$$

$$D = F_W\sin\alpha_m - F_a\cos\alpha_m \quad \text{...(iv)}$$

The lift and drag coefficient is given by

$$C_L = \frac{L}{\frac{1}{2}\rho c C_m^2} \quad \text{and} \quad C_D = \frac{D}{\frac{1}{2}\rho c C_m^2}$$

from equation (i), (ii) and (iii),

$$L = \rho s C_a^2(\tan\alpha_1 - \tan\alpha_2)\cos\alpha_m + \rho s C_a^2(\tan\alpha_1 - \tan\alpha_2)\frac{\sin^2\alpha_m}{\cos\alpha_m} - \frac{1}{2}\xi\rho s\frac{C_a^2\sin\alpha_m}{\cos^2\alpha_1}$$

$$C_L = \frac{L}{\frac{1}{2}\rho c C_m^2} = 2\left(\frac{s}{c}\right)\frac{C_a^2}{C_m^2}(\tan\alpha_1 - \tan\alpha_2)\cos\alpha_m + 2\left(\frac{s}{c}\right)\left(\frac{C_a^2}{C_m^2}\right)(\tan\alpha_1 - \tan\alpha_2)\frac{\sin^2\alpha_m}{\cos\alpha_m}$$

$$-\xi\left(\frac{s}{c}\right)\frac{C_a^2}{C_m^2}\frac{\sin\alpha_m}{\cos^2\alpha_1}$$

As per momentum equation

$$F_w = \dot{m}(C_{w1} - C_{w2}) = C_a\rho s(C_{w1} - C_{w2}) = \rho s C_a^2(\tan\alpha_1 - \tan\alpha_2)$$

$$\tan\alpha_1 - \tan\alpha_2 = \frac{C_{w1} - C_{w2}}{C_a} = \frac{\Delta C_w}{C_a}$$

Now, expression of C_L can be written as:

$$C_L = 2\left(\frac{s}{c}\right)\left(\frac{C_a}{C_m}\right)^2\frac{\Delta C_w}{C_a}\cos\alpha_m + 2\left(\frac{s}{c}\right)\left(\frac{C_a}{C_m}\right)^2\frac{\Delta C_w}{C_a}\frac{\sin^2\alpha_m}{\cos\alpha_m} - \xi\left(\frac{s}{c}\right)\left(\frac{C_a}{C_m}\right)^2\frac{\sin\alpha_m}{\cos^2\alpha_1}$$

$$C_L = 2\left(\frac{s}{c}\right)(\cos^2\alpha_m)\left(\frac{\Delta C_w}{C_a}\right)\left[\frac{\cos^2\alpha_m + \sin^2\alpha_m}{\cos\alpha_m}\right] - \xi\left(\frac{s}{c}\right)\cos^2\alpha_m\frac{\sin\alpha_m}{\cos\alpha_1}\quad\left[\frac{C_a}{C_m} = \cos\alpha_m\right]$$

$$C_L = 2\left(\frac{s}{c}\right)(\cos\alpha_m)\frac{\Delta C_w}{C_a} - \xi\left(\frac{s}{c}\right)\frac{\cos^3\alpha_m}{\cos\alpha_1}\tan\alpha_m$$

$$C_L = 2\left(\frac{s}{c}\right)\frac{\Delta C_w}{C_m} - \xi\left(\frac{s}{c}\right)\frac{\cos^3\alpha_m}{\cos\alpha_1}\tan\alpha_m$$

First expression derived

Now, let us consider equation (i), (ii) and (iv):

$$D = \rho s C_a^2(\tan\alpha_1 - \tan\alpha_2)\sin\alpha_m - \rho s C_a^2(\tan\alpha_1 - \tan\alpha_2)\tan\alpha_m\cos\alpha_m + \frac{1}{2}\xi\rho s\frac{C_a^2}{\cos^2\alpha_1}\cos\alpha_m$$

$$D = \rho s C_a^2(\tan\alpha_1 - \tan\alpha_2)(\sin\alpha_m - \sin\alpha_m) + \frac{1}{2}\xi\rho s\frac{C_a^2}{\cos^2\alpha_1}\cos\alpha_m$$

$$= \frac{1}{2}\xi\rho s\frac{C_a^2}{\cos^2\alpha_1}\cos\alpha_m$$

$$C_D = \frac{D}{\frac{1}{2}\rho c C_m^2} = \xi\left(\frac{s}{c}\right)\left(\frac{C_a}{C_m}\right)^2\frac{1}{\cos^2\alpha_1}\times\cos\alpha_m = \xi\left(\frac{s}{c}\right)\frac{\cos^3\alpha_m}{\cos^2\alpha_1}$$

Second expression derived.

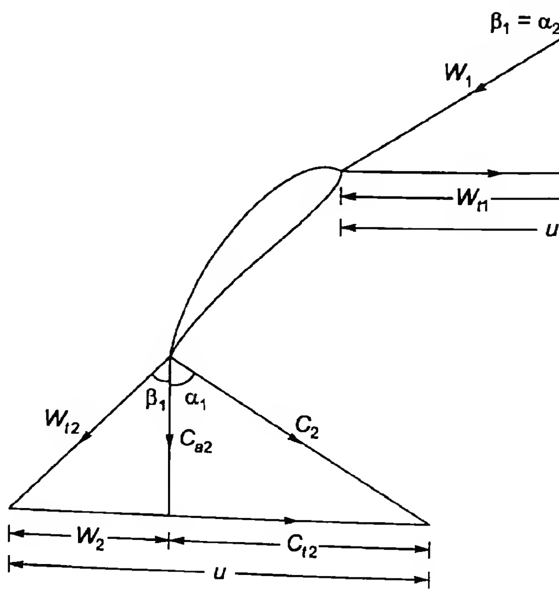
Q.49 An axial flow compressor compresses the air up to overall stagnation pressure 10 bars with overall stagnation isentropic efficiency of 88%. The inlet stagnation pressure and temperature are 1 bar and 300 K. The mean blade speed is 200 m/s. The degree of reaction is 0.5 at the mean radius with air angles of 30° and 10° at rotor inlet and outlet with axial direction respectively. The work done factor is 0.88. The hub-tip ratio is 0.4. The mass flow rate is 50 kg/s. Work out the following:

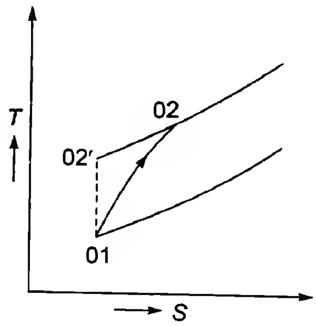
(i) Draw the inlet and outlet velocity triangles and show the compression process on T-S diagram
(ii) The stagnation polytropic efficiency
(iii) The number of stages
(iv) The blade height in first stage of the compressor.

[CSE (Mains) 2006 : 30 Marks]

Solution:

Given: η = 88%, P_{01} = 1 bar, T_{01} = 300 K, u = 200 m/s, R = 0.5, $\dot{m}$ = 50 kg/s, β_1 = 30°, β_2 = 10°





Isentropic and polytropic efficiencies are related by,

$$\eta_c = \frac{r^{\frac{r-1}{r}} - 1}{r^{\frac{r-1}{r \cdot \eta_p}} - 1}$$

$$\Rightarrow \quad 0.88 = \frac{10^{0.286} - 1}{10^{\frac{0.286}{\eta_p} - 1}}$$

$$\Rightarrow \quad (10)^{\frac{0.286}{\eta_p}} = 1 + \frac{10^{0.286} - 1}{0.88} = 2.06$$

$$\Rightarrow \quad \frac{0.286}{\eta_p} \log_e 10 = \log_e 2.06$$

$$\Rightarrow \quad \eta_p = 0.286 \times \frac{\log_e 10}{\log_e 2.06} \times 100 = \mathbf{91.12\%}$$

Since, the stages are symmetrical, degree of reactions is 50% and also $\alpha_1 = \beta_2$ and $\alpha_2 = \beta_1$.

From the velocity triangle above, $\quad u = C_a(\tan\beta_1 + \tan\beta_2)$

$$C_a = \frac{u}{\tan 30° + \tan 10°} = \frac{200}{\tan 30° + \tan 10°} = 265.36 \text{ m/s}$$

The Stage temperature rise, $\Delta T_s = \dfrac{\Omega u C_a}{C_p}(\tan\alpha_2 - \tan\alpha_1)$

$$= \frac{0.88 \times 200 \times 265.36}{1005}(\tan 30° - \tan 10°)$$

$$\Delta T_s = 18.636 \text{ K}$$

$$\frac{T_{02}}{T_{01}} = \left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma\eta_p}} = (10)^{\frac{0.286}{0.9112}} = 2.06$$

$$T_{02} = 300 \times 2.06 = 618 \text{ K}$$

Total temperature rise, $T_{02} - T_{01} = 618 - 300 = 318$ K

$$\text{Total number of stages} = \frac{\text{Total temperature rise}}{\text{Temperature rise in one stage}}$$

$$= \frac{318}{18.636} \approx 17$$

$$\text{Density of air at inlet, } \rho = \frac{P_1}{RT_1} = \frac{1\times10^5}{287\times300} = 1.164 \text{ kg/m}^3$$

Mass flow rate, $\dot{m}$ = Flow area × Density × Axial velocity = $\rho C_a(\pi D_m h)$ [h = blade height]

$$D_m h = \frac{\dot{m}}{\pi\rho C_a} = \frac{50\times10^6}{3.14\times1.1614\times265.36} = 51668.17 \text{ mm}^2 \quad ...(i)$$

where,

$$D_m = \frac{D_h + D_t}{2}$$

or,

$$D_m = \frac{D_t}{2}\left[\frac{D_h}{D_t}+1\right] \quad \left[\text{As given } \frac{D_h}{D_t} = 0.4\right]$$

$$D_m = \frac{1.4}{2}D_t = 0.7\,D_t$$

As,

$$h = \frac{D_t - D_h}{2} = \frac{1-0.4}{2}D_t = 0.3\,D_t$$

⇒

$$D_m h = 0.7\times0.3\times D_t^2 = 51668.17 \quad [\text{As } D_m = 0.7\,D_t \text{ and from equation (i)}]$$

$$D_t^2 = 246038.9$$

$$D_t = \mathbf{496 \text{ mm (Tip Diameter)}}$$

$$\text{Hub diameter, } D_h = \mathbf{0.4\,D_t = 198.4 \text{ mm}}$$

$$\text{Blade height, } h = \frac{D_t - D_h}{2} = 0.3\times496 = 148.8 \text{ mm}$$

Q.50 The first axial-flow air compressor stage without inlet guide vanes is operating at a speed of 15000 r.p.m. in the atmospheric conditions of T_0 = 288 K, P_0 = 1.01 bar. The rotor mean blade ring diameter is 0.34 m and hub to tip ratio is 0.5. Atmospheric air enters the stage with a velocity of 150 m/s. Consider constant axial velocity through the stage and take stage efficiency as 0.86, mechanical efficiency as 0.97, work done factor as 0.97, C_p = 1.005 kJ/kg/K and R = 0.287 kJ/kg/K. Sketching the axial compressor stage with velocity diagrams and labelling with most general notations used in practice, determine the following for attaining relative velocity ration across rotor (de Haller number) of 0.73:

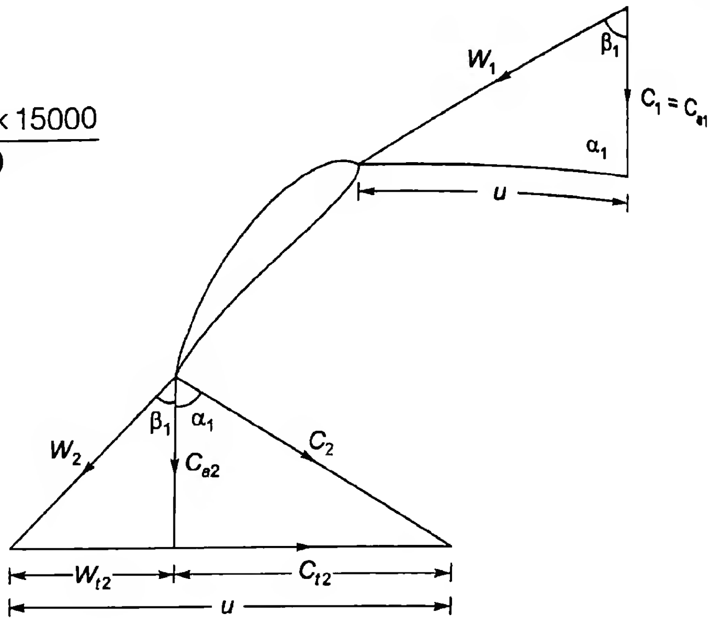
(i) Mass flow rate in kg/s
(ii) Maximum Mach number at rotor blade at entry
(iii) Angles made by relative and absolute velocity at rotor entry and exit with axial direction
(iv) Power required to drive the compressor
(v) Stage pressure ratio

[CSE (Mains) 2007 : 50 Marks]

Solution:

Given: T_o = 288 K, P_o = 1.01 bar, C_a = 150 m/s, work factor 'Ω' = 0.97, η_s = 86%, η_m = 97%

$$\text{Mean blade diameter, } d_m = \frac{d_{hub} + d_{tip}}{2} = 0.34 \text{ m}$$



$$\frac{d_{hub}}{d_{tip}} = 0.5$$

$$u = \frac{\pi d_m N}{60} = \frac{\pi \times 0.34 \times 15000}{60}$$

$$= 267 \text{ m/s}$$

$$\frac{d_h + d_t}{2} = 0.34$$

or $$d_h + d_t = 0.68$$

$$d_h = 0.5\, d_t$$

$\Rightarrow$ $$1.5\, d_t = 0.68$$

$$\text{Tip diameter, } d_t = 0.453 \text{ m}$$

$$\text{Hub diameter, } d_h = 0.2267 \text{ m}$$

$$\text{de Haller number, } \frac{W_2}{W_1} = 0.73$$

$$W_1 = \sqrt{C_1^2 + u^2} = \sqrt{(150)^2 + (267)^2} = 306.25 \text{ m/s} \quad \text{(From inlet velocity diagram)}$$

$$W_2 = 0.73 W_1 = 0.73 \times 306.25 = 223.56 \text{ m/s}$$

From exit velocity triangle,

$$\cos\beta_2 = \frac{C_{a2}}{W_2} = \frac{150}{223.56} = 0.67$$

$$\beta_2 = \cos^{-1}(0.67) = \mathbf{47.86^\circ}$$ Ans. (iii)

$$C_{t2} = u - W_{t2} = u - C_a \tan\beta_2 = 267 - 150 \tan 47.86^\circ = 101.22 \text{ m/s}$$

$$\tan\alpha_2 = \frac{C_{t2}}{C_{a2}} = \frac{101.22}{150}$$

$\Rightarrow$ $$\alpha_2 = \mathbf{34^\circ}$$ Ans. (iii)

$$\beta_1 = \tan^{-1}\left(\frac{u}{C_a}\right) = \tan^{-1}\left(\frac{267}{150}\right) = 60.67^\circ$$ Ans. (iii)

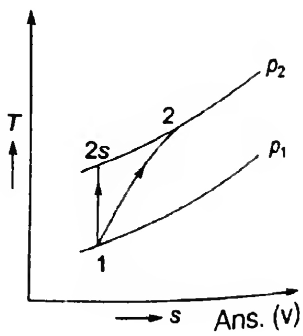
Increase in temperature per stage,

$$\Delta T_s = \frac{\Omega u C_a}{C_p}(\tan\beta_1 - \tan\beta_2) = \frac{0.97 \times 267 \times 150}{1005}(\tan 60.67^\circ - \tan 47.86^\circ)$$

$$= 26 \text{ K} = T_2 - T_1$$

$$T_1 = T_{01} - \frac{C_1^2}{2C_p} = 288 - \frac{(150)^2}{2 \times 1005} = 276.8 \text{ K}$$

$$T_2 = 276.8 + 26 = 302.8 \text{ K}$$



$$\text{Stage efficiency, } \eta_s = \frac{T_{2s} - T_1}{T_2 - T_1} = \frac{T_{2s} - T_1}{\Delta T_s}$$

$$T_{2s} = \frac{\Delta T_s}{\eta_s} + T_1 = \frac{26}{0.86 + 276.8} = 307 \text{ K}$$

$$\text{Pressure ratio, } \frac{p_2}{p_1} = \left(\frac{T_{2s}}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{307}{276.8}\right)^{\frac{1.4}{0.4}} = 1.437$$

Ans. (v)

Mass flow rate is given by, $\dot{m} = \frac{\pi}{4}\left(d_{tip}^2 - d_{hub}^2\right)C_a\rho_2$

$$\rho_2(\text{Density of air at exit}) = \frac{p_2}{RT_2} = \frac{1.01\times 1.437\times 10^5}{287\times 307} = 1.647 \text{ kg/m}^3$$

$$\dot{m} = \frac{\pi}{4}(0.453^2 - 0.2267^2)\times 150\times 1.647 = \mathbf{29.845\ kg/s} \qquad \text{Ans. (i)}$$

Mach number at rotor inlet is given by

$$M = \frac{W_1}{\sqrt{\gamma RT_1}} = \frac{306.25}{\sqrt{1.4\times 287\times 276.8}} = 0.9183 \qquad \text{Ans. (ii)}$$

Power required to drive the compressor is given by

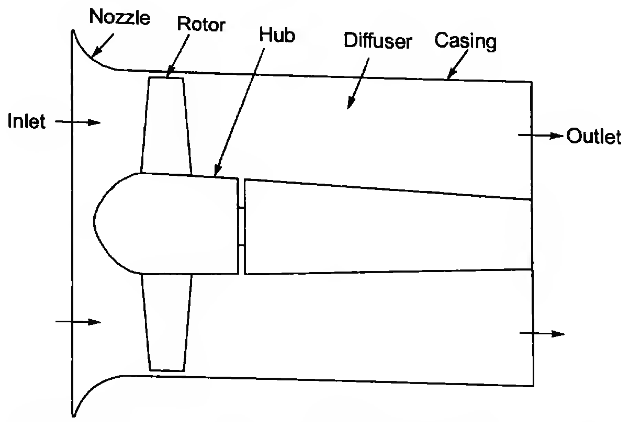
$$P = \frac{\Omega u C_a \dot{m}(\tan\beta_1 - \tan\beta_2)}{\eta_m}$$

$$= \frac{0.97\times 267\times 150\times 29.845(\tan 60.67^\circ - \tan 47.86^\circ)}{0.97\times 1000} = \mathbf{806.374\ kW}$$

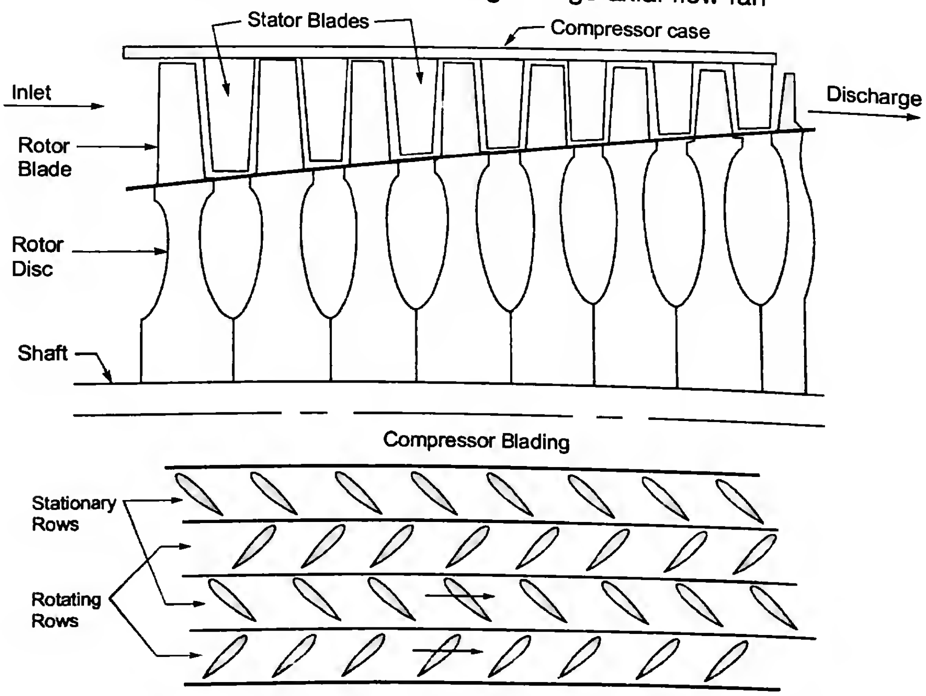
Q.51 Draw the Meridional view of a single stage axial flow fan and a multistage axial flow compressor. Describe an axial flow fan with its flow configuration.

[CSE (Mains) 2009 : 20 Marks]

Solution:



2D Meridional view of a single stage axial flow fan



2D Meridional view of a multistage axial flow compressor

Description of Axial Flow Fan.

An axial fan is a type of a compressor that increases the pressure of the air flowing through it. The blades of the axial flow fans force air to move parallel to the shaft about which the blades rotate. In other words, the flow is axially in and axially out, linearly, hence their name. The design priorities in an axial fan revolve around the design of the propeller that creates the pressure difference and hence the suction force that retains the flow across the fan. The main components that need to be studied in the designing of the propeller include the number of blades and the design of each blade. Their applications include propellers in aircraft, helicopters, hovercrafts, ships and hydrofoils. They are also used in wind tunnels and cooling towers. If the propeller is exercising propulsion, then efficiency is the only parameter of interest and other parameters like power required and flow rate are considered of no interest. In case the propeller is used as a fan, the parameters of interest includes power, flow rate, pressure rise and efficiency. An axial fan consists of much fewer blades i.e., two to six, as compared to ducted fans. Axial fans operate at high specific speed i.e., high flow rate and low head and hence adding more blades will restrict the high flow rate required for its operation. Due to fewer blades, they are unable to impose their geometry on the flow, making the rotor geometry and the inlet and outlet velocity triangles meaningless. Also the blades are made very long with varying blade sections along the radius.

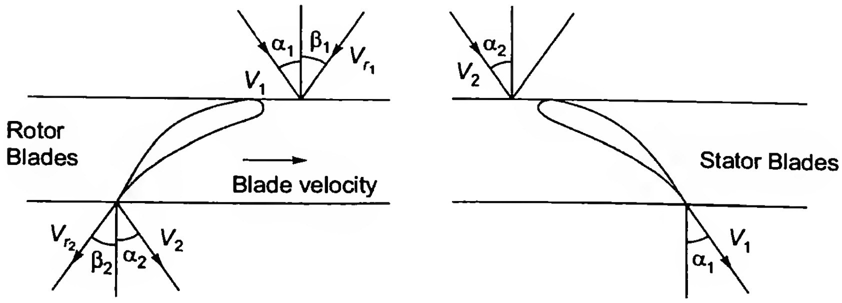
Q.52 Draw rotor and stator blades and velocity diagrams for an axial flow compressor stage and derive the expression for energy transfer in terms of blade speed U, axial velocity C_a and blade angles. Discuss the effect of variation of axial velocity along blade height on energy transfer. How the influence is taken into account?

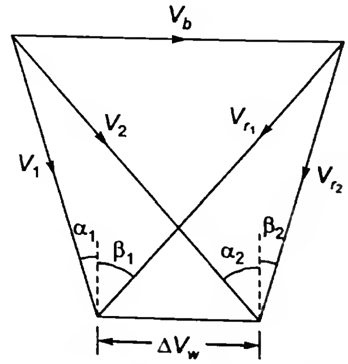
[CSE (Mains) 2011 : 15 Marks]

Solution:

In an axial flow compressor, air flows axially through the moving and fixed blades, with diffuser passages throughout which continuously increase the pressure and decrease the velocity.

Typical blades sections of an axial-flow compressor are :





Blade Velocity Diagram

Work done per unit mass or specific work input

$$W = U\left(v_{w_2} - v_{w_1}\right)$$

where, U = blade speed

v_w = whirl velocity

From blade velocity diagram, $V_{w1} = C_a \tan\alpha_1$

$V_{w2} = C_a \tan\alpha_2$

where, $V_{f1} = V_{f2} = C_a$ = Axial velocity

So, $W = UC_a[\tan\alpha_2 - \tan\alpha_1]$

Also, $(\tan\alpha_1 + \tan\beta_1 = \tan\alpha_2 + \tan\beta_2)$

$$W = UC_a[\tan\beta_1 - \tan\beta_2] \quad \ldots (i)$$

where, $(\beta_1, \beta_2 = \text{blade angles})$

As, $$\frac{U}{C_a} = \tan\alpha_1 + \tan\beta_1$$

$$\tan\beta_1 = \frac{U}{C_a} - \tan\alpha_1 \quad \ldots (II)$$

From (i) & (ii)

$$W = UC_a\left[-\tan\beta_2 + \left(\frac{U}{C_a} - \tan\alpha_1\right)\right] = U[-Ca\tan\beta_2 + U - C_a\tan\alpha_1]$$

$$W = U[U - C_a(\tan\alpha_1 + \tan\beta_2)]$$

Since the outlet angles of the stator and the rotor blades fix the value of α_1 and β_2 and hence the value of $(\tan\alpha_1 + \tan\beta_2)$. Any increase in the axial velocity along blade height (C_a) will result in decrease in energy transfer. If the compressor is designed for constant radial distribution of axial velocity (C_a), the effect of an increase in (C_a) in the central region of the annulus will be to reduce, the work capacity of blading in that area. However this reduction is somewhat compensated by an increase in energy transfer in the regions of the root and tip of the blading because of the reduction of axial velocity (C_a) at these parts of the annulus. However the net result is a loss in total work capacity because of the adverse effects of blade tip clearance and boundary layers on the annulus walls.

Q.53 Explain briefly the design and off-design characteristics of an axial flow compressor.

[CSE (Mains) 2012 : 10 Marks]

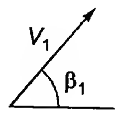
Solution:

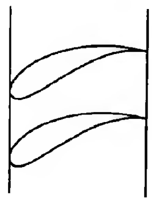
Design and off-Design characteristics of an Axial flow compressor:

In axial flow compressor, there are three parameters on which a stage performance will depend; loading coefficient (Ψ), flow coefficient (ϕ) which is (C_a by U) and the efficiency(η).

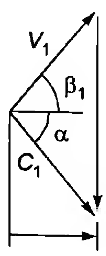
Consider velocity triangle for three different cases-one is a design condition and two off-design conditions.

Design condition : (Normal operation)





$$= \left(\frac{C_a}{U}\right)_{design}$$



The design condition which is normal operation has $\left(\frac{C_a}{U}\right) = \left(\frac{C_a}{U}\right)_{design}$.

This is the rotor blade and flow entering the rotor at a velocity V_1 which is relative velocity at an angle of (β_1).

Suppose, for a constant speed, the mass flow rate is reduced, then the ratio $\left(\frac{C_a}{U}\right)$ also reduces and this necessarily an off-design condition, when $\left(\frac{C_a}{U}\right) < \left(\frac{C_a}{U}\right)_{design}$ i.e. axial velocity decreases for same U. So, keeping U fixed, if axial velocity (Ca) reduces because mass flow rate is reduced, it leads to an increase in angle (β_1) and as (β_1) increase beyond a certain angle, it leads to positive incidence flow separation. There could be flow separation taking place on the suction surface of the rotor blade. There will be flow separation from suction surface when $\beta_1 > (\beta_1)_{design}$, which occurs when flow coefficient is less than flow coefficient design. The other part of this off-design condition is negative incident separation which will occur when

$\left(\frac{C_a}{U}\right) > \left(\frac{C_a}{U}\right)_{design}$, i.e. when $C_a > (C_a)_{design}$ for constant U, (β_1) becomes very low and as (β_1) decreases, there could be chances of flow separation from the pressure surface of the blade of the rotor blade. This is basically a negative incidence flow separation. Therefore, when flow coefficient $\left(\frac{C_a}{U}\right)$ is

Case 1 : $\left(\frac{C_a}{U}\right) = \left(\frac{C_a}{U}\right)_{design}$ (Design Condition)

Case 2 : $\left(\frac{C_a}{U}\right) < \left(\frac{C_a}{U}\right)_{design}$ (Positive incidence flow separation)

Case 3 : $\left(\frac{C_a}{U}\right) > \left(\frac{C_a}{U}\right)_{design}$ (Negative incidence flow separation)

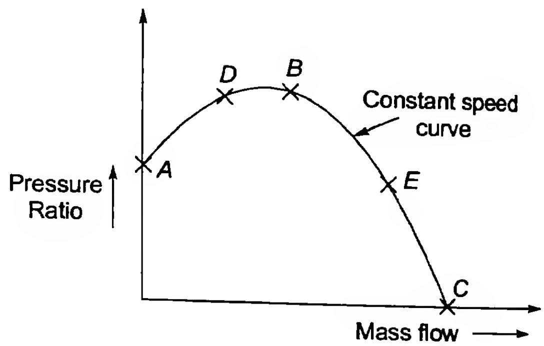
Q.54 Explain the phenomena of surge and choking in centrifugal compressors.

[CSE (Mains) 2012, 16 : 10 Marks]

Solution:

Surge and Choking in centrifugal compressors:

The performance curve of centrifugal curve is in centrifugal compressor the operating point 'A' could be obtained but a part of the curve between 'A' and 'B' could not be obtained due to surging. Surging is associated with sudden drop in delivery pressure and with violent aerodynamic pulsation transmitted through the machine. If the pressure of the air downstream of the compressor does not fall quickly enough, the air will tend to reverse its direction and will flow back in the direction of the resulting pressure gradient.



When this occurs, the pressure ratio drops rapidly causing a further drop in mass flow until the point 'A' is reached, where the mass flow is zero. Surging starts to occur in diffuser passages where flow is retarded by frictional forces near the vanes. Tendency to surge increases with number of diffuses vanes.

Choking : At some point 'E', the position is reached where no further increase in mass flow can be obtained with further opening of the control valve. The mass flow rate is maximum and is known as choking flow. It is obtained when the Mach no corresponding to relative velocity at inlet becomes unity.

Q.55 What is the pressure coefficient of a centrifugal compressor? Derive that

$\Psi_p = 1 - \phi_2 \cot \beta_2$, where ϕ_2 = flow coefficient.

[CSE (Mains) 2016 : 10 Marks]

Solution:

Let ψ_p be the pressure coefficient and

ϕ_2 = Flow coefficient

β_2 = Blade angle at outlet

$$\psi_p = \frac{H}{u_2^2 / g} = \text{Pressure coefficient}$$

where, H = Heat developed = $\dfrac{(V_{\omega_2}u_2 - V_{\omega_1}u_1)}{g}$

for centrifugal compressor,

$$\text{As } (V_{\omega 1} = 0), (V_1 = V_{f1}) \quad \text{[Axial inlet]}$$

$$H = \left(\frac{V_{\omega_2} u_2}{g}\right)$$

$$V_{\omega 2} = u_2 - V_{f2} \cot \beta_2$$

So,

$$H = \frac{(u_2 - V_{f2} \cot\beta_2) u_2}{g}$$

$$\psi_p = \frac{(u_2 - V_{f2}\cot\beta_2)u_2}{g \times (u_2^2 / g)} = \left(\frac{u_2 - V_{f2}\cot\beta_2}{u_2}\right) = 1 - \frac{V_{f2}}{u_2}\cot\beta_2 \quad \text{... (i)}$$

Let

$$\phi_2 = \text{Flow coefficient} = \frac{V_{f2}}{u_2} \quad \text{... (ii)}$$

Putting value of ϕ_2 in equation (i)

$$\psi_p = 1 - \phi_2 \cot \beta_2$$

Q.56 In a double-sided centrifugal compressor, the following data are given:

Overall diameter of impeller = 50 cm | Eye tip diameter = 30 cm
Eye root diameter = 15 cm | RPM = 15000
Total mass flow = 18 kg/s | Inlet total head temperature = 295 K
Total head isentropic efficiency = 78% | Power input factor = 1.04
Slip factor = 0.9

Assume that the velocity of air at inlet is 150 m/s and is axial, and remains constant across the eye annulus.

Find (i) the total head pressure ratio, (ii) the power required to drive the compressor and (iii) the inlet angles of the vanes at the root and tip of impeller eye. Draw the T-s diagram and velocity triangles.

[CSE (Mains) 2016 : 20 Marks]

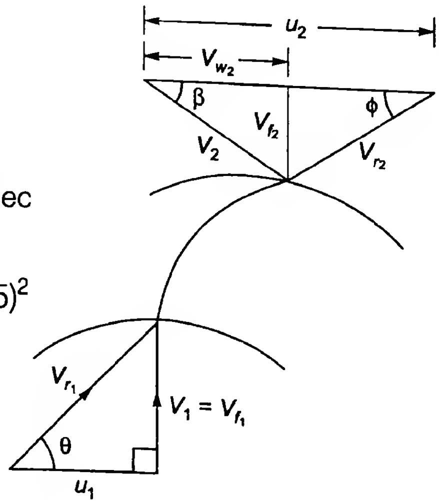
Solution:

Given: D_2 = 30 cm, D_1 = 15 cm, N = 15000 rpm, $\dot{m}$ = 18 kg/sec

T_1 = 295 k, η_{isen} = 78%, ϕ_ω = 1.04, ϕ_s = 0.9

$$V_1 = V_{f1} = V_{f2} = 150 \text{ m/sec}$$

$$\frac{T_{2s}}{T_1} = (r_p)^{\frac{\gamma-1}{\gamma}}$$



Also,

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi(0.3)15000}{60} = 235.5 \text{ m/sec}$$

Power required to drive the compressor

$$= \dot{m}\phi_s\phi_\omega u_2^2 = 18 \times 0.9 \times 1.04 \times (235.5)^2$$

(ii)

$$P = 0.94 \text{ MW}$$

Also,

$$\text{Power} = \dot{m}C_p(T_2 - T_1) = 0.94 \text{ MW}$$

$$18 \times 1.005 \times (T_2 - 295) = 0.94 \times 10^3$$

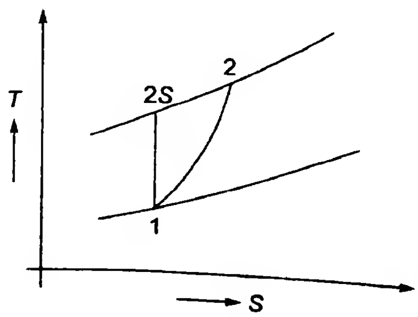
$$T_2 = 347 \text{ k}$$

Also,

$$\eta_{isen} = 0.78 = \frac{T_{2s} - T_1}{T_2 - T_1}$$

$$T_{2s} - T_1 = 0.78(347 - 295)$$

$$T_{2s} = 335.56 \text{ k}$$



Let (r_p) be the pressure ratio.

$$r_p = \left(\frac{T_{2s}}{T_1}\right)^{\gamma/\gamma-1} \left(\frac{335.56}{295}\right)^{1.4/0.4} = 1.56$$

$$u_1 = \frac{\pi D_1 N}{60} = \frac{(3.14)(0.15)(15000)}{60} = 117.75 \text{ m/sec}$$

Let θ and ϕ be the vane angles at root and tip of impeller eye respectively.

$$\tan\theta = \frac{V_1}{u_1} = \frac{150}{117.75}; \qquad \theta = 51.86°$$

Also,

$$\phi_s = \frac{V_{\omega 2}}{u_2} \Rightarrow V_{\omega 2} = \phi_s u_2 = 0.9 \times 235.5 = 211.95 \text{ m/sec}$$

$$\tan\phi = \left(\frac{V_{f_2}}{u_2 - V_{\omega_2}}\right) = \left(\frac{150}{235.5 - 211.95}\right)$$

$$\phi = 81.07°$$

5. Steam Turbines

Q.57 Show that in a 50% reaction turbine stage, the maximum stage efficiency is given by, $\frac{2\cos^2\alpha}{1+\cos^2\alpha}$ where, α is the nozzle angle.

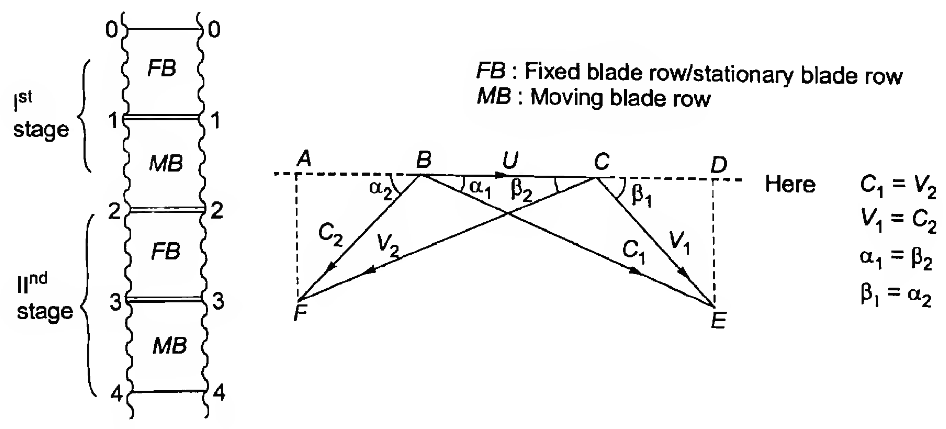
CSE (Mains) 2001 : 20 Marks]

or

Derive the expression for optimum ratio for blade velocity to steam velocity in the case of Parson's reaction steam turbine with the sketch of blade shape of a stage and velocity triangles.

[CSE (Mains) 2010 : 10 Marks]

Solution:



Combined velocity diagram for a moving blade of 50% reaction turbine.

50% degree of reaction turbine stage has symmetrical stationary and moving blades. The combined velocity diagram for a moving blade having section 1 – 1 as inlet section and section 2 – 2 as exit section is shown in figure above. 50% degree of reaction turbine has equal enthalpy drops occurring in stationary blade row and moving blade rows.

Let us assume that turbine has more than one symmetrical stage such that absolute velocity of steam leaving moving blade row equals to the velocity of steam entering fixed blade. Thus, from given velocity diagram steam enters fixed blade with velocity C_2 while leaves it with velocity C_1. In case of moving blades the relative velocities are to be considered i.e. velocity increasing from V_1 to V_2 from inlet to exit.

Rate of work done from reaction stage can be estimated similar to that of impulse stage.

$$\text{Work done} = m \cdot U \cdot \Delta C_w$$

where,

$$\Delta C_w = C_{w2} + C_{w1} = (C_1 \cos \alpha_1 + C_2 \cos \alpha_2)$$

For symmetrical blading, $C_2 = V_1,\ \alpha_2 = \beta_1,$

so,

$$\Delta C_w = C_1 \cos \alpha_1 + V_1 \cos \beta_1$$

or,

$$\Delta C_w = C_1 \cos \alpha_1 + (C_1 \cos \alpha_1 - U)$$

$$\Delta C_w = 2C_1, \cos \alpha_1 - U$$

Therefore, work done, $W = m \cdot U \cdot (2C_1 \cos \alpha - U)$

Diagram efficiency of reaction stage can be estimated by knowing the energy input to the moving blades and taking ratio of work done to energy input.

Energy input to moving blades, $E_{in} = m \cdot \dfrac{C_1^2}{2} + m \cdot \left(\dfrac{V_2^2 - V_1^2}{2}\right)$

$$E_{in} = \frac{m}{2} \cdot (C_1^2 + V_2^2 - V_1^2) = \frac{m}{2}(C_1^2 + C_1^2 - V_1^2), \text{ as } C_1 = V_2 = m\left(C_1^2 - \frac{V_1^2}{2}\right)$$

From velocity diagram, $V_1^2 = C_1^2 + U^2 - 2C_1 U \cos \alpha_1$

Diagram efficiency is also known as stage efficiency.

Substituting value of V_1 in energy input expression.

$$E_{in} = m \cdot \left\{C_1^2 - \frac{(C_1^2 + U^2 - 2C_1 U \cos \alpha)}{2}\right\} = m \cdot \left\{\frac{C_1^2 + 2UC_1 \cos \alpha - U^2}{2}\right\}$$

Thus, Diagram efficiency, $\eta_d = \dfrac{W}{E_{in}} = \dfrac{m \cdot U \cdot (2C_1 \cos \alpha - U)}{m \cdot \dfrac{\{C_1^2 + 2UC_1 \cos \alpha - U^2\}}{2}} = \dfrac{2U \cdot (2C_1 \cos \alpha - U)}{(C_1^2 + 2UC_1 \cos \alpha - U^2)}$

Substituting $(\rho) = \dfrac{U}{C_1}, \eta_d = \dfrac{2\rho \cdot (2 \cos \alpha - \rho)}{(1 + 2\rho \cos \alpha - \rho^2)}$

The maximum value of diagram efficiency and the optimum value of blade speed to steam velocity ratio, ρ can be estimated by differentiating with respect to ρ and equating to zero.

$$\frac{d\eta_d}{d\rho} = 0$$

or,

$$2 \cos \alpha - 2\rho = 0$$

or,

$$\rho = \cos \alpha$$

Substituting 'ρ' for getting maximum value of diagram efficiency, $\eta_{d,\max}$.

$$\eta_{d,\max} = \frac{2 \cos \alpha \cdot (2 \cos \alpha - \cos \alpha)}{(1 + 2\cos^2 \alpha - \cos^2 \alpha)} = \frac{2\cos^2 \alpha}{1 + \cos^2 \alpha} = \frac{2\cos^2 \alpha_1}{1 + \cos^2 \alpha_1}$$

Q.58 The velocity of steam entering a simple impulse turbine is 1000 m/s and the nozzle angle is 20°. The mean peripheral velocity of blades is 400 m/s. The blades are asymmetrical. If the steam is to enter the blades without shock, that will be the blade angles?

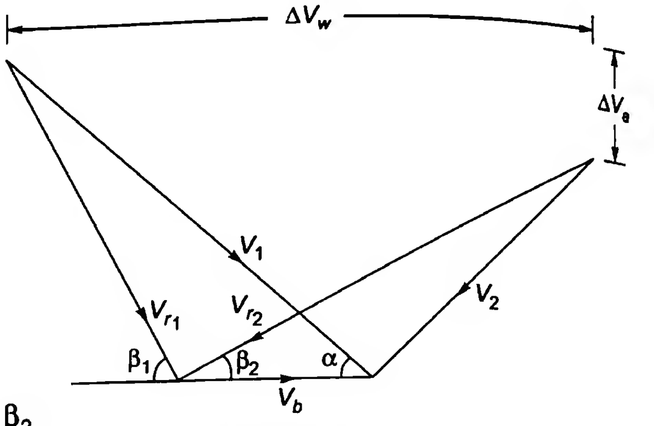
Neglecting the friction effects on the blades, calculate the tangential force on the blades and the diagram power for a mass flow of 0.75 kg/s. Calculate the axial thrust and diagram efficiency.

[CSE (Mains) 2001 : 30 Marks]

Solution:

Given: $V_1 = 1000$ m/s, $V_b = 400$ m/s, $\alpha = 20°$, $\beta_1 = \beta_2$ (for asymmetric blade), $\dot{m}_s = 0.75$ kg/s, $k_b = 0$

$$V_{r1}\sin\beta_1 = V_1\sin\alpha$$
$$V_{r2}\cos\beta_1 = V_1\cos\alpha - V_b$$



Velocity diagram

$$\beta_1 = \tan^{-1}\left[\frac{V_1\sin\alpha}{V_1\cos\alpha - V_b}\right]$$

$$\left[\text{As } V_{r2} = V_1\cos\alpha - V_b\right]$$

$$= \tan^{-1}\left[\frac{1000\times\sin 20°}{1000\cos 20° - 400}\right]$$

$$= \tan^{-1}\left[\frac{342}{940-400}\right] = 32.35° = \beta_2$$

Blade angles, $\beta_1 = \beta_2 = \mathbf{32.35°}$

$$V_{r1}\sin 32.35° = 342$$

$$V_{r1} = \tan^{-1}\frac{342}{\sin 32.35°} = 639.25 \text{ m/s} = V_{r2}$$

$$\Delta V_w = V_{r1}\cos\beta_1 + V_{r2}\cos\beta_2 = 2V_{r1}\cos\beta_1 = 2\times 639.25\times\cos 32.35° = 1080.07 \text{ m/s}$$

$$\Delta V_a = V_{r1}\sin\beta_1 + V_{r2}\sin\beta_2 = 0$$

Tangential thrust, $= \dot{m}_s\Delta V_\omega = 0.75\times 1080.07 = \mathbf{810.05\ N}$

Diagram power, W_D = Tangential thrust × Blade velocity = 810.05 × 400 = **324.02 kW**

$$\text{Diagram efficiency, } \eta_D = \frac{324.02}{\frac{1}{2}\times 0.75\times(1000)^2\times 10^{-3}} = \mathbf{0.864 \text{ or } 86.4\%}$$

Axial thrust $= \dot{m}_s\Delta V_a = 0$

Q.59 Steam expands in a steam turbine isentropically from inlet to exhaust having an enthalpy drop = 1200 kJ/kg. Assuming ideal conditions, determine the mean diameter of the wheel if the turbine were of:

(i) single impulse stage,
(ii) single 50% reaction stage,
(iii) one two-row Curtis stage and
(iv) ten 50% reaction stages.

Take the nozzle angle as 18° and blade speed as 4000 rpm.

[CSE (Mains) 2002 : 30 Marks]

Solution:

Given: Total enthalpy drop, $\Delta h = 1200$ kJ/kg, Nozzle angle, $\alpha = 18°$, Blade speed, $N = 4000$ rpm

(i) Optimum speed ratio for simple impulse stage,

$$\rho = \frac{\cos\alpha}{2}; \qquad u = \frac{V_1\cos\alpha}{2}$$

Absolute velocity is given by,
$$\frac{V_1^2}{2} = \Delta h \Rightarrow V_1 = (2\times\Delta h)^{1/2}$$

$$\Rightarrow \quad V_1 = (2\times 1200\times 10^3)^{1/2} = 1549.2 \text{ m/s}$$

$$\Rightarrow \quad u = \frac{\pi DN}{60} = \frac{V_1\cos\alpha}{2} = \frac{1549.2\times\cos 18°}{2}$$

$$D = 3.517 \text{ m}$$

(ii) Optimum speed ratio for single 50% reaction stage,

$$\rho = \cos\alpha \Rightarrow u = V_1 \cos\alpha$$

or

$$u = 1549.2 \cos 18° \text{ m/s}$$

$$\frac{\pi DN}{60} = 1473.376$$

$$D = \frac{60 \times 1473.376}{\pi \times 4000} = 7.034 \text{ m}$$

(iii) Optimum speed ratio for two-rwo curtis stage,

$$\rho = \frac{\cos\alpha}{4} \text{ or } u = \frac{V_1 \cos\alpha}{4}$$

$$u = 368.344 \text{ m/s}$$

$$\frac{\pi DN}{60} = 368.344$$

$$D = \frac{60 \times 368.344}{\pi \times 4000} \approx 1.76 \text{ m}$$

(iv) Total enthalpy drop for 10 stages = 1200 kJ/kg

$$\text{Enthalpy drop for one stage} = \frac{1200}{10} = 120 \text{ kJ/kg}$$

$$\text{Absolute velocity, } V_1 = 44.72\,(120)^{1/2} = 4.90 \text{ m/s}$$

Optimum speed ratio for 50% reaction turbine,

$$\frac{u}{V_1} = \cos\alpha$$

$$u = 4.90 \times \cos 18° = 465.92 \text{ m/s}$$

$$\frac{\pi DN}{60} = 465.92$$

$$D = \frac{60 \times 465.92}{\pi \times 4000} = 2.224 \text{ m/s}$$

Q.60 A salesman reports that he has a steam turbine that delivers 3 MW. The steam enters the turbine at 6.0 bar, 260°C and leaves the turbine at 0.15 bar and the required rate of steam flow is 12000 kg/hr.

(i) Find the maximum power output of turbine to justify his claim.

(ii) Also, verify his claim if the steam flow rate is 20,000 kg/hr.

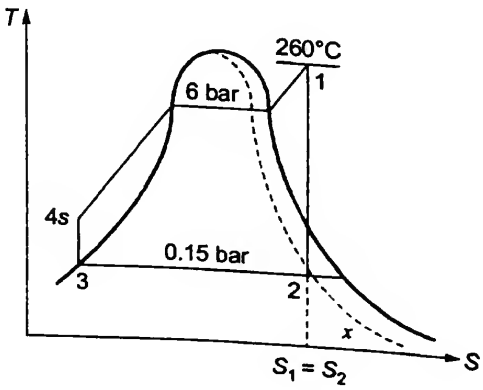
[CSE (Mains) 2003 : 20 Marks]

Solution:

Given: Steam flow rate $\dot{m} = 12000$ kg/hr

Superheated steam property at 6 bar & 260°C:

Saturated water property at 0.15 bar:



$$\text{Specific enthalpy, } h_1 = 2978.5 \text{ kJ/kg}$$

$$\text{Specific entropy, } s_1 = \text{ kJ/kgK}$$

$$h_{f2} = 226 \text{ kJ/kg}$$

$$h_{fg2} = 2373.2 \text{ kJ/kg}$$

$$h_{g2} = 2599.2 \text{ kJ/kg}$$

$$S_{f2} = 0.7549 \text{ kJ/kg}$$

$$S_{fg2} = 7.2544 \text{ kJ/kg}$$

$$S_{g2} = 8.0093 \text{ kJ/kg}$$

Assuming ideal condition, $S_1 = S_2$

$$7.223 = S_{f2} + xS_{fg2}$$

$$x = \frac{7.223 - 0.7549}{7.2544} = 0.8916$$

Enthalpy at turbine outlet, $h_2 = S_{f2} + xS_{fg2} = 226 + 0.8916 \times 2373.2 = 2341.97$ kJ/kg

(i) Maximum power output, $W = (h_1 - h_2)\dot{m} = (2978.5 - 2341.97) \times \frac{12000}{3600}$

$$= \mathbf{2121.76 \text{ kW or } 2.12 \text{ MW}}$$

Salesman claim is WRONG.

(ii) New output for steam flow rate of 20000 kg/hr,

$$W = (2978.5 - 2341.97) \times \frac{12000}{3600} = 3536.28 \text{ kw or } \mathbf{3.54 \text{ MW}}$$

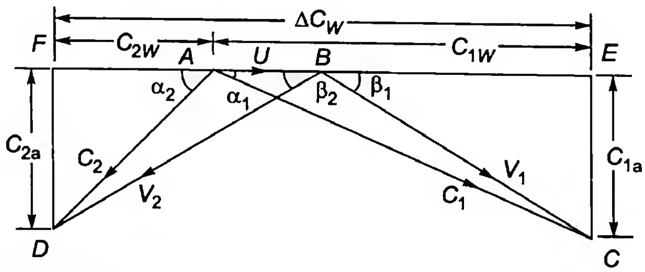
So, salesman claim is correct when steam flow rate is 20000 kg/hr.

Q.61 Derive an expression for the blade efficiency of a single row impulse steam turbine assuming equiangular blades in terms of nozzle angle α_1, blade velocity coefficient K and blade speed ratio. If $\alpha_1 = 20°$ and $K = 0.9$, what is the maximum blade efficiency and corresponding blade angles?
If the blade efficiency is 85% of the maximum value, what are the possible blade speed ratios for the same nozzle angle α_1, blade velocity coefficient, K and equiangular blades? Also, find the corresponding blade angles.

[CSE (Mains) 2003 : 30 Marks]

Solution:

Theory Part



Combined inlet and outlet velocity diagrams

ρ = Ratio of linear velocity of blade and absolute velocity at inlet of moving blade = $\frac{U}{C_1}$

K = Blade velocity coefficient (Ratio of relative velocity at exist and inlet) = $\frac{V_2}{V_1}$

Let the mas flow rate be m kg/s

$$\text{Tangential force } F_T = m \times (\text{change of tangential component of velocity or whirl velocity})$$

$$= m \times (-C_2 \cos\alpha_2 - C_1 \cos\alpha_1)$$

$$F_T = -m\,(C_2 \cos\alpha_1 + C_1 \cos\alpha_1) = -m \cdot \Delta C_w$$

Driving thrust on rotor will be reaction of this force and will be equal and opposite

$$\text{Driving thurst, } F_D = -F_T$$

$$F_D = m(C_2 \cos\alpha_2 + C_1 \cos\alpha_1) = m \cdot \Delta C_w$$

From velocity triangles,

$$(C_1 \cos\alpha_1 + C_2 \cos\alpha_2) = (V_1 \cos\beta_1 + V_2 \cos\beta_2)$$

or, $$\Delta C_w = \Delta V_w$$

Hence, $$F_D = m(C_2 \cos\alpha_2 + C_1 \cos\alpha_1) = m \cdot (V \cos\beta_1 + V_2 \cos\beta_2)$$

This driving thrust can be used for getting the rate of work done on the rotor.

$$\text{Rate of work done} = W = F_D \times U$$

$$W = m \cdot \Delta C_w \cdot U$$

Work done per unit of steam mass flow, $w = U \cdot \Delta C_w$

Rate of work done will be the power produced by the turbine stage.

$$W = m \cdot U \cdot (C_2 \cos\alpha_2 + C_1 \cos\alpha_1)$$

$$W = m \cdot U \cdot (V_1 \cos\beta_1 + V_2 \cos\beta_2) = m \cdot U \cdot V_1 \cos\beta_1 \left(1 + \frac{V_2 \cos\beta_2}{V_1 \cos\beta_1}\right)$$

$$= m \cdot U \cdot (C_1 \cos\alpha_1 - U)\left\{1 + \left(\frac{V_2}{V_1}\right) \cdot \left(\frac{\cos\beta_2}{\cos\beta_1}\right)\right\}$$

$$W = m \cdot U \cdot (C_1 \cos\alpha_1 - U) \,.\, \{1 + K \cdot C\}$$

where, $$K = \text{Blade velocity coefficient} = \frac{V_2}{V_1}$$

$$C = \text{Ratio of cosines of blade angles} = \frac{\cos\beta_2}{\cos\beta_1}$$

For perfectly smooth and symmetrical blade both K and C shall have unity value.

i.e. $K = 1$, $C = 1$

$$\text{Diagram efficiency, } \eta_D = \frac{\text{Rate of work done}}{\text{Energy supplied to rotor}}$$

$$\eta_D = \frac{m \cdot U \cdot \Delta C_w}{m \cdot \frac{C_1^2}{2}} = \frac{2U \cdot \Delta C_w}{C_1^2}$$

$$\text{Diagram or blading efficiency} = \frac{2 \cdot U \cdot \Delta C_w}{C_1^2}$$

$$\eta_D = \frac{2U \cdot (C_1 \cos\alpha_1 - U)(1 + KC)}{C_1^2} = 2\left(\frac{U}{C_1}\right)\left(\cos\alpha_1 - \frac{U}{C_1}\right)(1 + KC)$$

Here $\frac{U}{C_1}$, is non-dimensional form of velocity. Let us denoted it by ρ

i.e. $$\rho = \frac{U}{C_1}, \text{ so, } \eta_D = 2\rho\,(\cos\alpha_1 - \rho)\,(1 + KC)$$

So, $$\eta_D = 2\,\rho(\cos\alpha_1 - \rho)(1 + KC)$$

Diagram efficiency can be optimized with respect to non-dimensional velocity denoted by ρ. This non-dimensional velocity ρ is also called as 'blade-steam velocity ratio' or 'blade speed-steam velocity ratio' or 'blade-steam speed ratio'.

Differentiating η_D with respect to ρ, we get,

$$\frac{d\eta_D}{d\rho} = 2\,(\cos\alpha_1 - 2\rho)\,(1 + KC)$$

For a perfectly smooth and symmetrical blade, $K = 1$, $C = 1$

$$\frac{d\eta_D}{d\rho} = 4\,(\cos\alpha_1 - 2\rho)$$

Equating first differential to zero;

$$\cos\alpha_1 - 2\rho = 0$$

$$\rho = \frac{\cos\alpha_1}{2}$$

Second order differential of η_D with respect to ρ indicates that the diagram efficiency is maximum corresponding to the blade speed-steam velocity ratio given as $\frac{\cos\alpha_1}{2}$.

Hence, maximum diagram efficiency,

$$\eta_{D,\max} = \frac{\cos^2\alpha_1 \cdot (1+KC)}{2} \qquad \left[\text{Putting } \rho = \frac{\cos\alpha_1}{2} \text{ in equation (i)}\right]$$

Numerical Part

As given,

$$\alpha_1 = 20°,$$
$$k = 0.9$$

$$\eta_{D,\max} = \frac{(\cos 20°)(1+0.9\times 1)}{2} \qquad \left[C = \frac{\cos\beta_2}{\cos\beta_1}\right]$$

$$= \mathbf{0.8388 \text{ or } 83.88\%}$$

Blade efficiency, $\eta_b = 0.85 \times \eta_{D,\max} = 0.85 \times 0.8388 = 0.713$

$$\eta_b = 2\left(\frac{U}{C_1}\right)\left[\cos\alpha_1 - \frac{U}{C_1}\right](1+kC)$$

$$= 2\rho\,(\cos\alpha_1 - \rho)\,(1 + kC) = 2\rho\,(\cos 20° - \rho)\,(1 + 0.9)$$

$$0.713 = 3.8(.94 - \rho)$$

$$0.1876 = \rho(0.94 - \rho)$$

$$\rho^2 - 0.94\rho + 0.1876 = 0$$

$$\rho = \frac{0.94 \pm \sqrt{(0.94)^2 - 4\times 0.1876}}{2} = \frac{0.94 \pm \sqrt{0.1332}}{2}$$

$$= \frac{0.94 \pm 0.365}{2} = \mathbf{0.6525 \text{ or } 0.2875}$$

NOTE: Blade angles can be found out by drawing velocity diagrams to the scale.

Q.62 Explain what do you understand by specific speed of a turbine. Highlight its importance.
A centrifugal compressor develops a pressure ratio of 1.5 while running at 24,000 RPM and discharging 2.0 kg/s of air. The entry conditions are P_1 = 1.0 bar and T_1 = 290 K. Determine the specific speed. Take γ = 1.4, R = 287 J/kg K, C_p = 1.005 kJ/kg K.

[CSE (Mains) 2004 : 20 Marks]

Solution:

Theory Part: The specific speed of a turbine is defined as the speed of operation of a geometrically similar model of the turbine which is so proportional that it produces 1 kW power when operating under 1 m head. It is given by

$$N_s = \frac{N\sqrt{P}}{h^{5/4}}$$

where, P = Power (kW), N = Wheel speed (rpm), H = Heat on turbine (m)

It is to be noted that specific speed is not a non-dimensional number but its serves its purpose.

Importance: Specific speed plays an important role in the selection of turbine. It is always better to choose turbines of high specific speed as it means compact size of turbine, generator, power house etc. The basic concept of the turbine specific speed is to identify the optimum operating conditions for a given turbine design.

Numerical Part:

Given: $r_p = 1.5$, $N = 24000$ rpm, $\dot{m} = 2$ kg/s , $P_1 = 1$ bar , $T_1 = 290$ K

$$\text{Air density at entry, } \rho_1 = \frac{p_1}{RT_1} = \frac{1\times10^5}{287\times290} = 1.2 \text{ kg/m}^3$$

$$\text{Flow rate at entry, } Q = \frac{\dot{m}}{\rho_1} = \frac{2}{1.2} = 1.667 \text{ m}^3\text{/s}$$

$$\text{Rotational, } \omega = \frac{2\pi N}{60} = \frac{2\pi\times24000}{60} = 2513.27 \text{ rad/s}$$

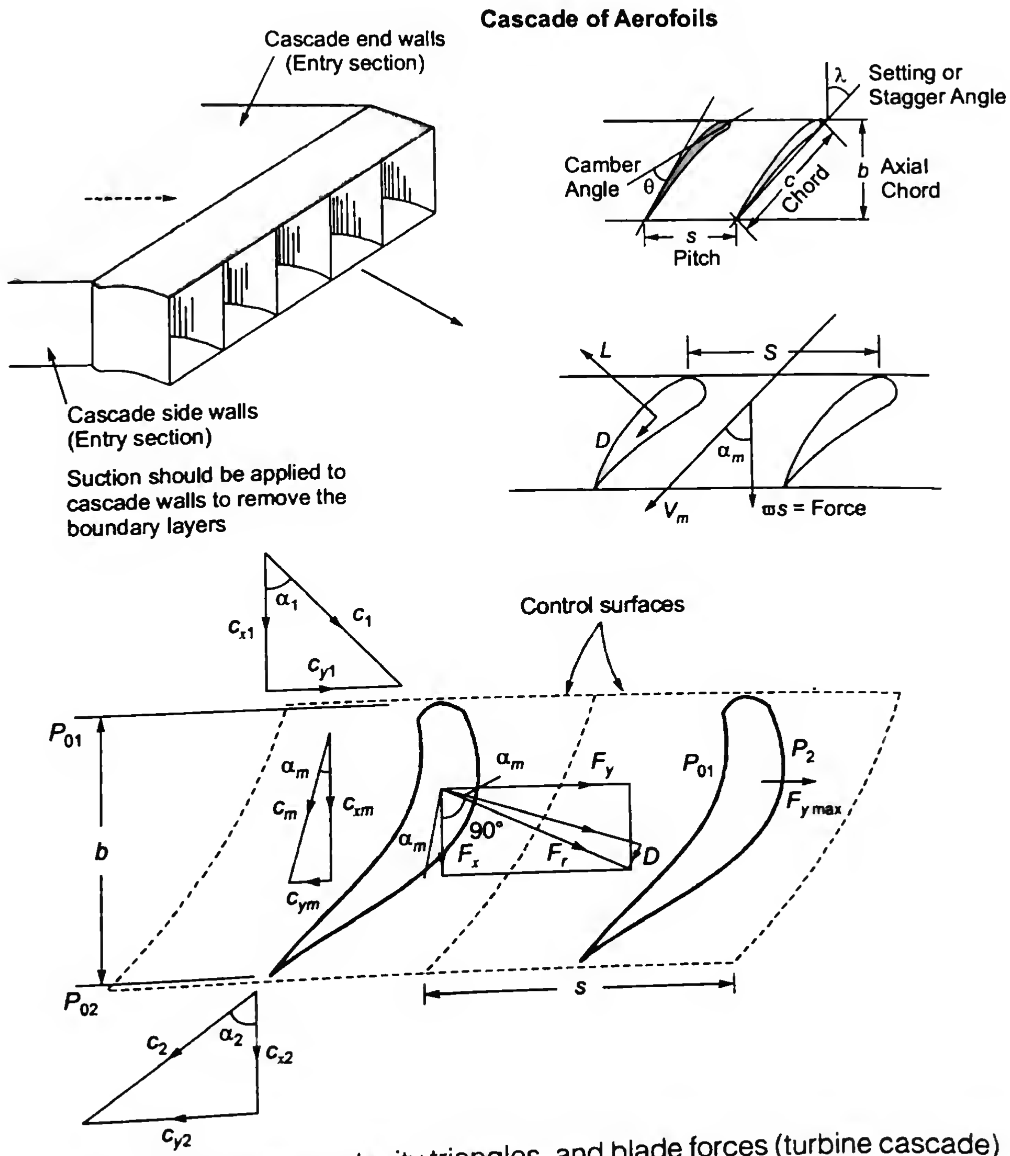
$$\Delta h_{os} \approx C_p(T_2 - T_1) = C_pT_1\left\{\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right\} = 1005\times290\,(1.5^{0.286} - 1) = 35835 \text{ J/kg}$$

$$\text{Specific speed, } N_s = \frac{\omega\sqrt{Q}}{(gH)^{3/4}} = \frac{2513.27\sqrt{1.667}}{(35835)^{0.75}} = \mathbf{1.246}$$

Q.63 Sketch the axial-flow turbine cascade. Show the velocity with its components at entry exit indicating the forces exerted by the flow on blades. State the expression of ideal lift in terms of flow angles and show that it is equal to $pC_m\Gamma$, where C_m is the mean velocity and Γ is the circulation.

[CSE (Mains) 2007 : 10 Marks]

Solution:



Inlet, outlet and mean velocity triangles, and blade forces (turbine cascade)

A mean velocity triangle for the flow through the cascade is defined by the following quantities:

$$C_{xm} = \frac{1}{2}(C_{x1} + C_{x2})$$

$$C_{ym} = \frac{1}{2}(C_{y2} - C_{y1})$$

$$\tan\alpha_m = \frac{C_{ym}}{C_{xm}}$$

For constant axial velocity, $\tan\alpha_m = \frac{1}{2}(\tan\alpha_2 - \tan\alpha_1)$

Expression for ideal lift can be written as follows:

$$L = F_y \cos\alpha_m + F_x \sin\alpha_m$$

$$F_y = \rho s C_{xm}^2(\tan\alpha_1 + \tan\alpha_2) = \frac{1}{2}\rho s(C_2^2 - C_1^2)$$

From the velocity triangle at the entry and exit,

$$C_2^2 - C_1^2 = C_{xz}^2 + C_{yz}^2 - C_{x1}^2 - C_{y1}^2 = C_{yz}^2 - C_{y1}^2 \qquad \text{(Assuming } C_{x1} = C_{xz}\text{)}$$

$$F_x = \frac{1}{2}\rho s(C_{yz}^2 - C_{y1}^2) = \frac{1}{2}\rho s C_{xm}^2(\tan^2\alpha_2 - \tan^2\alpha_1)$$

$$= \rho s C_{xm}^2 \frac{1}{2}(\tan\alpha_2 - \tan\alpha_1)(\tan\alpha_2 + \tan\alpha_1)$$

But, $\frac{1}{2}(\tan\alpha_2 - \tan\alpha_1) = \tan\alpha_m,$

Therefore, $F_x = \rho s C_{xm}^2 \tan\alpha_m(\tan\alpha_2 + \tan\alpha_1)$

Lift force can be expressed as,

$$L = F_y \cos\alpha_m + F_x \sin\alpha_m$$

$$= \rho s C_{xm}^2(\tan\alpha_1 + \tan\alpha_2)\cos\alpha_m + \rho s C_{xm}^2 \tan\alpha_m(\tan\alpha_2 + \tan\alpha_1)\sin\alpha_m$$

$$L = \rho s C_{xm}^2(\tan\alpha_1 + \tan\alpha_2)\left[\cos\alpha_m + \frac{\sin^2\alpha_m}{\cos\alpha_m}\right]$$

$$= \rho s C_{xm}^2(\tan\alpha_1 + \tan\alpha_2)\sec\alpha_m$$

$$= \rho s C_{xm}\sec\alpha_m(C_{x1}\tan\alpha_1 + C_{x2}\tan\alpha_2) = \rho s C_m(C_{y1} + C_{y2})$$

$$L = \rho C_m(C_{y1}s + C_{y2}s)$$

The circulation around the blade contained in the control surface is the line integral of the velocity around the closed circuit. This is given by

$$\Gamma = C_{y1}s + C_{y2}s$$

The line integral along the two curved branches of the circuit cancel each other.
So, lift can also be expressed as,

$$L = \rho C_m \Gamma \qquad \text{[Kutta - Joukowshi's relation]}$$

Q.64 A quarter-scale turbine model is tested under a head of 10.8 m. The full-scale turbine is required to work under a head of 30 m and to run at 7.14 rev/s. At what speed the model must run? If it (model) develops 100 kW and uses 1.085 m^3 of water per second at this speed, what power will be obtained from the full-scale turbine, its efficiency being 3% better than that of model? What is the dimensionless specific speed of full-scale turbine?

[CSE (Mains) 2007 : 20 Marks]

Solution:

Equating power coefficients (π term containing the power P) for the model and prototype, we can write

$$\frac{P_1}{\rho_1 N_1^3 D_1^5} = \frac{P_2}{\rho_2 N_2^3 D_2^5}$$

(Subscript 1 refers to the prototype 2 refers to the model)

Equating the head coefficient,

$$\frac{H_1}{(N_1 D_1)^2} = \frac{H_2}{(N_2 D_2)^2}$$

$$\frac{D_2}{D_1} = \left(\frac{H_2}{H_1}\right)^{1/2}\left(\frac{N_1}{N_2}\right)$$

$$N_2 = 4\times\left(\frac{10.8}{30}\right)^{1/2}\times 7.14 = \textbf{17.136 rps (rev/s)}$$

For the same fluid,

$$\rho_1 = \rho_2$$

$$P_1 = 100\times\left(\frac{7.14}{17.14}\right)^3 (4)^5 = 7407.4 \text{ kW or } \textbf{7.41 MW}$$

Alternatively power can be calculated as follows:

$$\text{Model efficiency} = \frac{\text{Power output}}{\text{Water power input}} = \frac{P_2}{\rho g Q H}\frac{100}{9.81\times 1.085\times 10.8}$$

$$\eta_2 = 0.87 \text{ or } 87\%$$

Prototype or full scale turbine, $\eta_1 = \eta_2 + 3 = 87 + 3 = 90\%$ [As given in qth]

$$\text{Full scale turbine efficiency} = \frac{P_1}{gQ_1H_1}$$

We know that, $\frac{Q_1}{N_1 D_1^3} = \frac{Q_2}{N_2 D_2^3}$

$$Q_1 = Q_2\left(\frac{N_1}{N_2}\right)\left(\frac{D_1}{D_2}\right)^3 = 1.085\times\frac{7.14}{17.14}\times 4^3 = 28.926 \text{ m}^3/\text{s}$$

Power output (turbine) = 0.9 × 9.81 × 28.926 × 30 = 7661.78 kW or **7.66 MW**

Dimensionless specific speed of a turbine is given by

$$K_{ST} = \frac{N\sqrt{P}}{\rho^{1/2}(gH)^{5/4}} = \frac{7.14(7.66\times 10^6)^{1/2}}{(1000)^{1/2}(9.81\times 30)^{5/4}} = 0.51265 \text{ rev or } \textbf{3.221 rad}$$

Q.65 A reaction turbine having identical blading delivers dry saturated steam at 3 bar. The velocity of steam is 100 m/sec. The mean blade height is 40 mm and the exit angle of the moving blade is 20°. At the mean radius, the axial flow velocity equals 3/4th of blade speed. For a steam flow rate of 10,000 kg/hr, determine:

(a) the rotor speed in rev/min,

(b) the power output of stage,

(c) the diagram efficiency,

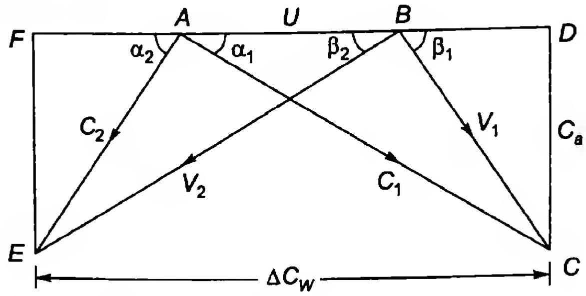
(d) the percentage increase in relative velocity in the moving blade due to expansion in these blades and

(e) the enthalpy drop of steam in the stage.

Specific volume of dry saturated steam at 3 bar is 0.6055 m³/kg.

[CSE (Mains) 2008 : 35 Marks]

Solution:



For 50% reaction turbine i.e. for identical bladings:

$$\alpha_1 = \beta_2,\ \alpha_2 = \beta_2,\ V_2 = C_1,\ V_1 = C_2,\ \alpha_1 = \beta_2 = 20°,$$

$$C_a = \frac{3}{4}U,\ C_1 = 100 \text{ m/s},\ \dot{m} = 10000 \text{ kg/hr}$$

$$C_{w1} = C_1 \cos\alpha_1 = 100 \cos 20° = 93.97 \text{ m/s}$$

From $\Delta ACD \times \sin\alpha_1 = \dfrac{C_a}{C_1}$

$$C_a = 100 \sin 20° = 34.2 \text{ m/s}$$

$$U = \frac{4C_a}{3} = \frac{4\times 34.2}{3} = 45.6 \text{ m/s}$$

Applying cosine rule to the ΔABC:

$$V_1^2 = C_1^2 + U^2 - 2C_1 U\cos\alpha_1$$
$$= (100)^2 + (45.6)^2 - 2\times 100\times 45.6 \cos 20° = 3509.363$$
$$V_1 = 59.24 \text{ m/s}$$

Now,
$$BD = C_{w1} - U = V_1\cos\beta_1$$

$$\cos\beta_1 = \frac{C_{w1}-U}{V_1} = \frac{93.97-45.6}{59.24} = 0.8165$$

$$\beta_1 = 35.26°$$

$$U + C_{w2} = V_2\cos\beta_2 = C_1\cos\alpha_1$$

$$C_{w2} = C_{w1} - U = 93.97 - 45.6 = 48.37 \text{ m/s}$$

Change in velocity of whirl, $\Delta C_w = 93.97 + 48.37 = 142.34$ m/s

Applying the continuity equation,

$$\pi Dl \times \text{velocity of flow} = \text{mass rate of flow} \times \text{specific volume}$$

$$D = \frac{\dot{m}\times V_s}{\pi l\times C_a} = \frac{10000}{3600}\times 0.6055\times\frac{1}{\pi}\times\frac{1}{0.04}\times\frac{1}{34.2} = 0.39136 \text{ m}$$

$$\text{Blade speed, } U = \frac{\pi DN}{60}$$

$$N = \frac{60V}{\pi D} = \frac{60\times 45.6}{\pi\times 0.39136} = \mathbf{2225.31 \text{ rpm}}$$

$$\text{Power output} = \frac{\dot{m}U\Delta C_w}{1000}\text{kW} = \frac{10000}{3600}\times\frac{45.6\times 142.34}{1000} = \mathbf{18.03 \text{ kW}}$$

$$\text{Diagram efficiency, } \eta_d = 2 - \frac{2}{1+2\rho\cos\alpha-\rho^2}$$

$$\text{where, speed ratio, } \rho = \frac{U}{C_1} = \frac{45.6}{100} = 0.456$$

$$\eta_d = 2 - \frac{2}{1+2\times 0.456\cos 20° - (0.456)^2}$$

$$\eta_d = 2 - \frac{2}{1.649} = \mathbf{0.7872 \text{ or } 78.72\%}$$

Relative velocity at the rotor outlet is,

$$V_2 = \frac{C_{a2}}{\sin\beta_2} = \frac{34.2}{\sin 20°} = 99.99 \text{ m/s}$$

Percentage increase in relative velocity in the moving blade is given by,

$$\left(\frac{V_2}{V_1} - 1\right) \times 100 = \left(\frac{99.99}{59.24} - 1\right) \times 100 = \mathbf{68.8\%}$$

Since the reaction is 50%, then

$$(\Delta h_{isen})_n = (\Delta h_{isen})_b = \frac{V_2^2 - V_1^2}{2} = \frac{99.99^2 - 59.24^2}{2000} = 3.24 \text{ kJ/kg}$$

Enthalpy drop of steam in the stage = 3.24 × 2 = **6.28 kJ/kg**

Q.66 Draw a schematic of a pass-out turbine and explain its working, Represent the relevant process on Enthalpy - Entropy coordinates.

[CSE (Mains) 2009 : 20 Marks]

Solution:

Pass out or extraction turbine: Pass out turbine refers to the steam turbine having provision for extraction of steam during expansion. Such provision is required because in combined heat and power requirement the steam available from back pressure turbine may be more than required one or the power produced may be less than the required value. Pass out turbine has arrangement for continuous extraction of a part of steam at the desired pressure for process heating and left out steam goes into low pressure section of turbine through a pressure control valve. In the low pressure section of turbine control mechanism is provided so that the speed of turbine and pressure of steam extracted remains constant irrespective of the variations in power produced and process heating.

The pass out turbines have to operate under widely varying load so its efficiency is quite poor. For facilitating the operation of pass out turbine from no extraction to full steam extraction conditions, nozzle control governing or throttle control governing are used.

In many cases, the power available from the back pressure turbine through which the whole of the heating steam flows is appreciably less than that required in a factory. This may be due to relatively high back pressure, or small heating requirement or both. Pass-out turbines are employed in those cases, where a certain quantity of steam is continuously extracted from the turbine at an intermediate stage for heating purposes at the desired temperature and pressure.

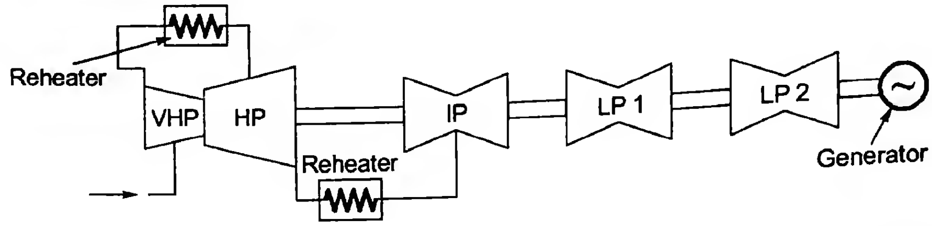
Q.67 Give a cylinder layout of a 500 MW steam turbine and explain the reasons of double flow cylinders used.

[CSE (Mains) 2010 : 20 Marks]

Solution:

Cylinder layout of a 500 MW steam turbine:

A typical turbine of (500 – 900 MW) output in a steam power plant would have one HP turbine, one intermediate pressure (IP) turbine and two (LP) turbines, rotating at 3000 or 3600 rpm, depending on the grid frequency. The IP and LP turbines are probably double flow type. These are double flow cylinders as the initial steam is at lower pressure and temperature, so the steam mass flow rate and volumetric flow rate are likely to be much higher for a given output. Thus to handle such large mass flow rate, double flow cylinders are utilized.



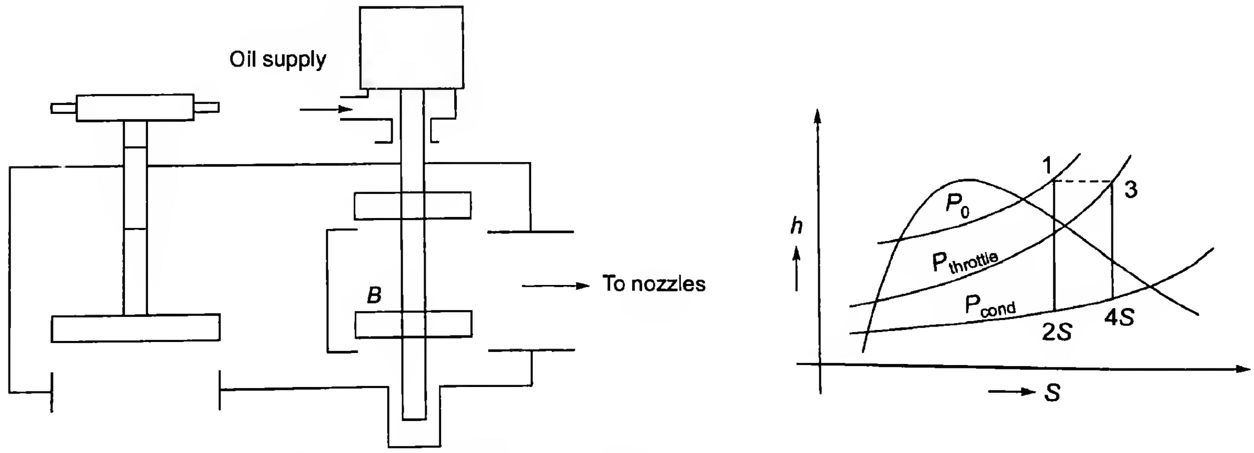
Q.68 What do you understand by governing of steam turbine? Explain the principle and working of throttle governing with the aid of a neat sketch. Compare the throttle governing with nozzle control governing with following aspects: (i) losses, (ii) use.
Sketch the efficiency against load with throttle governing.

[CSE (Mains) 2011 : 10 Marks]

Solution:

Steam turbine governing is the procedure of controlling the flow rate of steam to a steam turbine so as to maintain its speed constant as the load varies. The variation in load during the operation of a steam turbine can have a significant impact on its performance.

The principle and working of throttle governing is: In throttle governing the pressure of steam reduced at the turbine entry thereby decreasing the availability of energy. In this method steam is passed through a restricted passage thereby reducing its pressure across the governing value. As the load decreases and the shaft speed increases, the stop valve is partially closed to admit less steam to the turbine and to produce less power accordingly to the demand. Due to restriction of the passage in the value, steam is throttled say from P_0 to $P_{throttle}$. The specific ideal output of turbine thus reduces from $(h_1 - h_{2S})$ to $(h_3 - h_4s)$. With further closure of the value, $P_{throttle}$ will still be less to produce a still lower output. The throttle and stop valves are located in the steam supply line to the turbine. The stop valve is a hydraulically operated quick acting and shutting valve designed to be either fully open or shut. For small turbines, the stop value may be manually operated. The throttle valve is used to regulate steam flow during starting or stopping.

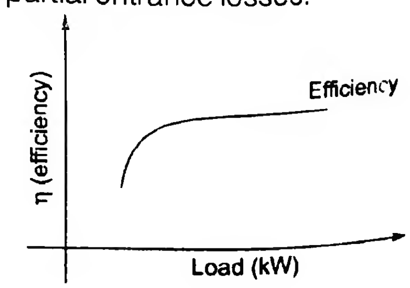


Comparison of throttle governing with nozzle control governing

(i) **Losses** : When throttle governing is done at low loads, the turbine efficiency is considerably reduced. The nozzle control is then a better method of governing. It has lesser losses due to frictional entrance of steam than nozzle control method where this nozzle control has large partial entrance losses.

Uses : Throttle Governing is usually working with both impulse and reaction turbines whereas nozzle control governing is working in impulse turbine and reaction turbines have initial impulse stages.

Graph of efficiency against load with throttle governing.

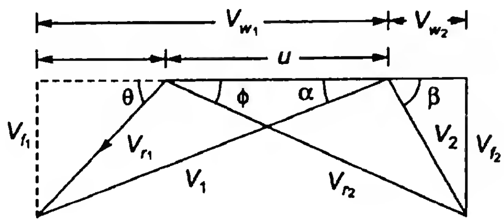


Q.69 At a stage in a reaction turbine, the pressure of steam is 34 kPa (v_g = 4.65 m³/kg) and dryness fraction is 0.95. For a flow rate of 36000 kg/hr, the stage develops 950 kW. The turbine runs at 3600 r.p.m. and velocity of flow is 0.72 times the blade velocity. The outlet angles of both stator and rotor blades are 20°. Determine at this stage the mean rotor diameter and height of blades.

[CSE (Mains) 2014 : 10 Marks]

Solution:

Given : At a stage in Reaction Turbine



v_g = 4.65 m³/kg, x = 0.95, $\dot{m}$ = 36000 kg/hr , Power = 950 kW,

N = 3600 rpm, $V_f = 0.72\,u$, $\alpha = \phi = 20°$

$$\dot{m} = 10 \text{ kg/sec}$$

$$\text{Power} = \dot{m}(V_{w_1} + V_{w_2})u$$

$$\tan\alpha = \frac{V_{f_1}}{V_{w_1}} \Rightarrow \left(V_{w_1} = \frac{V_{f_1}}{\tan 20°} = 2.75\,V_{f_1}\right)$$

Also,

$$V_{w_2} = V_{f_2}\cot\phi - u = V_{f_1}\cot 20° - u \qquad (\text{As } V_{f_1} = V_{f_2})$$

$$V_{w_2} = V_{f_1}(2.75) - u = (2.75V_{f_1} - u)$$

$$\text{Power} = \dot{m}\left[(2.75v_{f_1}) + (2.75v_{f_1} - u)\right]u$$

$$= \dot{m}\left[5.5v_{f_i} - u\right]u = \dot{m}[(5.5)(0.72u) - u] = \dot{m}[3.96u - u]u = 2.96\dot{m}u^2$$

$$\text{Power} = 950 \times 10^3 = (10 \times 2.96)\,u^2$$

$$u = 179.15 \text{ m/sec}$$

Also,

$$u = \frac{\pi D_m N}{60}$$

$$D_m = \frac{60 \times 179.15}{\pi(3600)} = 0.95 \text{ m}$$

Mean rotor diameter is 0.95 m

$$\text{Flow Velocity, } v_f = 0.72 \times u = 0.72 \times 179.15 = 128.9 \text{ m/sec}$$

$$\text{Volume flow of steam} = xv_g = 0.95 \times 4.65$$

$$v_1 = 4.4175 \text{ m}^3/\text{kg}$$

$$(\dot{m} \times v_1) = (\pi D_m H_b)V_{f_1}$$

$$(10 \times 4.4175) = (\pi \times 0.95 \times H_b) \times 128.9$$

$$H_b = 0.115 \text{ m}$$

$$\text{Height of blades } H_b = 0.115 \text{ m}$$

Q.70 A steam power station uses the following cycle:

Steam boiler outlet 150 bar, 550°C (h = 3450.4 kJ/kg, S = 6.523 kJ/kg K);

Reheat at 40 bar to 550°C (h = 3560.34 kJ/kg, s = 7.235 kJ/kg K);

Condenser at 0.1 bar (h_f = 191.8 kJ/kg, h_{fg} = 2392.05 kJ/kg, s_f = 0.649 kJ/kgK, s_{fg} = 7.5 kJ/kg K)

Assuming ideal processes, find quality of steam at turbine exhaust, cycle efficiency and steam flow rate per kWh.

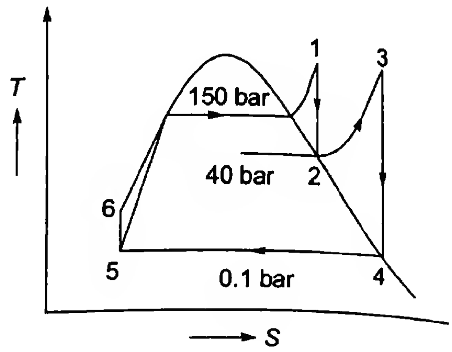
[CSE (Mains) 2014 : 10 Marks]

Solution:

Given: $h_1 = 3450.4$ kJ/kg, $S_1 = 6.523$ kJ/kgK

Reheat: $T_3 = 550°C = T_1$, $h_3 = 3560.34$ kJ/kg, $S_3 = 7.235$ kJ/kgK

Let the quality of steam at turbine exhaust be xy.



$\Rightarrow \quad S_3 = S_4 = 7.235 = S_f + x_y S_{fg}$

or $\quad 7.235 = 0.649 + x_y(7.5)$

$\Rightarrow \quad x_y = 0.878$

As, $\quad h_y = h_f + x_y h_{fg} = 191.8 + 0.878 \times 2392.05$

$h_y = 2292.33$ kJ/kg

$w_P =$ Pump work $= v(\Delta P) = 10^{-3} \times 150 \times 10^2 = 15$ kJ/kg

$h_5 = h_f = 191.8$ kJ/kg

Also, $\quad W_P = h_6 - h_5 = 15$ kJ/kg

$h_6 = 15 + 191.8 = 206.8$ kJ/kg

heat added, $Q_1 = (h_1 - h_6) + (h_3 - h_2)$

$Q_1 = (3450.4 - 206.8) + (3560.34 - 3065)$ [$h_2 = 3065$ kJ/kg from steam table]

$Q_1 = 3738.95$ kJ/kg

$w_T = (h_1 - h_2) + (h_3 - h_4) = (3450.4 - 3065) + (3560.34 - 2292.33)$

$w_T = 1653.41$ kJ/kg

$w_{net} = w_T - w_P = 1653.41 - 15 = 1638.41$ kJ/kg

$$\eta_{cycle} = \frac{w_{net}}{Q_1} = \frac{1638.41}{3738.95} = 0.4382 \text{ or } 43.82\%$$

$$\text{Steam rate} = \frac{3600}{1638.41} = 2.19 \text{ kg/kW-h}$$

Q.71 Even though velocity-compounded impulse turbines are less efficient, in the initial stages of high pressure turbines are normally velocity-compounded. Why? Plot the variation of pressure and velocities in velocity-compounded and pressure-compounded impulse turbines.

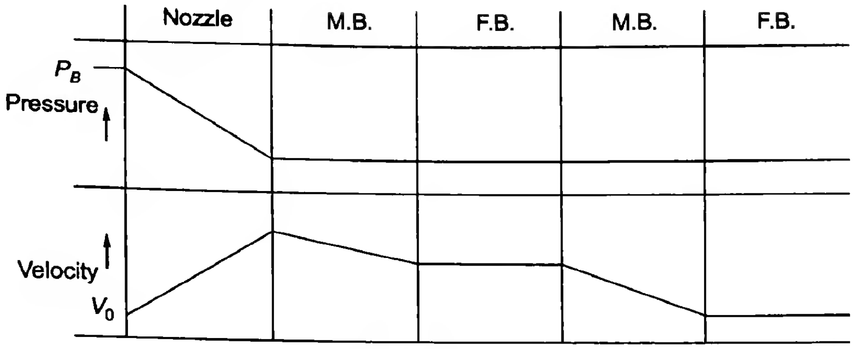
[CSE (Mains) 2014 : 20 Marks]

Solution:

Even though velocity-compounded impulse turbines are less efficient, but in the initial stages of high pressure turbines are normally velocity-compounded. The energy loss due to friction is proportional to the square of the velocity of a fluid (steam). Since the fluid velocity is highest for a 2-row curtis stage (velocity compounded), so energy loss due to friction is highest in this turbine. And these are less efficient. But in modern turbines, a 2-row curtis stage is generally used as the first stage. It is followed by a series of reaction stages. The use of initial curtis stage reduces the length of the rotor i.e. the number of stages required (since a 2-row curtis stage can replace about eight 50% reaction stages) and hence, the cost of the rotor significantly, at the expense of some loss in efficiency. It is often called the control stage. A large drop in enthalpy occurs in this curtis stage and remaining enthalpy drop occurs in the subsequent stages.

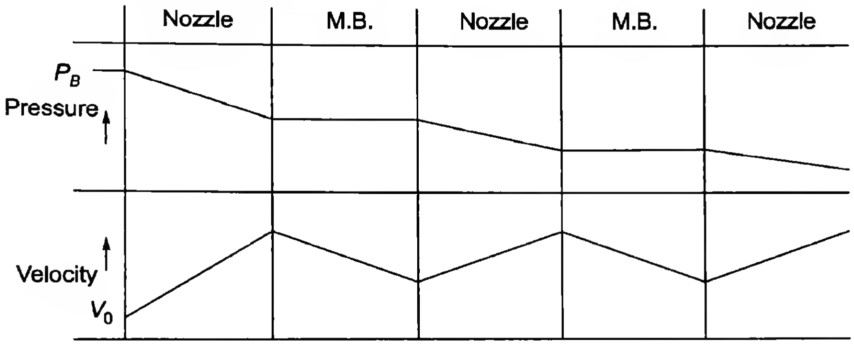
Variation of pressure and velocities in compounding.

Velocity Compounded impulse turbine



M.B. = Moving Blades
F.B. = Fixed/stationary blades

In pressure compounded impulse turbine



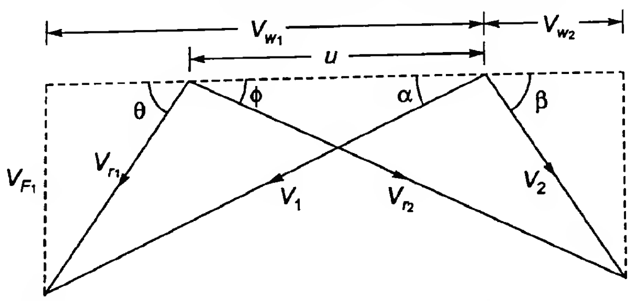
Q.72 The angles at inlet and discharge of the blading of a 50% reaction steam turbine are 35° and 20° respectively. The speed of rotation is 1500 rpm and at a particular stage, the mean ring diameter is 0.67 m, and the steam condition is 1.5 bar, 0.96 dry. Estimate (i) the required height of blading to pass 3.6 kg/s of steam, and (ii) the power developed by the ring.

Assume, at 1.5 bar pressure, $v_f = 0.001052\dfrac{m^3}{\text{kg}}$ and $v_{fg} = 1.15937\dfrac{m^3}{\text{kg}}$.

[CSE (Mains) 2015 : 20 Marks]

Solution:

50% Reaction steam turbine



$(\Delta V_a = 0)$

Given : $\theta = \beta = 35°$, $\alpha = \phi = 20°$, $N = 1500$ rpm, $D = 0.67$ m

$$\text{Blade speed, } u = \frac{\pi DN}{60} = \frac{\pi(0.67)(1500)}{60}$$

$$u = 52.6 \text{ m/sec}$$

At 1.5 bar pressure,
$$v_1 = v_f + x\, v_{fg} = 0.001052 + 0.96\,(1.15937) = 1.114 \text{ m}^3/\text{kg}$$

As,
$$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u} = \frac{V_1 \sin\alpha}{V_1 \cos\alpha - u}$$

or
$$\tan 35^\circ = \frac{V_1 \sin 20^\circ}{V_1 \cos 20^\circ - u}$$

$\Rightarrow$
$$0.66V_1 - 0.7u = 0.342\,V_1$$

$\Rightarrow$
$$V_1 = 2.2u = 2.2 \times 52.6 = 115.8 \text{ m/sec}$$
$$V_{f_1} = V_1 \sin\alpha = V_1 \sin 20^\circ = 115.8 \sin 20^\circ = 39.6 \text{ m/sec}$$

Let the required height of blading to pass $\dot{m} = 3.6$ kg/s of steam be h_b.

So,
$$(\dot{m}v_1) = (\pi D_m H_b)V_{f_1}$$
$$(3.6 \times 1.114) = \pi(0.67)H_b \times 39.6$$
$$H_b = 0.0481 \text{ m} = 48.13 \text{ mm}$$
$$\text{Power developed} = \dot{m}\left[V_{w_1} + V_{w_2}\right]u = \dot{m}\left[2V_1\cos\alpha - u\right]u$$
$$= 3.6[2 \times 115.8 \cos 20^\circ - 52.6]\,52.6$$
$$P = 31.25 \text{ kW}$$

Q.73 Dry saturated steam at 5 bar enters a convergent-divergent nozzle at a velocity of 100 m/s. The exit pressure is 1.5 bar. The throat and exit areas are 1280 mm² and 1600 mm² respectively. Assuming isentropic flow upto the throat and taking the critical pressure ratio as 0.58, estimate the mass flow rate. If the nozzle efficiency is 0.973, determine the exit condition of steam dryness fraction. Show the process on T-s and h-s diagrams.

Properties of Steam

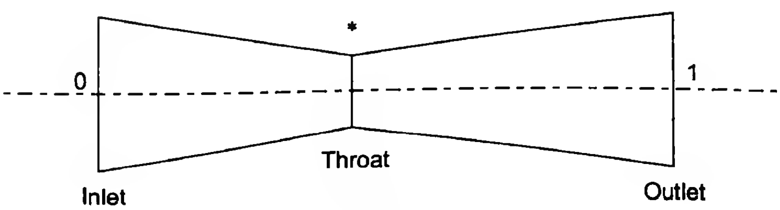
P (bar)	Enthalpy (kJ/kg)		Entropy (kJ/kg K)		Volume (m³/kg)	
	h_f	h_{fg}	s_f	s_{fg}	v_f	v_{fg}
5.0	640.23	2108.5	1.8607	4.9606	0.00109	0.3708
2.9	556	2168	1.660	5.344	0.00107	0.6253
1.5	467.11	2226.5	1.4336	5.7897	0.00105	1.158

[CSE (Mains) 2015 : 20 Marks]

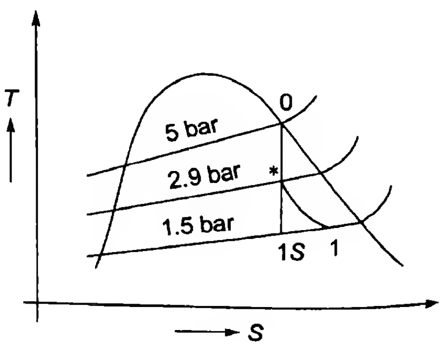
Solution:

Given; $V_1 = 100$ m/sec, $P_1 = 5$ bar, $P_2 = 1.5$ bar, $A^* = 1280$ mm², $A_2 = 1600$ mm²

$$\frac{P^*}{P_1} = 0.58 \Rightarrow P^* = 0.58 \times 5$$
$$P^* = 2.9 \text{ bar}$$



Nozzle



from table given:
$$h_0 = 2108.5 + 640.23 = 2748.73 \text{ kJ/kg}$$

$$S_0 = 4.9606 + 1.8607 = 6.821 \text{ kJ/kg-K}$$

So,

$$S^* = 6.8213 = 1.660 + x^*(5.344)$$

$$x^* = 0.966$$

$$h^* = 556 + 0.966\,(2168)$$

$$h^* = 2650.28 \text{ kJ/kg}$$

Energy equation will give,

$$h_0 + \frac{V_0^2}{2} = h^* + \frac{V^{*2}}{2}$$

$$2748.73 + \frac{(100)^2}{2000} = 2650.28 + \frac{V^{*2}}{2000}$$

$$\text{Velocity, } V^* = 456 \text{ m/sec}$$

$$v^* = (v_f)^* + x^*(v_{fg})$$

$$v^* = 0.00107 + 0.966\,(0.6253)$$

$$v^* = 0.605 \text{ m}^3\text{/kg}$$

$$\text{Mass flow rate, } \dot{m} = \frac{A^* V^*}{v^*}$$

$$\dot{m} = \frac{1280 \times 10^{-6} \times 456}{0.605}$$

$$\dot{m} = 0.965 \text{ kg/sec}$$

$$\text{Nozzle efficiency, } \eta = 0.973 = \frac{h^* - h_1}{h^* - h_{1s}}$$

$$0.973 = \frac{2650.28 - h_1}{2650.28 - h_{1s}} \qquad ...(1)$$

Also,

$$S_0 = S_{1s} = 6.8213 = 1.4336 + x_{1s}(5.7897)$$

$$x_{1s} = 0.93$$

$$h_{1s} = 467.11 + 0.93 \times 2226.5$$

$$h_{1s} = 2537.75 \text{ kJ/kg} \qquad ...(2)$$

From (1) and (2)

$$0.973 = \frac{2650.28 - h_1}{2650.28 - 2537.75}$$

$$h_1 = 2540.8 \text{ kJ/kg}$$

Let the exit condition dryness fraction be x_1.

$$h_1 = (h_f)_{1.5\text{ bar}} + x_1(h_{fg})_{1.5\text{ bar}}$$

$$2540.8 = 467.11 + x_1(2226.5)$$

$$x_1 = 0.932$$

6. Boilers, Condensers and Accessories

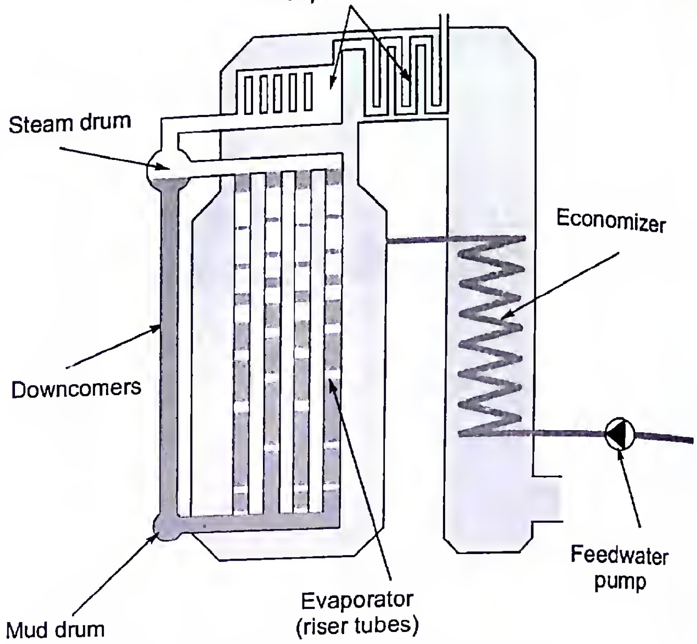
Q.74 Clearly explain as to how the circulation is maintained in modern boilers having operating pressures 100 bar, 180 bar and 240 bar respectively? What is circulation ratio? What is the normal range of circulation ratio in case of utility boilers and industrial boilers?

[CSE (Mains) 2001 : 30 Marks]

Solution:

Natural Circulation for Boiler Operating Pressure at 100 bar: The water/steam circulation begins from the feed water tank, from where feed water is pumped.

The feed water pump (refer figure shown below) raises the pressure of the feed water to the desired boiler pressure. In practice, the final steam pressure must be under 170 bar in order for the natural circulation to work properly. The feed water is then preheated in the economizer almost up to the boiling point of the water at the current pressure. To prevent the feed water from boiling in the economizer pipes, the economizer temperature is on purpose kept about 10 degrees under the boiling temperature.



Natural Circulation Principle at 100 bar

From the economizer the feed water flows to the steam drum of the boiler. In the steam drum the water is mixed well with the existing water. This reduces thermal stresses within the steam drum. The saturated water flows next from the steam drum through downcomer tubes to a mud drum (header). There are usually a couple of downcomer tubes, which are unheated and situated outside the boiler.

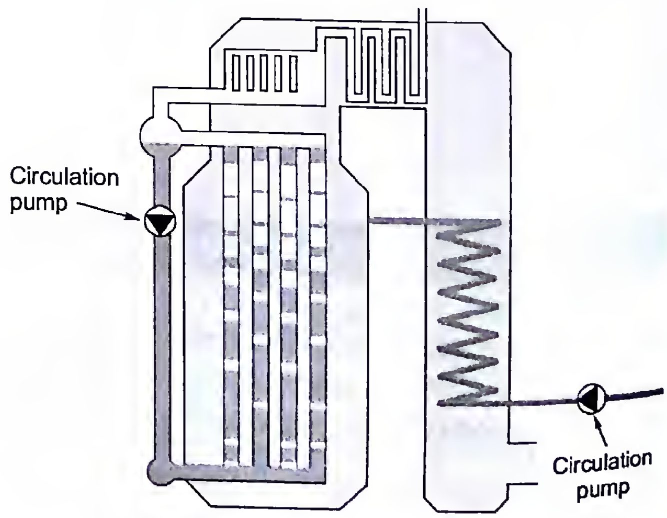
The name "mud drum" is based on the fact that a part of the impurities in the water will settle and this 'mud' can then be collected and removed from the drum. The saturated water continues from the header to the riser tubes and partially evaporates. The riser tubes are situated on the walls of the boiler for efficient furnace wall cooling.

The rises tubes are sometimes also called generating tubes because they absorb heat efficiently to the water/steam mixture. The riser tubes forms the evaporator unit in the boiler. After risers, the water/steam mixture goes back to the steam drum. In the steam drum water and steam are separated: the saturated water will return to the downcomer tubes and the saturated steam will continue to the superheater tubes. The purpose of this separation is to protect the inside of the superheater tubes and turbine for impurity deposition.

The steam from the steam drum continues to the superheater, where it is heated beyond its saturation point. After the last superheater stage the steam exits the boiler. This type of circulation is called natural circulation, since there is no water circulation pump in the circuit. The circulation happens by itself due to the water/steam density differences between the downcomers and risers.

Natural circulation boilers are only suitable for subcritical pressure levels (practically for steam pressures under 180 bar in the steam drum). This is due to the lack of density difference in supercritical steam, and thus the lack of a driving force.

Forced Circulation for Boiler Operating Pressure at 180 bar: In contrast to natural circulation boilers, forced circulation is based on pump-assisted internal water/steam circulation. The circulation pump is the main difference between natural and forced circulation boilers. In the most common forced circulation boiler type, the Lamont boiler, the principles of forced circulation is basically the same as for natural circulation, except for the circulation pump.



Forced Circulation Principle at 180 bar

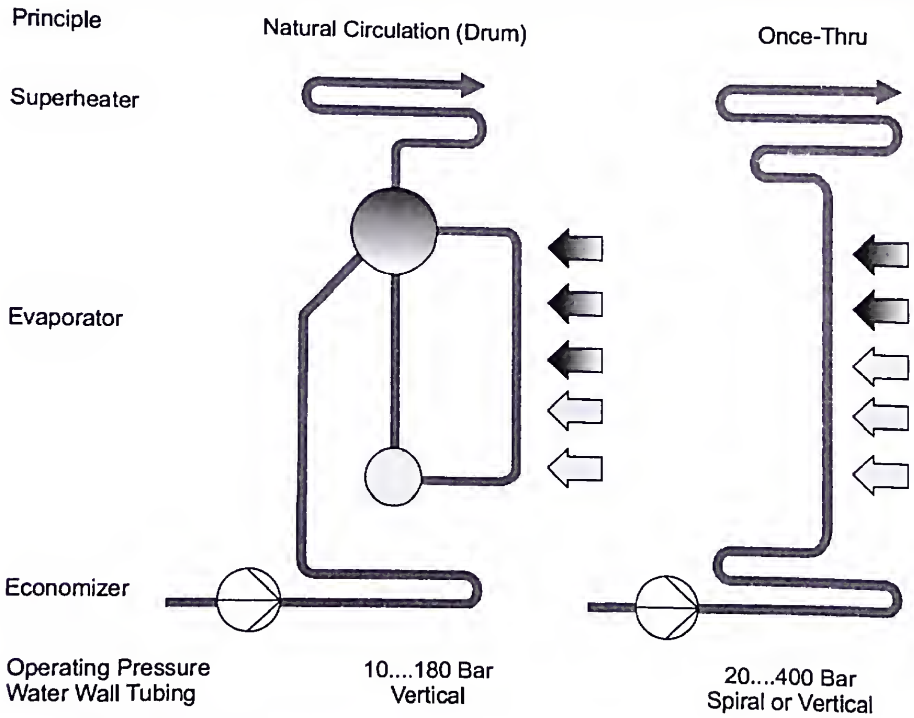
Due to the use of circulation pump, the operation pressure level of forced circulation boiler can be slightly higher than a natural circulation boiler, but since the steam/water separation in the steam drum is based on the density difference between steam and water, these boilers are not either suitable for supercritical pressures (>221 bar). Practically the maximum operation pressure for a forced circulation boiler is 190 bar and the pressure drop in the boiler is about 2-3 bar.

The water/steam circulation begins from the feed water tank, from where feed water is pumped.

The feedwater pump raises the pressure of the feedwater to the desired boiler pressure.

In practice, the final steam pressure is below 190 bar, in order to keep the steam steadily in the subcritical region.

Once Through Circulation for Boiler Operating Pressure at 240 bar: Beyond the critical pressure, phase transformation is absent, and hence once through system is adopted. A once-through (or universal pressure) boiler can be simplified as a long, externally heated tube . There is no internal circulation in the boiler, thus the circulation ratio for once-through boilers is 1. In contrast to other water tube boiler types (natural and controlled circulation), once through boilers do not have a steam drum. Once-through boilers are also called universal pressure boilers because they are applicable for all pressures and temperatures. However, once through boilers are usually large sized boilers with high subcritical or supercritical steam pressure.



The circulation ratio R is defined as

$$R = \text{(mass flow rate of circulation water)/(evaporation rate)} = GR/GS$$
$$\cong G_R/x_e\, G_R + 1/x_e$$

where G_R (kg/s) is the mass flow rate of circulation water, G_S(kg/s) is the evaporation rate, and x_e is the quality at the outlet of the evaporator tube. The evaporation rate G_S is expressed as $x_e \times G_R$ under the assumption that the water at the tube inlet is in the saturated state. Circulation Ratio for utility boilers is between 6 to 9. Circulation Ratio for industrial boilers is between 8 to 30

Q.75 **A forced draught fan discharges 1200 m^3 of air per minute through the outlet of 2 m^2 area and maintains a steady pressure of 110 mm of water. The temperature of air is 27°C. Calculate the power of the motor to drive the fan if the efficiency of the fan is 85%. Take the density of air at 27°C as 1.25 kg/m^3.**

[CSE (Mains) 2002 : 20 Marks]

Solution:

Given: Fan cross section $A = 2$ m^2, Density of air, $\rho = 1.25$ kg/m^3

$$\text{Volume of air to be handled, } Q = 1200 \text{ m}^3\text{/minute} = \frac{1200}{60} = 20 \text{ m}^3\text{/s}$$

[As $Q = v \times A$]

$$\text{Velocity of supplied air, } V = \frac{20}{2} = 10 \text{ m/s}$$

$$\text{Velocity head, } P_v = \frac{\rho V^2}{2} = 1.25 \times \frac{100}{2} = 62.5 \text{ N/m}^2$$

$$\text{Static pressure head, } P_s = 110 \text{ mm of water} = 0.11 \times 9810 = 1079.1 \text{ N/m}^2$$

Total pressure head to be produced by fan,

$$\Delta P = P_v + P_s = 62.5 + 10791.1 = 1141.6 \text{ N/m}^2$$

$$\text{Power reqired by FD fan} = \frac{Q\Delta P}{\eta_{FD} \times 1000} = \frac{20 \times 1141.6}{0.85 \times 1000} = \mathbf{26.86 \text{ kW}}$$

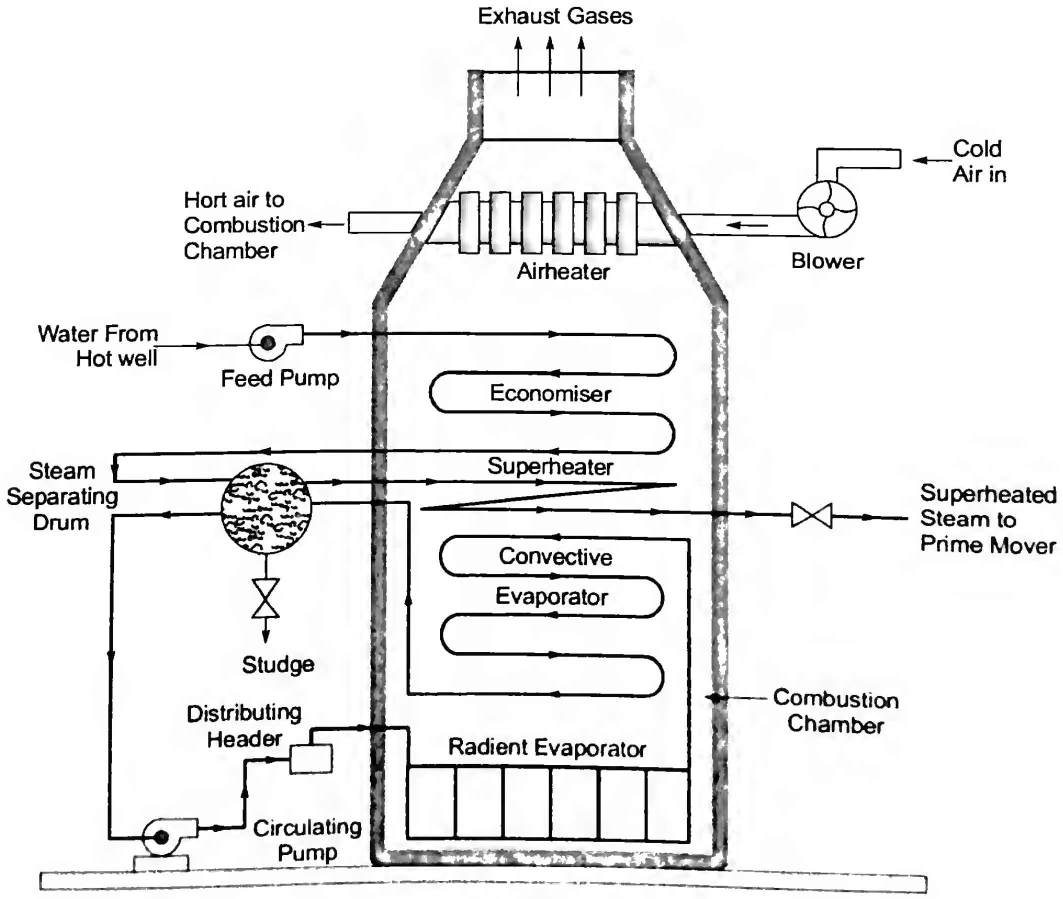
Q.76 Discuss the difference between natural circulation, forced circulation and one-through boilers. Explain, with the help of a sketch, the working of a La Mont boiler.

[CSE (Mains) 2002 : 30 Marks]

Solution:

[Refer Question Number 74]

Working Principle of Lamont Boiler: Lamont boiler is a forced circulation, internally fired water tube boiler. The fuel is burn inside the boiler and the water is circulating by a centrifugal pump through evaporator tubes. The working of this boiler is as follow. A feed pump forces the water into the economizer where the temperature of water increases. This water forced into the evaporator tube by using a centrifugal pump driven by steam turbine. Water passes 10 - 15 times into the evaporator tube. The mixture of saturated steam and water is formed inside the tube.



Lamont Boiler

This mixture is sent to the steam separator drum which is situated outside the boiler. Steam from the separator sends to the super heater, where the saturated steam converts into superheated steam. The water is again sent to the economizer where it again passes by the evaporator tubes.

The air from the air preheater enter into the furnace where fuel burn. The flue gases first heat the evaporator tube then passes by the super heater. These gases from the super heater again use to preheat the air into air preheater before exhaust into atmosphere.

This working pressure of this boiler is above 170 bar and have the steam generation capacity of about 50000 kg/hour at temperature 773 K.

Advantages:

1. It is a high pressure boiler.
2. It is flexible in design.
3. This boiler can be reassembled in natural circulation boiler.
4. It can easily start.
5. It has high steam generation capacity of about 50 ton/ hour.
6. This boiler has higher heat transfer rate.

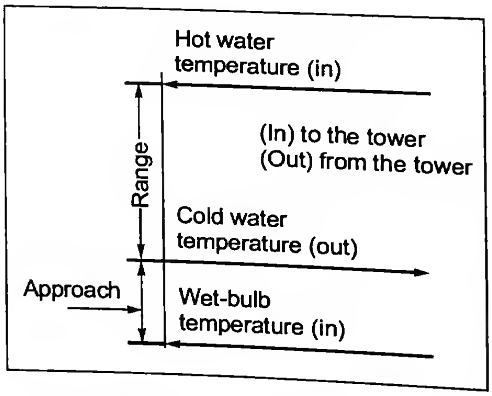
Q.77 What is the role of a cooling tower in a power plant? What is meant by range and approach and what purpose does the fill serve in a cooling tower?

[CSE (Mains) 2003 : 20 Marks]

Solution:

A cooling tower is a heat rejection device that rejects waste heat to the atmosphere through the cooling of a water stream to a lower temperature. Cooling tower perform the release of heat from the hot water coming from the condenser. In a power plant condenser acts as the sink and steam from the turbine is being dumped into the condenser. Cooling water is sent into to the condenser to condense this steam. As a result, the cooling water temperature rises. To reuse this water again for condenser cooling, the absorbed heat has to be released and cooled. Thermal power plants use cooling towers to cool the circulating water used for condenser cooling. Cooling tower cool the warm water discharged from the condenser and feed the cooled water back to the condenser. They reduce the cooling water demand in the power plant.

Cooling towers are a very important part of many chemical plants. The primary task of a cooling tower is to reject heat into the atmosphere. They represent a relatively inexpensive and dependable means of removing low-grade heat from cooling water. The make-up water source is used to replenish water lost to evaporation. Hot water from heat exchangers is sent to the cooling tower. The water exits the cooling tower and is sent back to the exchangers or to other units for further cooling.



"Range" is the difference between the cooling tower water inlet and outlet temperature.

"Approach" is the difference between the cooling tower outlet cold water temperature and ambient wet bulb temperature. Although, both range and approach should be monitored, the 'Approach' is a better indicator of cooling tower performance.

Purpose of Fill in Cooling Towers: Most towers employ fills (made of plastic or wood) to facilitate heat transfer by maximizing water and air contact. Fill can either be splash or film type. With splash fill, water falls over successive layers of horizontal splash bars, continuously breaking into smaller droplets, while also wetting the fill surface. Plastic splash fill promotes better heat transfer than the wood splash fill. Film fill consists of thin, closely spaced plastic surfaces over which the water spreads, forming a thin film in contact with the air. These surfaces may be flat, corrugated, honeycombed, or other patterns. The film type of fill is the more efficient and provides same heat transfer in a smaller volume than the splash fill.

Q.78 Discuss the trends in the design of modern power plant boilers. Explain the zones of heat transfer and location of evaporators, superheaters and reheater.

[CSE (Mains) 2003 : 20 Marks]

Solution:

Introducing modern boiler concepts in the design of thermal power stations is nowadays becoming mandatory, not only from an economic point of view of new investments, but also as a significant and pro-active step towards the reduction of greenhouse gases & dust emissions by the enhancement of efficiency.

The increase in the cycle efficiency in modern power station is mainly achieved by increasing the steam parameters. The technological steps in boiler design are therefore shifting steam from sub critical to super critical and ultra supercritical parameters.

A boiler which generates steam at a pressure of 85 kgf/cm^2 or above is termed as a "high pressure boiler". The present tendency is towards the use of high pressure boilers in power plants. The modern high pressure boilers used for power generation have capacities of 40 to 1600 tonnes/hr of superheated steam with a pressure up to 210 kgf/cm^2 and a temperature of about 650°C. One of the largest modern steam power plants in the world is in Japan with a steam capacity of 1600 Tonnes/hr. In India, the Trombay power plant has a steam generating capacity of 550 tonnes/hr, Ramagundam power power plant with 320 tonnes/hr and Bokaro plant with 160 tonnes/hr.

Advantages of High-pressure Boilers:

1. High-pressure boilers use the forced circulation of water which ensures the positive circulation of water and increased evaporative capacity.
2. They require less heat of vaporization.
3. They are compact and thus require less floor space.
4. Due to the high velocity of water, the tendency of scale formation is minimized.
5. All parts are uniformly heated and the danger of overheating is minimized.
6. The steam can be raised quickly to meet the variable load requirements without the use of complicated control devices.
7. The plant efficiency is increased.
8. With the use of high-pressure boilers, the steam generation is economical.

Zones of Heat Transfer in Boiler: A steam boiler is designed to absorb the maximum amount of heat released from the process of combustion. Heat transfer within steam boiler is accomplished by three methods: radiation, convection, and conduction. The heating surface in the furnace area receives heat primarily by radiation. The remaining heating surface in the steam boiler receives heat by convection from the hot flue gases. Heat received by the heating surface travels through the metal by conduction Heat is then transferred from the metal to the water by convection.

The relative percentage of each heat transfer within steam boiler is dependent on the type of steam boiler, the designed transfer surface, and fuels.

Convection from heated gases around the heated boiler tubes and circulating boiler feed water within the boiler tubes. This could be the superheated gases from the combustion of the fuel, or superheated fluids from another process somewhere outside of the boiler area.

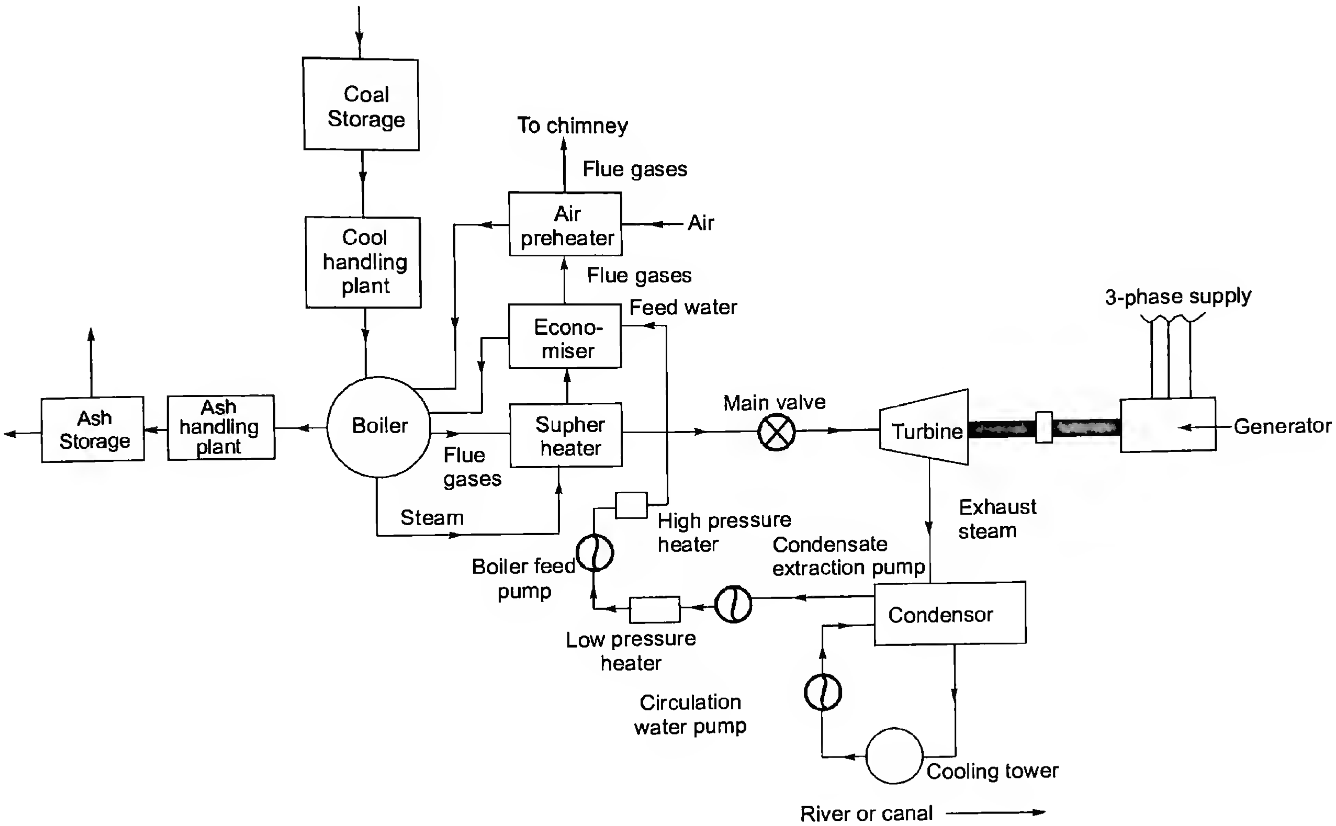
Conduction through the surface of the boiler tubes. The tubes get heat up, and then transfer the heat into the boiler feed water.

Radiation of heat absorbed into the boiler refractory, then absorbed by the boiler feed water inside the boiler tubes. This occurs mostly in the cases where fuel is combusted rather than superheated process fluids coming from a separate process.

An **evaporator** operates at a temperature lower than this. Boiling can take place throughout a liquid, but evaporation only takes place at an open surface. An evaporator usually has some flow of gas past the surface to take away the vapor being produced. This is often achieved by pumping the liquid up to a high level and letting it run down, but there are quite a lot of other designs.

Superheater: It is integral part of boiler and is placed in the path of hot flue gases from the furnace. The heat recovered from the flue gases is used in superheating the steam before entering into the turbine (i.e., prime mover).Its main purpose is to increase the temperature of saturated steam without raising its pressure.

Reheater: The reheater functions similar to the superheater in that it serves to elevate the steam temperature. Primary steam is supplied to the high pressure turbine. After passing through the high pressure turbine, the steam is returned to the steam generator for reheating (in a reheater) after which it is sent to the low pressure turbine. A second reheat cycle may also be provided.



Q.79 Explain the working principle of Electrostatic Precipitator (ESP). Discuss the factors which affect its efficiency.

[CSE (Mains) 2005 : 20 Marks]

Solution:

Electrostatic precipitation is a method of dust collection that uses electrostatic forces, and consists of discharge wires and collecting plates. A high voltage is applied to the discharge wires to form an electrical field between the wires and the collecting plates, and also ionizes the gas around the discharge wires to supply ions. When gas that contains an aerosol (dust, mist) flows between the collecting plates and the discharge wires, the aerosol particles in the gas are charged by the ions. The Coulomb force caused by the electric field causes the charged particles to be collected on the collecting plates, and the gas is purified. This is the principle of electrostatic precipitation, and Electrostatic precipitator apply this principle on an industrial scale. The particles collected on the collecting plates are removed by methods such as (1) dislodging by rapping the collecting plates, (2) scraping off with a brush, or (3) washing off with water, and removing from a hopper. Dust collection efficiency is affected by the following factors:

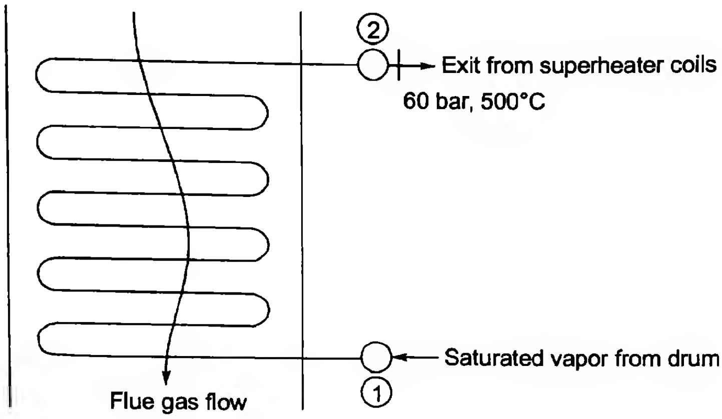
1. Electrical resistivity of dust
2. Particle size distribution
3. Aerosol concentration

Q.80 A superheater is to be designed using metallic coils (heat flux 150 kW/m^2) of inside diameter 50 mm and wall thickness 5 mm. The steam leaving the superheater coils is at 60 bars, 500°C and flows at a velocity of 10 m/s. If the steam mass flow rate is 90 kg/s, find the number and length of coils. For steam at 60 bars, take the following values – dry saturated steam h = 2784.3 kJ/kg, at 500°C superheated steam temperature h_{sup} = 3422.2 kJ/kg and specific volume V_{sup} = 0.05665 m^3/kg. The steam enters the superheater as dry and saturated.

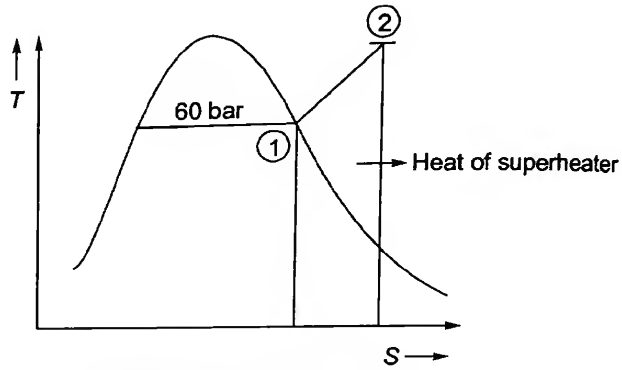
[CSE (Mains) 2006 : 20 Marks]

Solution:

Given: Heat flux $\dot{q}$ = 150 KW/m^2, d_i = 50 mm = 0.05 m, velocity V = 10 m/s, $\dot{m}_s$ = 90 kg/s



Schematic of Superheater coils



Process on T-s coordinate

As given, $h_1 = hg$ = 2784.3 kJ/kg, h_2 = 3422.2 kJ/kg and specific volume, v_2 = 0.05665 m^3/kg

Heat absorption rate in superheater coils = $\dot{m}_s(h_2 - h_1)$ = 90(3422.2 – 2784.3) = 57411 kW = $\dot{Q}$

$$\text{Surface area required} = \frac{57411}{150} = 382.74 \text{ m}^2 \quad [\text{As, } \dot{Q} = \dot{q} \times A]$$

$$\text{From continuity equation, } \dot{m}_s = \rho_1 A_1 V_1 = \rho_2 A_2 V_2 = \left(n\frac{\pi}{4}d_i^2\right)\frac{V_2}{v_2} = 90 \text{ kg/s}$$

[where 'n' is the number of superheater coils]

$$n = \frac{4 \times 90 \times 0.05665}{\pi \times (0.05)^2 \times 10} = 262.55 \text{ or } 263$$

Number of superheater coils, n = 263

As, Surface area, A_0 = 382.74 = $n \pi d_0 l$

d_0 = outer diameter = 50 + 2 × 5 = 60 mm [As thickness is 5 mm]

$$\text{Length of one coil} = \frac{382.74}{263 \times \pi \times 0.06} = 7.72 \text{ m}$$

Q.81 Why is the use of dust collector necessary in case of thermal power plant? Discuss the location of electrostatic precipitator with reference to sulphur content of coal and flue gas temperature.

[CSE (Mains) 2007 : 20 Marks]

Solution:

Thermal Power plants emit fly ash as well as other gases like Carbon dioxide(CO_2), Carbon Monoxide (CO), Sodium Oxides (NO_x), Sulphur oxides(SO_x) ,etc. to the atmosphere and, in turn, the atmosphere gets polluted. The pollution to this effect is dangerous to our living society as well as to the plant life. The reason is the use of fossil fuel like coal to run the thermal power plants. Now-a-days, more numbers of old coal based power plants, are still in existence with high emission. High ash content or deterioration in quality of coal reserve is

the major focus for our environmental sustainability and thus use of dust collector necessary in case of thermal power plant. In many thermal power plant plants, fly ash generated in the coal combustion process is carried as dust in the hot exhaust gases. These dust-laden gases are allowed to pass through an electrostatic precipitator that collects most of the dust. Cleaned gas then passes out of the precipitator and then through a stack to the atmosphere. Precipitators typically collect 99.9% or more of the dust from the gas stream. Precipitators function by electro-statically charging the dust particles in the gas stream coming out of the boiler after coal combustion. The charged particles are then attracted to and deposited on the collector plates. When enough dust has accumulated, the collectors are hammered to dislodge the dust, causing it to fall with the force of gravity to hoppers below. The dust is then removed by a conveyor, slurry form, vacuum trapped system etc. for disposal or recycling. Depending upon dust characteristics, coal composition (primarily ash content) and the gas volume to be treated, there are many different sizes, types and designs of electrostatic precipitators.

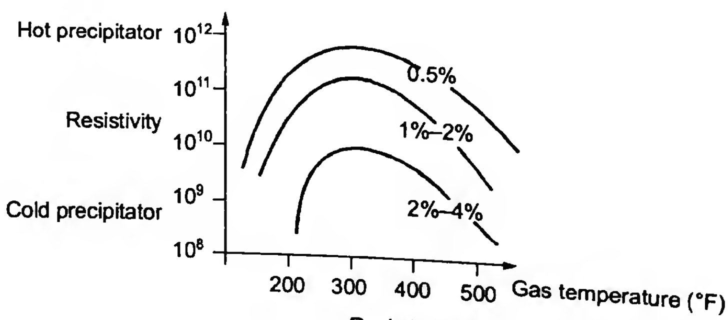
Factors Affecting the Performance of ESP

There are many factors which influence the performance of an ESP. These are discussed below:

- **Corona:** Corona ionizes gas molecules and charges dust particles. At higher temperature, flue gas density reduces. Lower density of the flue gas initiate corona at lower voltage. Corona power is the power required to energise the discharge electrode and create corona. Corona power is expressed in watts per 1000 cubic metre per hour or watts per 1000 actual cubic feet per minute (ACFM). It is approximately 60 to 300 watts per 1000 cubic metre per hour.
- **Resistivity of particulates:** It is a main factor that influences an ESP performance. This is the measurement of resistance to electrical conduction. Resistivity is the electrical resistance of a dust sample of 1 cm^2 in cross section and 1 cm in thickness. The unit of resistivity is ohm centimetre (Ω cm) and it is normally 10^8 to 10^{12} for coal fly ash. The charged particulates are supposed to transfer their charge to the collecting plate. It is difficult to charge the particulates having high resistance. But once they are charged, they do not give up their acquired charge to the collecting plate easily. The particles having low resistivity can be charged easily and give up their acquired charge easily.

If the resistivity is too low, the particulates discharge their charge too quickly an escape from the collecting plate and come back to the gas stream again. This phenomenon is called as re-entrain. If the resistivity is too high, particles, do not give up their charge completely to the collecting electrode. So, the charge is accumulated on the collecting plate and leads to back corona. This back corona reduces ionization and particles escape with the flue gas. The particles also remain strongly attracted to the collecting plate, as they are still having charge. So, it is difficult to rap off.

The major factors that influence ash resistivity are temperature, carbon particles and chemical composition (sodium and sulphur trioxide). Carbon particulates in the fly ash reduce resistivity. The resistivity of ash versus temperature for different sulphur containing coal is shown in figure. The resistivity is maximum between 300°F to 400°F. Resistivity of low sulphur coal is more.



Resistivity of ash.

To make the resistivity normal, gas conditioning is required. Small amount of sulphur trioxide (SO_3) is injected into the flue gas when low sulphur coal is used to boiler to reduce flue gas resistivity. The critical sulphur level of ash particle for better performance of ESP is 0.5% of the flue gas. 10 ppm to 20 ppm of SO_3 is sufficient in flue gas to maintain the resistivity of fly ash below the critical level.

In case of high sulphur coal, SO_3 level is more, so the resistivity is less. To maintain the resistivity, ammonia (NH_3) is injected into the flue gas for conditioning.

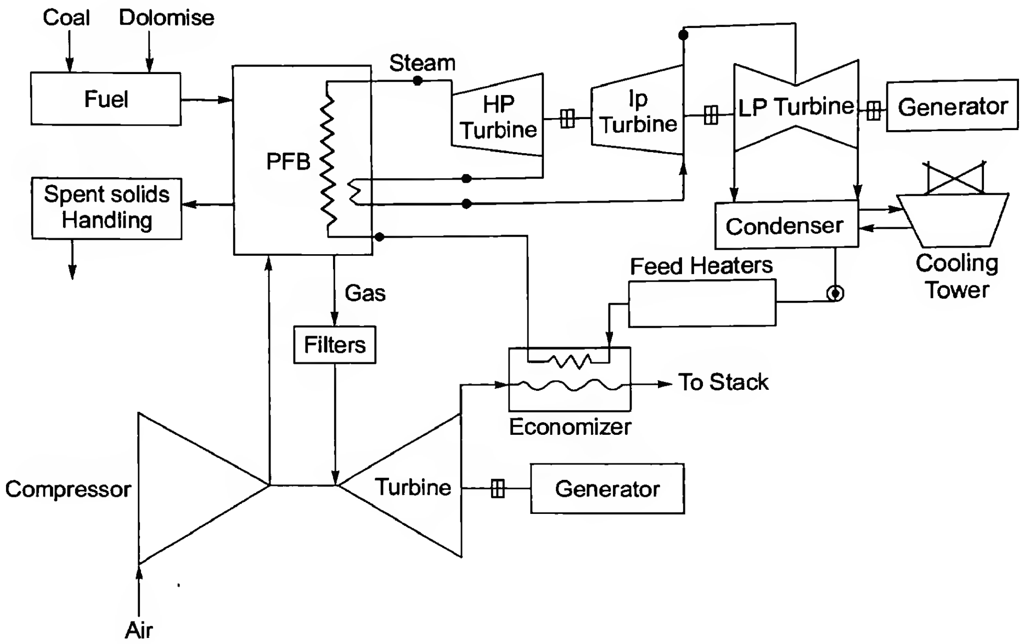
Flue gas velocity: It is having a significant effect on the dust collection efficiency of ESP. Efficiency increases when flue gas velocity decreases.

Q.82 Explain with a neat diagram the working of a supercharged boiler. What are its advantages over conventional boilers? Why economisers are essentially used irrespective of the fuel used in boiler furnace?

[CSE (Mains) 2008 : 20 Marks]

Solution:

Working of a supercharged boiler:



Combustion in supercharged boiler carried by pressurized combustion method at 5-6bars. Heated feed water of economizer and boiler drum are mixed and supplied to evaporating tubes where it is changed into wet steam and additional heating is carried out in evaporating tube and steam is discharged in boiler drum. Approximately dry steam as of drum is superheated in superheater before it is supplied to steam turbine. The high pressure exhaust gases from boiler are used to run the gas turbine. Power developed by the gas turbine is used to run compressor and other auxiliaries. Turbine exhaust gases are used to heat feed water in the economizer and lastly these are exhausted to surrounding by chimney.

In a supercharged boiler, the combustion is carried out under pressure in the combustion chamber by supplying the compressed air. The exhaust gases from the combustion chamber are used to run the gas turbine as they are exhausted to high pressure. The gas turbine runs the air compressor to supply the compressed air to the combustion chamber.

Advantages:

1. Owing to very high overall heat transfer co-efficient the heat transfer surface required is hardly 20 to 25% of the heat transfer surface of a conventional boiler.
2. The part of the gas turbine output can be used to drive other auxiliaries.
3. Small heat storage capactiy of the boiler plant gives better response to control.
4. Rapid start of the boiler is possible.
5. Comparatively less number of operators are required.

An economizer is a mechanical device which is used as a heat exchanger by preheating a fluid to reduce energy consumption. In a steam boiler, it is a heat exchanger device that heats up fluids or recovers residual heat from the combustion product i.e. flue gases in thermal power plant before being released through the chimney. Flue gases are the combustion exhaust gases produced at power plants consist of mostly nitrogen, carbon dioxide, water vapor, soot carbon monoxide etc. Hence, the economizer in thermal power plants, is used to economise the process of electrical power generation. The recovered heat is in turn used to preheat the boiler feed water, that will eventually be converted to super-heated steam. For higher boiler efficiencies, the feed water is preheated by economizer, using the waste heat in the flue gas.

Thus economizers are essentially used irrespective of the fuel used in boiler furnace.

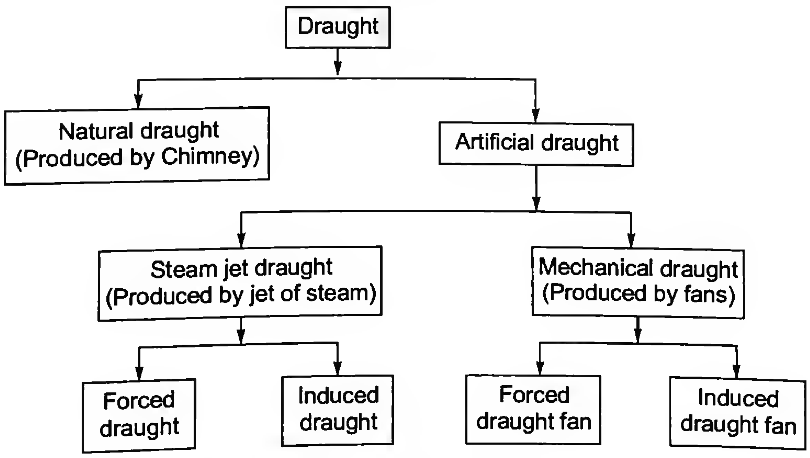
Q.83 How do you classify draught? What are the advantages of the forced draught over the induced draught? What are the limitations of chimney draught? What are the different losses taken into consideration in designing the draught system?

[CSE (Mains) 2008 : 20 Marks]

Solution:

The draught is one of the most essential systems of thermal power plant which supplies required quantity of air for combustion and removes the burnt products from the system. To move the air through the fuel bed and to produce a flow of hot gases through the boiler, economizer, preheater and chimney require a difference of pressure. This difference of pressure for to maintaining the constant flow of air and discharging the gases through the chimney to atmosphere is known as draught. Draught can be obtained by use of chimney, fan, steam or air jet or combination of these. When the draught is produced with the help of chimney only, it is known as Natural Draught and when the draught is produced by any other means except chimney it is known as artificial draught.

Natural draught is the draught produced by a chimney alone. It is caused by the difference in weight between the column of hot gas inside the chimney and column of outside air of the same height and cross section. Being much lighter than outside air, chimney flue gas tends to rise, and the heavier outside air flows in through the ash pit to take its place. It is usually controlled by hand-operated dampers in the chimney and breeching connecting the boiler to the chimney. Here no fans or blowers are used. The products of combustion are discharged at such a height that it will not be a nuisance to the surrounding community.



Mechanical draught : It is draught artificially produced by fans. Three basic types of draughts that are applied are :

Balanced draught: Forced-draught (F-D) fan (blower) pushes air into the furnace and an induced draught (I-D) fan draws gases into the chimney thereby providing draught to remove the gases from the boiler. Here the pressure is maintained between 12.442 Pa to 24.88 Pa of water gauge below atmospheric pressure in the case of boilers and slightly positive for reheating and heat treatment furnaces.

Induced draught: An induced-draught fan draws enough draught for flow into the furnace, causing the products of combustion to discharge to atmosphere. Here the furnace is kept at a slight negative pressure below the atmospheric pressure so that combustion air flows through the system.

Forced draught: The Forced draught system uses a fan to deliver the air to the furnace, forcing combustion products to flow through the unit and up the stack.

Advantages of the forced draught over the induced draught are:

1. Easy control of combustion and evaporation
2. Increase in evaporative power of a boiler
3. Improvement in the efficiency of the plant
4. Reduced chimney height
5. Prevention of smoke
6. Capability of consuming low grade fuel
7. Better accessibility of fans and upper bearings for maintenance
8. The fuel consumption per kW due to artificial draught is 15 % less

Limitations of chimney draught are as follows:

1. The maximum pressure available for producing natural draught by chimney is hardly 10 to 20 mm of water under the normal atmospheric and flue gas temperatures.
2. The available draught decreases with increase in outside air temperature and for producing sufficient draught, the flue gases have to be discharged at comparatively high temperatures resulting in the loss of overall plant efficiency. And thus maximum utilization of Heat is not possible.
3. As there is no through mixing of air and fuel in the combustion chamber due to low velocity of air therefore combustion is very poor. This increases the specific fuel consumption.
4. The chimney has no flexibility to create more draught under peak load conditions because the draught available is constant for a particular height of chimney and the draught can be increased by allowing the flue gases to leave the combustion chamber at higher temperatures. This reduces the overall efficiency of the plant.
5. Nearly 20% heat released by the fuel is lost to the flue gases. The chimney draught is only used for very small boilers. Nowadays the chimney is never used for creating draught in thermal power plants as it has no flexibility, the total draught produced is insufficient for high generating capacity.

Different losses taken into consideration in designing the draught system are as follows:

1. The frictional resistance offered by the flues and gas passages to the flow of flue gases.
2. Loss near the bends in the gas flow circuit
3. Loss due to friction head in equipment like grate, economizer, superheater etc.
4. Loss due to imparting velocity to the flue gas.

Q.84 Draw a schematic diagram of bubbling bed fluidized boiler and explain its working.

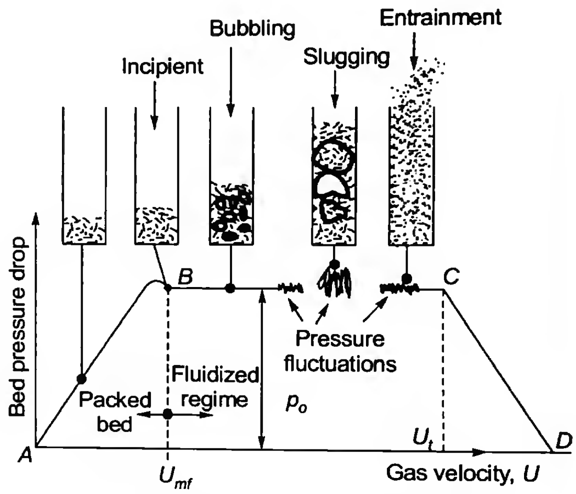
[CSE (Mains) 2009 : 15 Marks]

Solution:

Working of bubbling bed fluidized boiler: When an evenly distributed air or gas is passed upward through a finely divided bed of solid particles such as sand supported on a fine mesh, the particles are undisturbed at low velocity. As air velocity is gradually increased, a stage is reached when the individual particles are suspended in the air stream – the bed is called "fluidized". With further increase in air velocity, there is bubble formation, vigorous turbulence, rapid mixing and formation of dense defined bed surface. The bed of solid particles exhibit the properties of a boiling liquid and assumes the appearance of a fluid – "bubbling fluidized bed". At higher velocities, bubbles disappear, and particles are blown out of the bed. Therefore, some amounts of particles have to be recirculated to maintain a stable system – "circulating fluidized bed". Fluidization depends largely on the particle size and the air velocity.

The mean solids velocity increases at a slower rate than does the gas velocity. The difference between the mean solid velocity and mean gas velocity is called as slip velocity. Maximum slip velocity between the solids and the gas is desirable for good heat transfer and intimate contact. If sand particles in a fluidised state is heated to the ignition temperatures of coal, and coal is injected continuously into the bed, the coal will burn rapidly and bed attains a uniform temperature. The fluidised bed combustion (FBC) takes place at about 840°C to 950°C. Since this temperature is much below the ash fusion temperature, melting of ash and associated problems are avoided.

The lower combustion temperature is achieved because of high coefficient of heat transfer due to rapid mixing in the fluidised bed and effective extraction of heat from the bed through in-bed heat transfer tubes and walls of the bed. The gas velocity is maintained between minimum fluidisation velocity and particle entrainment velocity. This ensures stable operation of the bed and avoids particle entrainment in the gas stream.



Variation of bed pressure drop with superficial velocity

Combustion process requires the three "T"s that is Time, Temperature and Turbulence. In FBC, turbulence is promoted by fluidisation. Improved mixing generates evenly distributed heat at lower temperature. Residence time is many times greater than conventional grate firing.

Thus an FBC system releases heat more efficiently at lower temperatures.

Since limestone is used as particle bed, control of sulphur dioxide and nitrogen oxide emissions in the combustion chamber is achieved without any additional control equipment. This is one of the major advantages over conventional boilers. There are three basic types of fluidised bed combustion boilers:

1. Atmospheric classic Fluidised Bed Combustion System (AFBC)
2. Atmospheric circulating (fast) Fluidised Bed Combustion system(CFBC)
3. Pressurised Fluidised Bed Combustion System (PFBC).

Q.85 The following observations refer to a surface condenser:

Mass flow rate of condensate = 20 kg/min
Mean temperature of condensation = 35°C
Barometer reading = 1.03 kg/cm^2
Outlet cooling water temperature = 30°C
Mass flow rate of cooling water = 800 kg/min
Condenser vacuum = 0.95 kg/cm^2
Inlet cooling water temperature = 20°C
Temperature of the hot well = 29°C

Calculate:

(i) Weight of air per unit volume of condenser.
(ii) Entering condition of steam to the condenser.
(iii) Vacuum efficiency of the condenser.

Properties of saturated steam:

Temperature (°C)	P (MPa)	Sp. volume (m^3/kg)		Entropy (kJ/kg)		Entropy (kJ/kg-K)	
		v_f	v_g	h_f	h_g	s_f	s_g
35	0.0056	0.001	25.245	146.56	2565.4	0.5049	8.3543

(Use R = 0.287 kJ/kg-K)

[CSE (Mains) 2009 : 25 Marks]

Solution:

Given: Mass flow rate of condenstate $m_s = 20$ kg/min, $\dot{m}_w = 800$ kg/min

Specific volume of steam at mean condensate temperature of 35°C = 25.245 m^3/kg

$$\text{Partial pressure of steam at 35°C} = 0.0056 \text{ MPa} = 0.056 \text{ bar}$$

$$1 \text{ bar} = 1.02 \text{ kg/cm}^2$$

$$\text{Partial pressure of steam} = 0.056 \times 1.02 = 0.05712 \text{ kg/cm}^2$$

$$\text{Absolute condenser pressure} = \text{Barometric pressure} - \text{Vaccum reading}$$

$$= 1.03 - 0.95 = 0.08 \text{ kg/cm}^2$$

$$\text{Partial pressure of air} = 0.08 - 0.05712 = 0.02288 \text{ kg/cm}^2$$

$$\text{Partial pressure of air (in bar)} = 0.02288 \times 0.98 = 0.02243 \text{ bar}$$

Mass of air present per kg of uncondensed vapour,

$$m_a = \frac{P_a V}{RT} = \frac{0.02243 \times 10^5 \times 25.245}{287 \times (273 + 35)} = \mathbf{0.64\ kg}$$

$$\text{Absolute condenser pressure} = 0.08 \times 0.98 = 0.0784 \text{ bar} \approx 0.08 \text{ bar}$$

At this pressure, $h_f = 173.9$ kJ/kg, $h_{fg} = 2403.1$ kJ/kg

Enthalpy of condensate corresponding to hot well temperature of 29°C = 121.6 kJ/kg

Also, heat lost by steam = heat gained by water

$$m_s [(h_f + x\, h_{fg}) - h_{\text{hot well}}] = m_w C_{pw} (t_{w2} - t_{w1})$$

$$20 [(173.9 + x \times 2403.1) - 121.6] = 800 \times 4.186 (30 - 20)$$

$$52.3 + (2403.1)x = 1674.4$$

$$x = 0.675$$

Hence, entering condition of steam to the condenser = **0.675 or 67.5%**

Saturation pressure corresponding to 35°C = 0.056 bar

$$\text{Vaccum efficiency} = \frac{\text{Condenser vaccum}}{\text{Barometric reading} - \text{pressure of steam}} = \frac{0.95 \times 0.98}{1.03 \times 0.98 - 0.056}$$

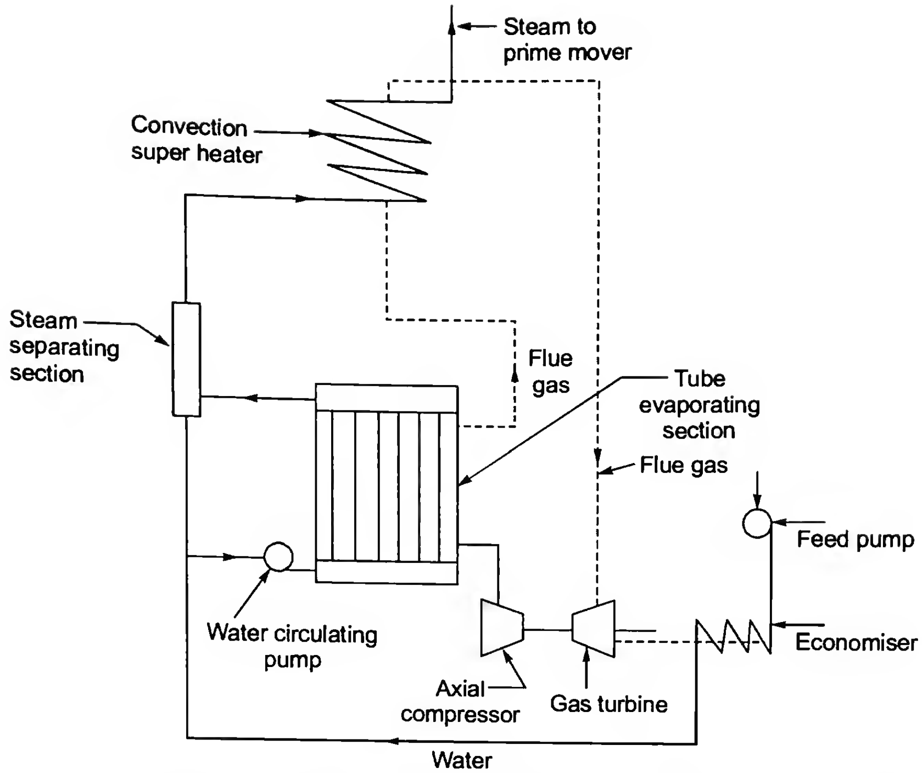
$$= \mathbf{0.9765 \text{ or } 97.65\%}$$

Q.86 In what way is Velox boiler different from La Mont boiler? Describe the working of the Velox boiler with a schematic.

[CSE (Mains) 2009 : 20 Marks]

Solution:

Parameters	**Velox Boiler**	**La Mont Boiler**
Combustion	This boiler makes use of pressurized combustion wherein the heat is transferred from the gas on account of supersonic flow of gas.	This boiler uses conventional combustion process.
Circulation	This boiler is a high pressure, forced circulation, pressurized forced combustion but with limitation of firing with oil or gas under pressure with little or no ash problem. Air compressor is driven by gas turbine.	This boiler works on basic principle of forced convection. If the water is circulate by a pump inside the tube, the heat transfer rate from gas to the water is increases. This boiler is the first force circulation boiler.
Performance	This boiler is very compact steam generating plant of great flexibility. It is capable of quick starting and its thermal efficiency is about 90 to 95%.	Working pressure of this boiler is above 170 bar and have the steam generation capacity of about 50 t /h at temperature 773 K.



It is a well known fact that when the gas velocity exceeds the sound-velocity, the heat is transferred from the gas at a much higher rate than rates achieved with sub-sonic flow. The advantage of this theory is taken to effect the large heat transfer from a smaller surface area in this boiler.
This boiler makes use of pressurised combustion.

The gas turbine drives the axial flow compressor which raises the incoming air from atmosphere pressure to furnace pressure. The combustion gases after heating the water and steam flow through the gas turbine to the atmosphere. The feed water after passing through the economiser is pumped by a water circulating pump to the tube evaporating section. Steam separated in steam separating section flows to the superheater, from there it moves to the prime mover.

The size of the Velox boiler is limited to 100 tonnes/h because 600 B.H.P. is required to run the air compressor at this output. The power developed by the gas turbine is not sufficient to run the compressor and therefore some power from external source must be supplied.

Advantages:

1. The boiler is very compact and has greater flexibility.
2. Very high combustion rates are possible.
3. It can be quickly started.
4. Low excess air is required as the pressurised air is used and the problem of draught is simplified.

Q.87 Draw a schematic of a Benson boiler and explain its working principle, pointing out its speciality.

[CSE (Mains) 2009 : 20 Marks]

or

With the help of a sketch discuss the working principle of a high pressure Benson boiler with advantages.

[CSE (Mains) 2010 : 10 Marks]

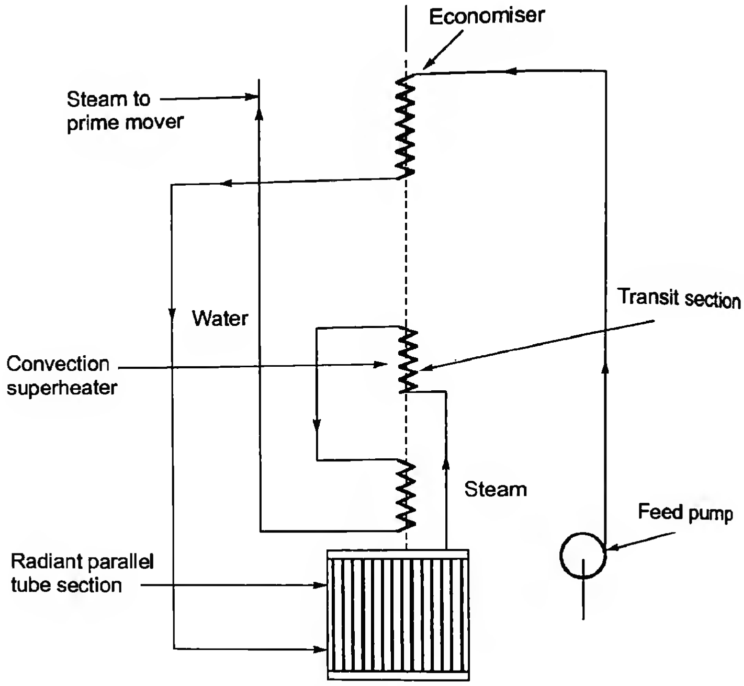
Solution:

In the LaMont boiler, the main difficult experienced is the formation and attachement of bubbles on the inner surfaces of the heating tubes.

The attached bubbles to the tube surfaces reduce the heat flow and steam generation as it offers high thermal resistance than water film. Benson in 1922 argued that if the boiler pressure was raised to critical pressure (225 atm), the steam and water have the same density and therefore, the danger of bubble formation can be easily eliminated. The first high pressure Benson boiler was put into operation in 1927 in West Germany. This boiler too makes use of forced circulation and uses oil as fuel. Its chief novel principle is that it eliminates the latent heat of water by first compressing the feed to a pressure of 235 bar, it is then above the critical pressure and its latent heat is zero.

Figure shows a schematic diagram of a Benson boiler. This boiler does not use any drum. The feed water after circulation through the economic tubes flows through the radiant parallel tube section to evaporate partly. The steam water mixture produced then moves to the transit section where this mixture is converted into steam. The steam is now passed through the convection superheater and finally supplied to the prime mover.

Boilers having as high as 650°C temperature of steam had been put into service. The maximum working pressure obtained so far from commercial Benson boiler is 500 atm. The Benson boilers of 150 tonnes/h generating capacity are in use.



Benson Boiler

Advantages of a Benson Boiler

The Benson boiler posses the following advantages:

1. It can be erected in a comparatively smaller floor area.
2. The total weight of a Benson boiler is 20% less than other boilers, since there are no drums. This also reduces the cost of the boiler.
3. It can be started very quickly because of welded joints.
4. Natural convection boilers require expansion joints but these are not required for Benson boiler as the pipes are welded.
5. The furnace walls of the boiler can be more efficiently protected by using smaller diameter and closed pitched tubes.
6. The transfer of parts of the boiler is easy as no drums are required and majority of the parts are carried to the site without pre-assembly.
7. It can be operated most economically by varying the temperature and pressure at partial loads and overloads. The desired temperature can also be maintained constant at any pressure.
8. The blow-down losses of the boiler are hardly 4% of natural circulation boiler of the same capacity.
9. Explosion hazards are not severe as it consists of only tubes of small diameter and has very little storage capacity.
10. The superheater in a Benson boiler is an integral part of forced circulation system, therefore no special starting arrangement for superheater is required.

Q.88 Discuss the purpose of drum used in boiler and show internal details for mechanism of separation of moisture in drum.

[CSE (Mains) 2010 : 10 Marks]

Solution:

The function of drum used in boiler are:

1. To store water and steam sufficiently to meet varying load requirement.
2. To aid in circulation.
3. To separate steam or vapour from water-steam mixture, discharged by the risers.
4. To provide enough surface area for liquid-vapour disengagement.
5. To maintain a certain desired ppm in the drum water by phosphate injection and blow down.

Separation of steam from steam-water mixture discharged by risers is one of the important functions of drum. At low pressures (upto 20 bar) gravity separation is simply used. But at high pressures a positive means of steam-water separation is required. Mechanical separators like baffles, screens and cyclones which are housed inside the drum for separation of steam-water mixture are known as drum internal. Baffles plates are acting as primary separators. They change or reverse the steam flow direction, thus assisting gravity separation. Screens made of wire mesh act as secondary separators where the individual wires attract and intercept the fine droplets, just as fabric filters attract dust from gases. Cyclone separators utilize the centrifugal forces for separations of the two-phase mixtures, which is entered tangentially to direct the water downward and to make the steam flow upward. The steam then goes through the zig-zag path in corrugated plates, called dryers or scrubber, on the way out to help remove the last traces of moisture.

Q.89 In a steam power plant, the steam generator generates steam at the rate of 120 *t/h* at a pressure of 100 bar and temperature of 500°C. The calorific value of fuel used by steam generator is 41 MJ/kg with an overall efficiency of 85%. In order to have efficient combustion, 17 kg of air per kg of fuel is used for which a draught of 25 mm of water gauge is required at the base of stack. The flue gases leave the steam generator at 240°C. The average temperature of gases in the stack may be taken as 200°C and the atmospheric temperature is 30°C. Work out the following:

(i) The height of stack required.

(ii) The diameter of stack at its base.

(iii) Draw the draught distribution considering balanced draught system in a steam generator and mention the advantages of balanced draught.

Take the following steam properties for solution:

h = 3375 kJ/kg, h_f = 632.2 kJ/kg.

[CSE (Mains) 2010 : 20 Marks]

Solution:

Given: $\dot{m}_s$ = 120 t/h = 33.33 kg/sec, CV = 41 mJ/kg, $\eta_{combustion}$ = 0.85, m = Mass (kg of air per kg fuel)

m = 17, h = Draught of 25 mm of water gauge, T_g = 200°C, T_a = 30°C

(i) Let H be the height of stack required.

$$h = 353H\left[\frac{1}{T_a} - \frac{m+1}{m}\frac{1}{T_g}\right]$$

$$25 = 353H\left[\frac{1}{303} - \frac{18}{17}\frac{1}{473}\right]$$

$$H = 66.7 \text{ m}$$

(ii) Let the diameter of stack at its base be d.

$$\text{Mass of flue gases} = \text{Mass of fuel} + \text{Mass of air}$$

$$m_g = m_F + m_a$$

Energy balance in steam generator,

$$\dot{m}_s(h - h_F) = \dot{m}_F \times CV\,\eta_{combustion}$$

$$\Rightarrow \quad 33.33(3375 - 632.2) = \dot{m}_F \times 41 \times 10^3 \times 0.85$$

$$\Rightarrow \quad \dot{m}_F = 2.62 \text{ kg/sec}$$

$$\therefore \quad \dot{m}_g = 17\dot{m}_F + \dot{m}_F = 18\dot{m}_F = 18 \times 2.62 = 47.16 \text{ kg/sec}$$

Also,

$$\rho_g = 353\left(\frac{m+1}{m}\right)\frac{1}{T_g} = 353\left(\frac{17+1}{17}\right) \times \frac{1}{473} = 0.79 \text{ kg/m}^3$$

$$\text{Volume flow rate of flue gas} = \frac{47.16}{0.79} = 59.69 \text{ m}^3\text{/sec} \qquad \left[\text{As } V = \frac{\dot{m}_g}{\rho_g}\right]$$

$$\text{Flue gas velocity, } V_g = \sqrt{2gH_g}$$

where,

$$H_g = H\left[\frac{m}{m+1}\frac{T_g}{T_a} - 1\right] = 66.7\left[\frac{17}{18} \times \frac{473}{303} - 1\right] = 31.64 \text{ m}$$

$$\Rightarrow \quad V_g = \sqrt{2 \times 9.81 \times 31.64} = 25 \text{ m/sec}$$

$$\text{As volume flow rate of flue gas} = \frac{\pi}{4}d^2 \times V_g = 59.69$$

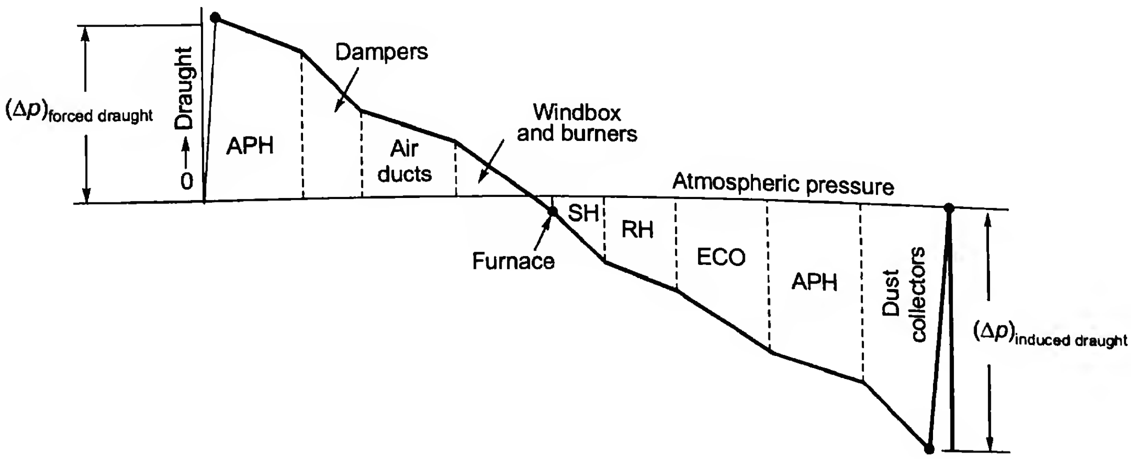
$$\Rightarrow \quad d^2 = \left(\frac{59.69 \times 4}{\pi \times 25}\right) = 3.04152$$

$$\therefore \quad d = 1.74 \text{ m}$$

Advantage of balanced draught: The balanced draught is a combination of forced and induced draught.

1. If the forced draught is used alone, then the furnace cannot be opened either for firing or inspection because the high pressure inside the furnace will try to blow out suddenly and there is every chance of blowing out the fire completely and furnace stops.
2. If the induced draft is used alone, then also furnace cannot be opened because of the cold air will try to rush into furnace as the pressure inside the furnace is below atmospheric pressure.
   This reduces the effective draught and dilutes the combustion. To over come both the difficulties mentioned above either using forced draught or induced draught alone, balanced draught has an advantage of avoiding such problems and is always preferred.

Draught distribution in balanced draught



Q.90 State the factors affecting the amount of draught produced in the boiler and discuss merits and demerits of induced draught system with reference to:

(i) boiler efficiency
(ii) fan maintenance
(iii) power to drive the fan
(iv) leakages.

[CSE (Mains) 2011 : 10 Marks]

Solution:

Large amounts of air are needed for combustion of the fuel. The gaseous combustion product in huge quantity have also to be removed continuously from the boiler furnace. To produce the required flow of either air or combustion gas, a pressure differential is needed. The term draught or draft is used to define the static pressure in the furnace, in the various ducts and the stack. There are two ways of producing draught:

(a) Natural Draught
(b) Mechanical Draught

The various factors affecting the amount of drought produced in the boiler are:

1. Density of atmospheric air.
2. Temperature of atmospheric air.
3. Height of chimney
4. Average gas temperature Tg. Higher is the Tg, higher is the draught produced.
5. Power input to the fans.
6. Efficiency of the FD and ID fans.
7. Exit velocity of the flue Gases.

Merit and Demerits of ID system with reference to

1. **Boiler Efficiency :** Induced draught system handle hot combustion gases and their power requirements are therefore greater, reducing the boiler efficiency.
2. **Fan maintenance :** Because induced draught fans require water cooled bearings and handle hot combustion gases, their maintenance is more severe than forced drought fans.
3. **Power to drive the fan :** ID fans require more power than FD fans.
4. **Leakages :** Tendency to air leak into the boiler furnace is greatly reduced in FD fan leakages of air and gases are more pronounced in ID fans.

Q.91 (i) Mention the unique features of modern high pressure boiler and describe the operation of LaMont boiler with a sketch.

(ii) Representing on h-ϕ diagram, explain supersaturated expansion of steam in a nozzle. Indicate Wilson line on diagram and state effects of supersaturation.

[CSE (Mains) 2011 : 10 Marks]

Solution:

(i) The unique features of high pressure boilers are :

(a) **Method of Water Circulation:** In all modern high pressure boiler plants, the water circulation is maintained with the help of a pump which forces the water through the boiler plant.

(b) **Type of Tubing:** In most of the high pressure boilers, the water circulated through the tubes and their external surfaces are exposed to the flue gases. In most cases, several sets of the tubings are used.

(c) **Improved Method of Heating:** The following improved methods of heating may be used :

(i) The saving of heat by evaporation of water above critical pressure of steam.
(ii) The heating of water can be made by mixing the superheated steam.

LaMont Boiler: **[Refer Question No. 76]**

(ii) Supersaturated expansion of steam in a nozzle **[Refer Question No. 20]**

Q.92 A Rankine cycle based power plant is designed with super heat and reheat:

(i) Sketch T-s diagram.

(ii) Explain the effect of lowering/raising condensor pressure.

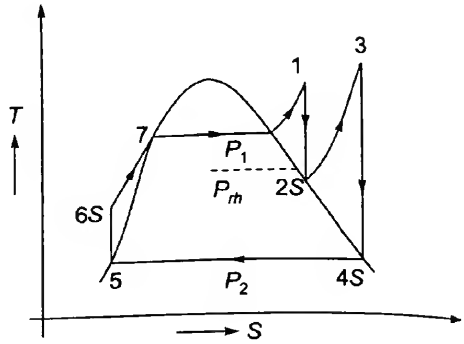
(iii) Explain the effect of lowering/raising boiler pressure.

[CSE (Mains) 2011 : 15 Marks]

Solution:

Consider a Rankine cycle with superheat and reheat:

(i) T – S Diagram

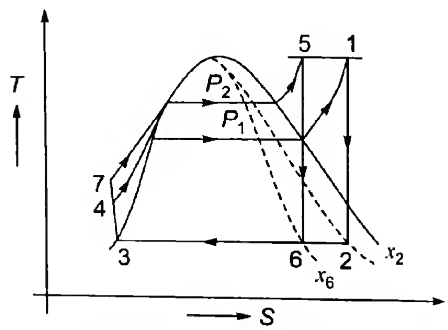


(ii) **Effect of lowering//increasing condenser pressure:** There is a considerable improvement in cycle efficiency with the decrease of condenser pressure. Such a decrease mainly depends on the available cooling water temperature (t_{c1}) and thus on the climatic conditions of the place. A lower cooling water temperature gives lower condenser pressure (higher vacuum).

Thus, with identical steam conditions, cycle and similar equipment, the thermal efficiency of a condensing steam power plant will be less in a warm region than in a cold region.

(iii) **Effect of lowering/Raising boiler pressure:**

The maximum temperature of stream that can be used is fixed from metallurgical considerations, i.e. the materials used for the manufacture of the components which are subjected to the high-temperature steam. It is called the metallurgical limit.



Effect of increase of pressure on Rankine Cycle

When the maximum temperature is fixed by this limit, as the operating boiler pressure at which heat is added in the boiler increases from P_1 to P_2, the mean temperature of heat addition increases. But when the turbine inlet pressure increases from P_1 to P_2, the ideal expansion line of steam shifts to the left and the moisture content at the turbine exhaust increases.

If the moisture content of steam in later stages of turbine is high, the entrained water particles along with the vapour coming out of nozzles with high velocity strike the blades and erode their edges, as a result of which the life of blade decreases, from the consideration of the erosion of blades in later stages of turbine, the maximum moisture content at turbine exhaust is not allowed to exceed 12% or quality of steam to fall below 88%.

Q.93 A boiler receives a flow of 5000 kg/hr liquid water at 5 MPa, 20°C and it adds energy to the flow to exit state of 450°C, 4.5 MPa. Determine the necessary minimum pipe flow area for inlet and outlet pipes. The velocity of water is maintained under 20 m/s. If this boiler is to operate on Moon, discuss two major design changes in this type of boiler, Given properties of water:

$V_{inlet} = 0.001$ m³/kg $\qquad$ $V_{exit} = 0.716$ m³/kg

[CSE (Mains) 2012 : 12 Marks]

Solution:

Given : Mass flow rate of liquid water $\dot{m}$ = 5000 kg/hr = 1.38 kg/sec

Let the minimum pipe flow area for inlet and outlet pipes be A_{inlet} and A_{outlet}.

$$\text{Velocity of water } (V_{max}) = 20 \text{ m/sec.}$$

and
$$v_{inlet} = 0.001 \text{ m}^3/\text{kg}$$

So,
$$\text{Volume flow rate at inlet } (Q_{inlet}) = \dot{m}\, v_{inlet}$$
$$Q_{inlet} = (1.38 \times 0.001) \text{ m}^3/\text{sec} = 1.38 \times 10^{-3} \text{ m}^3/\text{sec}$$

Also, $$Q_{inlet} = A_{inlet} \times V$$

$$\Rightarrow \quad (A_{inlet})_{minimum} = \frac{Q_{inlet}}{V_{max}} = \frac{1.38 \times 10^{-3}}{20} = 70 \text{ mm}^2$$

$$\Rightarrow \quad (V_{inlet})_{actual} = \frac{1.38 \times 10^{-3}}{70 \times 10^{-6}} = 19.71 \text{ m/sec}$$

Similarly, $$V_{exit} = 0.0716 \text{ m}^3\text{/kg}$$

$$Q_{exit} = \dot{m}V_{exit} = (1.38 \times 0.0716) m^3 / \text{sec} = 0.098 \text{ m}^3\text{/sec}$$

$$(A_{exit})_{minimum} = \frac{Q_{exit}}{V_{max}} = \frac{0.098}{20} = 4942 \text{ mm}^2 \qquad [\text{As } Q_{exit} = (A_{exit})_{min} \times V_{max}]$$

$$\Rightarrow \quad (V_{exit})_{actual} = \left(\frac{0.098}{4942 \times 10^{-6}}\right) = 19.83 \text{ m/sec}$$

Q.94 Hot gases inside a chimney are at 430°C and the chimney height is 32 metres. The temperature of outside air is 28°C. The furnace is supplied with 17 kg of air per kg of coal burnt. Calculate:
(i) draught in mm of water; (ii) draught height in metres of hot gases.

[CSE (Mains) 2013 : 10 Marks]

Solution:

Given: T_g(Hot gases temperature) = 430°C, Chimney height (H) = 32 m, T_a = 28°,
m = 17 kg of air per kg of coal burnt.

(i) Draught height in mm of water.

$$h = 353H\left[\frac{1}{T_a} - \left(\frac{m+1}{m}\right)\frac{1}{T_g}\right] = (353 \times 32)\left[\frac{1}{301} - \left(\frac{17+1}{17}\right)\frac{1}{703}\right]$$

$$h = 20.52 \text{ mm}$$

(ii) Draught height in terms of hot gases

Let the height of hot gas column producing the draught be H_g.

$$H_g = H\left[\left(\frac{m}{m+1}\right)\frac{T_g}{T_a} - 1\right] = 32\left[\left(\frac{17}{18}\right) \times \frac{703}{301} - 1\right] = 38.58 \text{ m}$$

Q.95 Determine the expression for the ratio of chimney gas temperature to outside air temperature in terms of mass flow rate.

[CSE (Mains) 2014 : 10 Marks]

Solution:

Mass of lot gases flowing through the chimney

$$\dot{m}_g = \rho_g A V_g$$

Since the density of hot gases is inversely proportional to its temperature,

$$\rho_g = \frac{C_1}{T_g}, \text{ where } C_1 = \text{constant}$$

$$\text{and Velocity of gases } (V_g) = C\sqrt{2gH_g}$$

So, $$\dot{m}_g = \frac{C_1}{T_g} \times A \times C \times \sqrt{2gH_g}$$

where, H_g = hot gas column in m.

As H_g can be written in terms of height of chimney (H) –

$$H_g = H\left[\frac{m}{m+1}\frac{T_g}{T_a} - 1\right] \quad \text{where, } m = \text{mass of air (kg)/kg of fuel}$$

$$\text{Draught } (\Delta P) - gH(\rho_a - \bar{\rho}_g) \quad ...(i)$$

Assuming volume of products of combustion is equal to the volume of air supplied for combustion, both volumes being measures at the same temperature. Thus, volume of 1 kg of flue gas at NTP,

$$v_a = \frac{0.287 \times 273}{101.325} = 0.7733 \text{ m}^3/\text{kg} \quad \left[\text{as } V_a = \frac{RT}{P}\right]$$

Volume of m kg of air per kg fuel at temperature T_a.

$$v_a = (0.7733\,m)\frac{T_a}{273}$$

$$\Rightarrow \quad \rho_a = \frac{1}{0.7733} \times \frac{273}{T_a} = 1.293\left(\frac{273}{T_a}\right)\text{kg/m}^3$$

The mass of the flue gas will be $(m + 1)$ kg and its temperature is T_g.

$$\rho_g = \frac{m+1}{0.7733 \times m \times (T_g/273)} = 1.293\left(\frac{273}{T_g}\right)\left(\frac{m+1}{m}\right) = \frac{353}{T_g}\left(\frac{m+1}{m}\right)$$

From (i)

Draught produced

$$(\Delta P) = 353gH\left[\frac{1}{T_a} - \frac{m+1}{m}\frac{1}{T_g}\right] \quad ...(ii)$$

Also,

$$(\Delta P) = \rho_g\, gH_g = \frac{353}{T_g}\left(\frac{m+1}{m}\right)gH_g \quad ...(iii)$$

From (ii) and (iii)

$$\frac{353}{T_g}\left(\frac{m+1}{m}\right)gH_g = 353\,gH\left[\frac{1}{T_a} - \left(\frac{m+1}{m}\right)\frac{1}{T_g}\right]$$

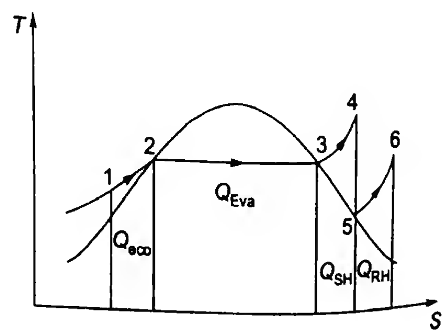
$$H_g = H\left(\frac{m}{m+1}\frac{T_g}{T_a} - 1\right) \text{ or } \frac{T_g}{T_a} = \left(\frac{H_g}{H} + 1\right)\left(\frac{m+1}{m}\right)$$

Q.96 Explain with a sketch how heat is absorbed at various stages from feed water to steam generation in water tube boilers.

[CSE (Mains) 2015 : 15 Marks]

Solution:

In a water-tube boiler, feedwater is heated, in three kinds of heat exchanger i.e. economiser, evaporator (downcomer-riser circuit) and superheaters. Feedwater from the h.p. heater enters the economiser where it is heated by the outgoing flue gases till it is saturated liquid at that pressure and then it is fed to the drum. Saturated water falls through the downcomer into the bottom header and moves up through the riser where water is partially boiled back into the drum.

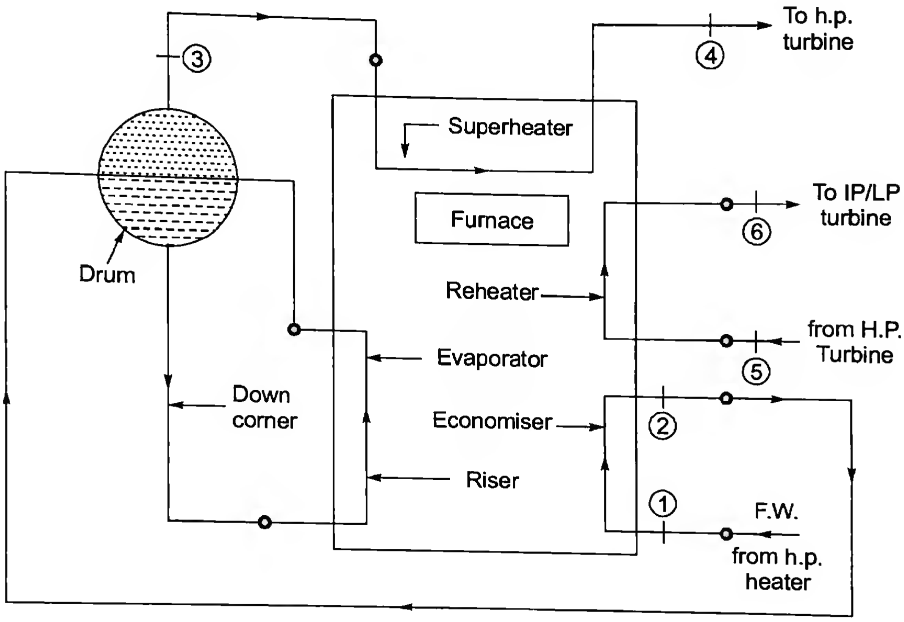


Saturated steam from the drum goes to the superheaters for being heated to the desired temperature. For each kg of steam formed, the heats absorbed in the economiser (in the liquid phase), in the

evaporator (liquid to vapour transition) and in the superheater (in the vapour or gas phase) is given by

$$Q_{eco} = h_2 - h_1$$
$$Q_{eva} = h_3 - h_2$$
$$Q_{SH} = h_4 - h_3$$
$$Q_{RH} = h_6 - h_5$$

The use of large number of feedwater heaters means a smaller economiser and a high pressure means a smaller boiler surface (risers). Thus a modern high pressure steam generators require more superheating and reheating surfaces and less boiler surface.



Heat Absorption in a water tube boiler

Q.97 What is the function of an economiser in a thermal power plant? Why are the economiser tubes often provided with fins on the gas side? Explain.

[CSE (Mains) 2016 : 10 Marks]

Solution:

An economizer is a heat exchanger which raises the temperature of the feed water leaving the highest pressure feedwater heater to about the saturation temperature corresponding to the boiler pressure. This is done by the hot flue gases exiting the last superheater or reheater at a temperature varying from 370°C to 540°C. Economizers makes the utilization of hot flue gases in heating feedwater higher efficiency and better economy and hence this heat exchanger is known as economizer. Modern economizers are open designed to allow some boiling of the feedwater in the outlet sections, upto 20 per cent quality at full power, less at part loads. They are often termed as "steaming economizers".

Economizer tubes are commonly 45-70 mm in outside diameter and are made in vertical coils of continuous tubes connected between inlet and outlet headers with each section formed into several horizontal paths connected by 180° vertical bends. The coils are installed at a pitch of 45 of 50 mm spacing, which depends on the type of fuel and ash characteristics.

The gas-side heat transfer coefficient is much less than the water-side heat transfer coefficient. To compensate this, the outer surface of the tubes (Gas-side) are often provided with fins to increase the surface area of the heat transfer and improves effectiveness of economizer.

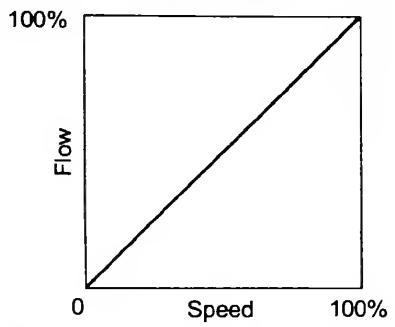
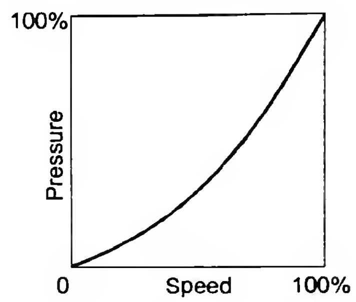
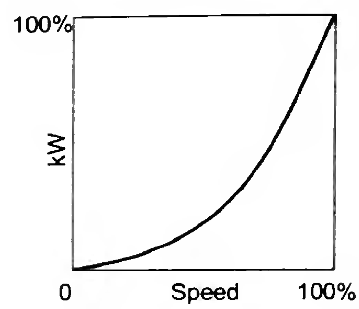
7. Compressible Flow

Q.98 What are the fan laws? Discuss atleast three methods which are used to control fan output (Q) in a power plant.

[CSE (Mains) 2001 : 20 Marks]

Solution:

The Fan Laws are the basic proportional relationships between fan speed, flow, pressure, and power. They are most useful for determining the impact of extrapolating from a known fan performance to a desired performance. The most common change made to a fan is that of altering its rotational speed. The fans operate under a predictable set of laws concerning speed, power and pressure. A change in speed (RPM) of any fan will predictably change the pressure rise and power necessary to operate it at the new RPM

Flow α Speed	**Pressure α (Speed)²**	**Power α (Speed)³**
		
$\frac{Q_1}{Q_2} = \frac{N_1}{N_2}$	$\frac{SP_1}{SP_2} = \left(\frac{N_1}{N_2}\right)^2$	$\frac{kW_1}{kW_2} = \left(\frac{N_1}{N_2}\right)^3$
Varying the RPM by 10% decreases or increases air delivery by 10%	Reducing the RPM by 10% decreases the static pressure by 19% and an increase in RPM by 10% increases the static pressure by 21%	Reducing the RPM by 10% decreases the power requirement by 27% and an increase in RPM by 10% increases the power requirementby 33%

Where Q– flow, SP – Static pressure, kW – Power and N-speed (RPM)

Once a fan system is designed and installed, the fan operates at a constant speed. There may be occasions when a speed change is desirable, i.e., when adding a new run of duct that requires an increase in air flow (volume) through the fan. There are also instances when the fan is oversized and flow reductions are required. Following three methods can be used to control fan output in a power plant.

1. **Pulley Change :** When a fan volume change is required on a permanent basis, and the existing fan can handle the change in capacity, the volume change can be achieved with a speed change. The simplest way to change the speed is with a pulley change. For this, the fan must be driven by a motor through a V belt system. The fan speed can be increased or decreased with a change in the drive pulley or the driven pulley or in some cases, both pulleys.

2. **Damper Controls:** Some fans are designed with damper controls . Dampers can be located at inlet or outlet. Dampers provide a means of changing air volume by adding or removing system resistance. This resistance forces the fan to move up or down along its characteristic curve, generating more or less air without changing fan speed. However, dampers provide a limited amount of adjustment, and they are not particularly energy efficient.

3. **Variable Speed Drives:** Although, variable speed drives are expensive, they provide almost infinite variability in speed control.

Variable speed operation involves reducing the speed of the fan to meet reduced flow requirements. Fan performance can be predicted at different speeds using the fan laws. Since power input to the fan changes as the cube of the flow, this will usually be the most efficient form of capacity control. However, variable speed control may not be economical for systems, which have infrequent flow variations. When considering variable speed drive, the efficiency of the control system (fluid coupling, eddy-current, VFD, etc.) should be accounted for, in the analysis of power consumption.

Q.99 Explain clearly what do you understand by Fanno flow. Show its plot on *h* - *s* diagram and give its characteristics.

Air flows in an insulated duct with a Mach number of 0.2. The initial temperature and pressure are 290 K and 20 bar respectively. Determine:

(i) the pressure and temperature at a section of the duct where the Mach number is 0.8.

(ii) the distance between these two points if the duct diameter is 10 cm and friction factor is 0004.

(iii) what will be the maximum length of the duct to avoid choking?

Use the following table:

M	P/P*	T/T*	$4\bar{f}L_{max}/D$
0.2	5.455	1.19	14.533
0.8	1.289	1.064	0.073

[CSE (Mains) 2005 : 30 Marks]

or

Show Fanno line in adiabatic flow with friction on *h-s* diagram and explain the physical significance.

[CSE (Mains) 2010 : 20 Marks]

or

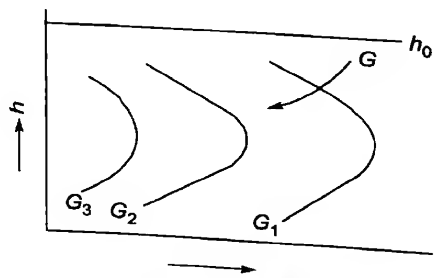
Explain what you mean by Fanno flow. Clearly mention the assumptions made and governing equations involved in the Fanno flow.

[CSE (Mains) 2016 : 10 Marks]

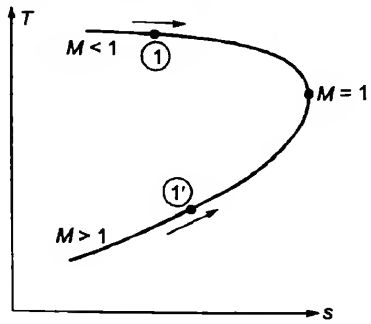
Solution:

Theory Part: Fanno Line:

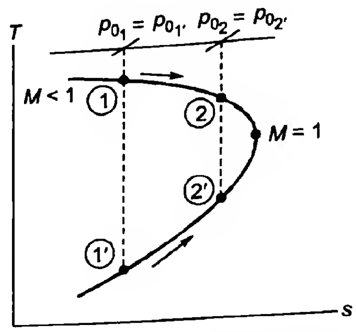
- The line representing the locus of points with the same mass velocity and stagnation enthalpy is called a Fanno line.
- It is a one-dimensional model for adiabatic flow in a constant area duct with friction.
- Steady, 1-d, constant area, adiabatic flow with no external work but with friction
- It is a combination of continuity and energy equations.
- Conserved quantities during Fanno flow:
  - since adiabatic, no work: h_o = constant
  - since Area =constant ; mass flux = ρv = constant
  - combining: $h_o = h + (\rho v)^2/2\rho$ =constant
- Velocity change (due to friction) associated with entropy change
- Friction can only increase entropy leading to maximum entropy when $M = 1$



Fanno line on h-s diagram



Schematic T-s diagram for frictional adiabatic (Fanno-line) in a constant-area duct.

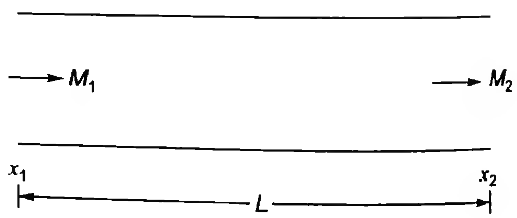


Schematic of Fanno-line flow on T-s plane, showing reducing inlocal isentropic stagnation pressure caused by friction

Property	Subsonic $M < 1$	Supersonic $M < 1$	Obtained from:
Stagnation temperature	T_0 = Constant	T_0 = Constant	Energy equation
Entropy	$s\uparrow$	$s\uparrow$	$T\,ds$ equation
Stagnation pressure	$p_0\downarrow$	$p_0\downarrow$	T_0 = constant; $s\uparrow$
Temperature	$T\downarrow$	$T\uparrow$	Shape of Fanno line
Velocity	$V\uparrow$	$V\downarrow$	Energy equation, and trend of T
Mach number	$M\uparrow$	$M\downarrow$	Trends of V, T and definition of M
Density	$\rho\downarrow$	$\rho\uparrow$	Continuity equation, and effect on V
Pressure	$p\downarrow$	$p\uparrow$	Equation of state and effects on ρ, T

Numerical Part

(i)
$$P_2 = \frac{(P/P^*)_2}{(P/P^*)_1}P_1 = \frac{1.289}{5.455}\times 2 = \mathbf{0.4726\ bar}$$



$$T_2 = \frac{(T/T^*)_2}{(T/T^*)_1}T_1 = \frac{1.064}{1.19}\times 290 = \mathbf{259.3\ K}$$

(ii) As,
$$\frac{4\bar{f}L}{D} = \left(\frac{4\bar{f}L_{max}}{D}\right)_{M_1=0.2} - \left(\frac{4\bar{f}L_{max}}{D}\right)_{M_2=0.8} = 14.533 - 0.073$$

$$\Rightarrow \quad \frac{4\bar{f}L}{D} = 14.46$$

$$\Rightarrow \quad \text{Length, } L = \frac{14.46\times 0.1}{4\times 0.004} = \mathbf{90.375\ meters}$$

[D = 10 cm, f = 0.004]

(iii) Limiting value of Mach number is unity to avoid the chocking,

$$\frac{4\bar{f}L}{D} = \left(\frac{4\bar{f}L}{D}\right)_{M_1=0.2} - \left(\frac{4\bar{f}L}{D}\right)_{M_2=1}$$

$$\Rightarrow \quad \frac{4\bar{f}L_{max}}{D} = 14.533 - 0\ ;\ L_{max} = \frac{14.533\times 0.1}{4\times 0.004} = \mathbf{90.83\ meters}$$

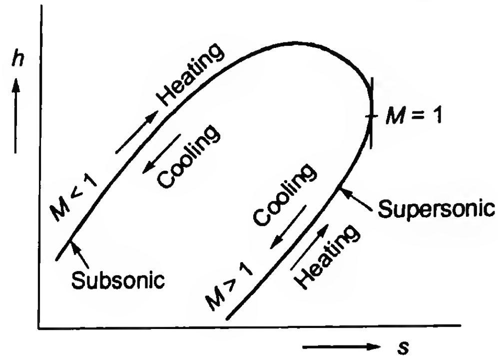
Q.100 Define Rayleigh flow. Give one practical example of Rayleigh flow.

Show that the Mach numbers at the maximum enthalpy and maximum entropy points on the Rayleigh line are $\frac{1}{\sqrt{\gamma}}$ and 1.0 respectively.

[CSE (Mains) 2006 : 6 + 14 = 20 Marks]

Solution:

Rayleigh flow: Rayleigh flow refers to frictionless, non-adiabatic flow through a constant area duct where the effect of heat addition or rejection is considered. Rayleigh flow is a model describing a frictionless flow with heat transfer through a pipe of constant cross sectional area.



Practical Example of Rayleigh Flow: Heat transfer process in heat exchangers and combustion chambers by ignoring frictional effects.

Part 2

Governing Equation

The energy balance on the control volume reads

$$Q = C_p(T_{02} - T_{01})$$

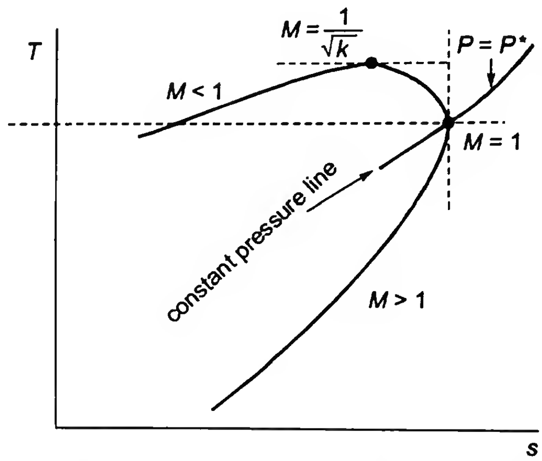
The momentum balance reads,

$$A(P_1 - P_2) = \dot{m}(V_2 - V_1)$$

The mass conservation reads, $\rho_1 U_1 A = \rho_2 U_2 A = \dot{m}$

Equation of state, $\frac{P_1}{\rho_1 T_1} = \frac{P_2}{\rho_2 T_2}$

$$\frac{P_2}{P_1} = \frac{1 + kM_1^2}{1 + kM_2^2}$$



The temperature Entropy Diagram For Rayleigh Line

The density ratio can be expressed in term of mass conservation as,

$$\frac{\rho_1}{\rho_2} = \frac{U_2}{U_1} = \frac{\frac{U_2}{\sqrt{kRT_2}}\sqrt{kRT_2}}{\frac{U_1}{\sqrt{kRT_1}}\sqrt{kRT_1}} = \frac{M_2}{M_1}\sqrt{\frac{T_2}{T_1}}$$

$$\frac{T_2}{T_1} = \frac{1 + kM_1^2}{1 + kM_2^2}\frac{M_2}{M_1}\sqrt{\frac{T_2}{T_1}}$$

Transferring the temperature ratio to left hand side and squaring results in,

$$\frac{T_2}{T_1} = \left[\frac{1 + kM_1^2}{1 + kM_2^2}\right]\left(\frac{M_2}{M_1}\right)^2$$

The Rayleigh line exhibits two possible maximums one for $dT/ds = 0$ and for $ds/dT = 0$. The second maximum can be expressed as $dT/ds = \infty$ The second law is used to find the expression for derivative.

$$\frac{s_1 - s_2}{C_p} = \ln\frac{T_2}{T_1} - \frac{k-1}{k}\ln\frac{P_2}{P_1} = 2\ln\left[\frac{(1 + kM_1^2)}{(1 + kM_2^2)}\frac{M_2}{M_1}\right] + \frac{k-1}{k}\ln\left[\frac{1 + kM_2^2}{1 + kM_1^2}\right]$$

Let the initial condition M_1 and s_1 are constant then the variable parameters are M_2 and s_2. A derivative of equation results in

$$\frac{1}{C_p}\frac{ds}{dM} = \frac{2(1-M^2)}{M(1+kM^2)}$$

Take the derivative of the equation when letting the variable parameters be T_2, and M_2 results in

$$\frac{dT}{dM} = \text{constant} \times \frac{1-kM^2}{(1+kM^2)^3}$$

Combining equations and by eliminating dM results in

$$\frac{dT}{ds} = \text{constant} \times \frac{M(1-kM^2)}{(1-M^2)(1+kM^2)^2}$$

On T-s diagram a family of curves can be drawn for a given constant. For every curve, several observations can be generalized. The derivative is equal to zero when $1 - kM^2 = 0$ or $M = 1/\sqrt{k}$ or when $M \to 0$. The derivative is equal to infinity, $dT/ds = \infty$ when $M = 1$. From thermodynamics, increase of heating results in increase of entropy. And cooling results in reduction of entropy. Hence, when cooling applied to a tube the velocity decreases and heating applied the velocity increases. The peculiars point of $M = 1/\sqrt{k}$ when additional heat is applied the temperature is decreasing. The derivative is negative, $dT/ds < 0$, yet not this point is not the choking point. The chocking is occurred only when $M = 1$ because it violates the second law. The transition to supper sonic flow occurs when the area changes, some what similarly to Fanno flow, yet, chocking can be explained by the fact increase of energy must be accompanied by increase of entropy. But the entropy of supersonic flow is lower and therefore it is not possible (the maximum entropy at $M = 1$).

Q.101 With the help of diagrams, explain the following:

(i) Normal shock **(ii) Oblique shock**

(iii) Attached shock and **(iv) Detached shock**

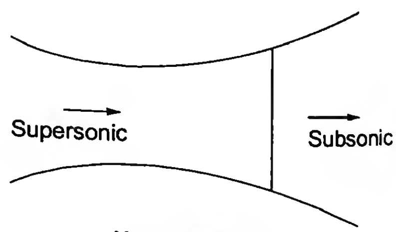
Explain why shock-waves cannot develop in a subsonic flow.

[CSE (Mains) 2008 : 20 Marks]

Solution:

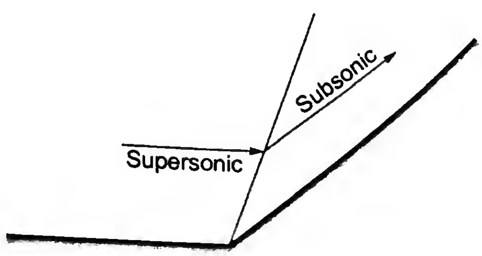
(i) Normal shock: Normal shocks are a fundamental type of shock waves. The waves which are perpendicular to the flow, are called normal shocks. Normal shocks only happen when the flow is supersonic.

These shocks decelerate the flow to subsonic speeds. These shock waves are highly localised irreversibilities in the flow. Within the distance of mean free path of a molecule, the flow passes from a supersonic to a subsonic state, the velocity decreases abruptly, and the pressure rises sharply.



Normal shock

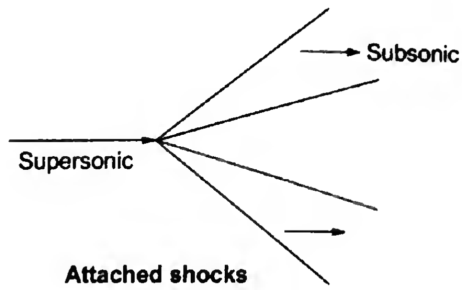
(ii) Oblique shocks: An oblique shock wave unlike a normal shock, is inclined with respect to the incident upstream flow direction. It will occur when a supersonic flow encounters a corner that effectively turns the flow into itself and compresses. The upstream streamlines are uniformly deflected after the shock wave. The most common way to produce an oblique shock wave is to place a wedge into supersonic, compressible flows.



Similar to a normal shock wave, the oblique shock wave consists of a very thin region across which nearly discontinuous changes in the thermodynamic properties of gas occurs. While the upstream and downstream flow directions are unchanged across a normal shock, they are different for flow across an oblique shock wave. It is always possible to convert an oblique shock wave into a normal shock by Galilean transformation.

(iii) **Attached shock:** These shocks appear as "attached" to the tip of a sharp body moving at supersonic speeds. Examples are supersonic wedges and cones with small apex angle. if the wedge angle is less than this detachment angle, an attached oblique shock occurs.

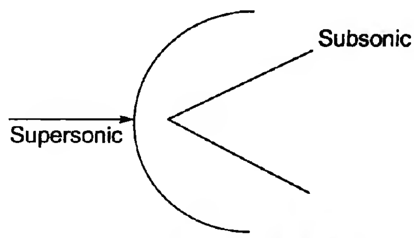
(iv) **Detached shock:** Detached shocks are curved and form a small distance in front of the body. Directly in front of the body, they stand at 90° to the on coming flow and then curve around the body. These follow the "strong-shock" solutions of the analytic equations, meaning that for some oblique shocks very close to the deflection angle limit, the downstream Mach number is subsonic.



Attached shocks

A detached shock wave occurs when the maximum deflection angle is exceeded. A detached shock is commonly seen on blunt bodies. Examples of detached shock include space return vehicles, bullets, Bow shock of a magnetosphere, etc.

Shock waves cannot develop in subsonic flows. Shocks are introduced to increase the pressure and hence it is a deceleration process. Therefor shocks are possible only when the fluid velocity is maximum. In a subsonic flow, the velocity of fluid is less than the critical velocity and hence deceleration is not possible. Thus, shock waves cannot develop in subsonic flow.



Detached shock

Q.102 Derive an expression for the Mach number after a normal shock wave occurring in a nozzle. Show the trend of this Mach number, (in the form of an *x-y* plot) with respect to the Mach number value before the shock.

[CSE (Mains) 2009 : 15 Marks]

Solution:

Expression of the Mach number after a normal shock in a nozzle:

Let a normal shock wave is occurring in a nozzle.

M_y is Mach number after the shock M_x is Mach number before the shock.

Similarly, P_x, T_x be the pressure and temperature just before the shock. P_y, T_y be the pressure and temperature just after the shock.

As,
$$\rho_x V_x = \rho_y V_y \text{ (continuity equation)}$$

$$\frac{P_x}{RT_x}V_x = \frac{P_y}{RT_y}V_y$$

$$\frac{T_y}{T_x} = \frac{P_y}{P_x}\frac{V_y}{V_x} = \frac{P_y}{P_x}\frac{M_yC_y}{M_xC_x} = \frac{P_y}{P_x}\frac{M_y}{M_x}\sqrt{\frac{T_y}{T_x}} \qquad [\text{As } C = \sqrt{\gamma RT}\,]$$

$$\frac{T_y}{T_x} = \left(\frac{P_y}{P_x}\right)^2\left(\frac{M_y}{M_x}\right)^2 \qquad \text{...(i)}$$

Also,
$$\frac{T_y}{T_x} = \frac{1+\left(\frac{\gamma-1}{2}\right)M_x^2}{1+\left(\frac{\gamma-1}{2}\right)M_y^2} \qquad \text{...(ii)}$$

Putting value of $\left(\frac{T_y}{T_x}\right)$ from (ii) in (i),

$$\frac{P_y}{P_x} = \frac{M_x}{M_y}\sqrt{\frac{1+\left(\frac{\gamma-1}{2}\right)M_x^2}{1+\left(\frac{\gamma-1}{2}\right)M_y^2}} \quad \text{...(iii)}$$

Since,

$$P_x + \rho_x V_x^2 = P_y + \rho_y V_y^2$$

$$\Rightarrow \quad P_x + \frac{\gamma P_x}{\gamma R T_x} V_x^2 = P_y + \frac{\gamma P_y}{\gamma R T_y} V_y^2$$

$$\Rightarrow \quad P_x\left(1+\gamma M_x^2\right) = P_y(1+\gamma M_y^2)$$

or,

$$\frac{P_y}{P_x} = \frac{1+\gamma M_x^2}{1+\gamma M_y^2} \quad \text{...(iv)}$$

From (iii) and (iv),

$$\frac{M_x}{M_y}\sqrt{\frac{1+\left(\frac{\gamma-1}{2}\right)M_x^2}{1+\left(\frac{\gamma-1}{2}\right)M_y^2}} = \frac{1+\gamma M_x^2}{1+\gamma M_y^2}$$

$$\Rightarrow \quad \left(\frac{M_x}{M_y}\right)^2\left(\frac{1+\left(\frac{\gamma-1}{2}\right)M_x^2}{1+\left(\frac{\gamma-1}{2}\right)M_y^2}\right) = \left(\frac{1+\gamma M_x^2}{1+\gamma M_y^2}\right)^2$$

or

$$\frac{1+\left(\frac{\gamma-1}{2}\right)M_x^2}{1+\left(\frac{\gamma-1}{2}\right)M_y^2} = \left[\frac{M_y}{M_x}\times\left(\frac{1+\gamma M_x^2}{1+\gamma M_y^2}\right)\right]^2$$

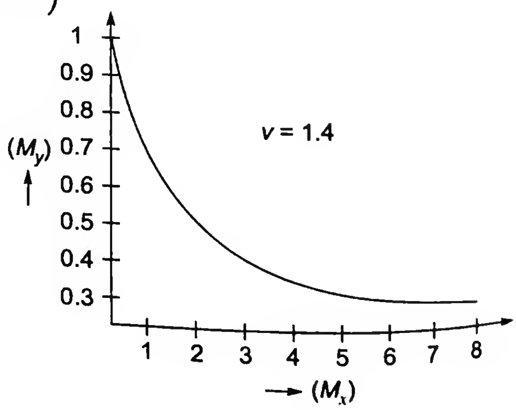
or

$$\left(M_x+\gamma M_x M_y^2\right)^2\left(1+\left(\frac{\gamma-1}{2}\right)M_x^2\right) = \left(M_y+\gamma M_x^2 M_y\right)^2\left(1+\frac{\gamma-1}{2}M_y^2\right)$$

Rearranging this equation, we get

$$M_y^2 = \frac{M_x^2+\frac{2}{\gamma-1}}{\frac{2\gamma}{\gamma-1}M_x^2-1}$$

Variation of downstream Mach No. (M_y) with respect to upstream Mach No. (M_x).



Q.103 For normal shock wave derive the following expression:

$$\frac{P_{oy}}{P_x} = \left[1+\frac{\gamma-1}{2}M_x^2\right]^{\gamma/(\gamma-1)} \cdot \left[\frac{2\gamma}{\gamma+1}M_x^2 - \frac{\gamma-1}{\gamma+1}\right]$$

where x and y are the conditions before and after the shock wave.

[CSE (Mains) 2010 : 10 Marks]

Solution:

For a normal shock wave:

To derive:
$$\frac{P_{oy}}{P_x} = \left[1-\frac{\gamma-1}{2}M_y^2\right]^{\frac{\gamma}{\gamma-1}} \cdot \left[\frac{2\gamma}{\gamma+1}M_x^2 - \frac{\gamma-1}{\gamma+1}\right]$$

where x and y are the conditions before and after the shock wave.

As,
$$\frac{P_{oy}}{P_x} = \frac{P_{oy}}{P_y}\cdot\frac{P_y}{P_x} \quad \text{... (i)}$$

$\Rightarrow$
$$\frac{P_{oy}}{P_y} = \left(\frac{T_{oy}}{T_y}\right)^{\frac{\gamma}{\gamma-1}} \quad \text{[For adiabatic process]}$$

and
$$\frac{T_{oy}}{T_y} = 1+\left(\frac{\gamma-1}{2}\right)M_y^2$$

So,
$$\frac{P_{oy}}{P_y} = \left[1+\left(\frac{\gamma-1}{2}\right)M_y^2\right]^{\frac{\gamma}{\gamma-1}} \quad \text{... (ii)}$$

Also,
$$\frac{P_y}{P_x} = \frac{1+\gamma M_x^2}{1+\gamma M_y^2} \quad \text{... (iii)}$$

and
$$M_y^2 = \frac{M_x^2 + \frac{2}{\gamma-1}}{\frac{2\gamma}{\gamma-1}M_x^2 - 1} \quad \text{... (iv)}$$

Putting the value of M_y^2 from equation (iv) in equation (iii), we get

$$\frac{P_y}{P_x} = \frac{1+\gamma M_x^2}{1+\gamma\left[\frac{M_x^2+\frac{2}{\gamma-1}}{\frac{2\gamma}{\gamma-1}M_x^2-1}\right]} = \frac{(1+\gamma M_x^2)\times\left(\frac{2\gamma}{\gamma-1}M_x^2-1\right)}{\left(\frac{2\gamma}{\gamma-1}M_x^2-1+\gamma M_x^2+\frac{2\gamma}{\gamma-1}\right)}$$

$$= \frac{(1+\gamma M_x^2)\times\left(\frac{2\gamma}{\gamma-1}M_x^2-1\right)}{\left(\frac{2\gamma}{\gamma-1}+\gamma\right)M_x^2+\left(\frac{2\gamma}{\gamma-1}-1\right)} = \frac{(1+\gamma M_x^2)\times\left(\frac{2\gamma}{\gamma-1}M_x^2-1\right)}{\frac{\gamma(\gamma+1)}{\gamma-1}M_x^2+\left(\frac{\gamma+1}{\gamma-1}\right)}$$

$$= \frac{(1+\gamma M_x^2)\times\left(\frac{2\gamma}{\gamma-1}M_x^2-1\right)}{\left(\frac{\gamma+1}{\gamma-1}\right)(\gamma M_x^2+1)} = \left(\frac{2\gamma}{\gamma+1}\right)M_x^2-\left(\frac{\gamma-1}{\gamma+1}\right) \quad \text{... (v)}$$

Putting the values of $\left(\frac{P_{oy}}{P_x}\right)$ from equation (v) and $\left(\frac{P_{oy}}{P_y}\right)$ from equation (ii) in equation (i)

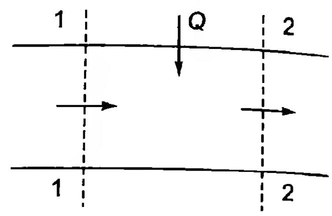
$$\frac{P_{oy}}{P_x} = \frac{P_{oy}}{P_y}\times\frac{P_y}{P_x} = \left[1+\left(\frac{\gamma-1}{2}\right)M_y^2\right]^{\frac{\gamma}{\gamma-1}}\cdot\left[\frac{2\gamma}{\gamma+1}M_x^2-\frac{\gamma-1}{\gamma+1}\right]$$

Q.104 Air flow is entering to a frictionless duct of 0.3 m diameter at a velocity of 580 m/s and a Mach number of 2. A 100 kJ/kg of heat is added to the flow and Mach number of 1.2 is attained. Determine
(i) change in enthalpy of flow,
(ii) change in kinetic-energy of the flow,
(iii) change in static pressure in terms of inlet pressure
Comment on result and show the process on T-f diagram.

[CSE (Mains) 2011 : 20 Marks]

Solution:

Given: Air flow through frictionless duct with Heat Transfer (Rayleigh flow)
Inlet conditions are



$$V_1 = 580 \text{ m/sec}$$
$$M_1 = 2$$
$$\text{Diameter, } D_1 = D_2 = 0.3 \text{ m}$$
$$Q_{1-2} = 100 \text{ kJ/kg} \quad \text{(Heat addition)}$$

Outlet conditions: $M_2 = 1.2$

From 1st law of thermodynamics,

$$h_1 + \frac{V_1^2}{2} + Q = h_2 + \frac{V_2^2}{2}$$

and

$$\frac{P_2}{P_1} = \frac{1+\gamma M_1^2}{1+\gamma M_2^2} = \frac{1+(1.4\times 2^2)}{1+(1.4\times 1.2^2)}$$

$$\frac{P_2}{P_1} = \frac{6.6}{3.016} = 2.188$$

(iii) Change in static pressure in terms of inlet pressure,

$$P_2 - P_1 = (2.188\,P_1 - P_1) = 1.188\,P_1$$

Also,

$$M_1 = \frac{V_1}{\sqrt{\gamma R T_1}}$$

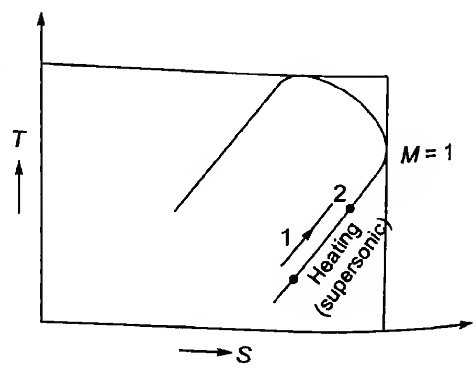
$$2 = \frac{580}{\sqrt{1.4\times 287\times T_1}} \qquad \text{[As } M_1 = 2\text{]}$$

$$\Rightarrow \quad T_1 = 209.3 \text{ K}$$

As,

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)\left(\frac{M_2}{M_1}\right)^2 = (2.188)^2\left(\frac{1.2}{2}\right)^2 = 1.723$$

$$\Rightarrow \quad T_2 = 1.723 \times 209.3 = 360.72 \text{ k}$$



Also

$$M_2 = \frac{V_2}{\sqrt{\gamma R T_2}}$$

$$\Rightarrow \quad V_2 = 456.85 \text{ m/sec}$$

(i) Change in enthalpy $= (h_2 - h_1)$

$$= Q + \left(\frac{V_1^2 - V_2^2}{2}\right) = 100 + \left(\frac{580^2 - 456.85^2}{2000}\right)$$

$$= 163.85 \text{ kJ/kg}$$

(ii) Change in kinetic energy of flow,

$$\left(\frac{V_1^2 - V_2^2}{2}\right) = \left(\frac{580^2 - 456.85^2}{2000}\right) = 63.85 \text{ kJ/kg}$$

Q.105 How would you define the strength of shock wave? What do you mean by weak shock? Also find the expression for the strength of shock in terms of density ratio.

[CSE (Mains) 2012 : 10 Marks]

Solution:

The strength of the shock wave is defined as the ratio of the pressure increase to the initial pressure, i.e.

$$P(\text{strength}) = \frac{P_y - P_x}{P_x} = \frac{P_y}{P_x} - 1$$

Where,

P_y = Pressure downstream of shock
P_x = Pressure upstream of shock

Weak shocks are shocks across which the differences in velocity, pressure and density are all very small. Thus, it can be approximated as ($V_x \approx V_y$), ($V_y - V_x = dv$), ($\rho_y \approx \rho_x$) ($\rho_y - \rho_x = d\rho$), etc. These weak shocks are almost isentropic (reversible).

Now,

$$P(\text{strength}) = \frac{P_y}{P_x} - 1 \quad \ldots \text{(i)}$$

Also according to Rankine - Hugoniot Equation,

$$\frac{P_y}{P_x} = \frac{\left(\frac{\gamma+1}{\gamma-1}\right)\left(\frac{\rho_y}{\rho_x}\right) - 1}{\left(\frac{\gamma+1}{\gamma-1}\right) - \frac{\rho_y}{\rho_x}} \quad \ldots \text{(ii)}$$

Putting value of $\left(\frac{P_y}{P_x}\right)$ in equation (i) strength of shock wave in terms of density ratio,

$$P_{(\text{strength})} = \frac{\left(\frac{\gamma+1}{\gamma-1}\right)\left(\frac{\rho_y}{\rho_x}\right) - 1}{\left(\frac{\gamma+1}{\gamma-1}\right) - \frac{\rho_y}{\rho_x}} - 1$$

$$P_{(\text{strenth})} = \frac{\left(\frac{\gamma+1}{\gamma-1}\right)\left(\frac{\rho_y}{\rho_x}\right) - 1 - \left(\frac{\gamma+1}{\gamma-1}\right) + \frac{\rho_y}{\rho_x}}{\left(\frac{\gamma+1}{\gamma-1}\right) - \frac{\rho_y}{\rho_x}} = \frac{\left(\frac{\gamma+1}{\gamma-1}\right)\left(\frac{\rho_y}{\rho_x} - 1\right) + \left(\frac{\rho_y}{\rho_x} - 1\right)}{\left(\frac{\gamma+1}{\gamma-1}\right) - \frac{\rho_y}{\rho_x}}$$

$$P = \frac{\left(\frac{\gamma+1}{\gamma-1} + 1\right)\left(\frac{\rho_y}{\rho_x} - 1\right)}{\left(\frac{\gamma+1}{\gamma-1}\right) - \frac{\rho_y}{\rho_x}} = \frac{\left(\frac{2\gamma}{\gamma-1}\right)\left(\frac{\rho_y}{\rho_x} - 1\right)}{\left(\frac{\gamma+1}{\gamma-1}\right) - \frac{\rho_y}{\rho_x}}$$

Q.106 A stream of air flows in an insulated tube of constant cross-sectional area of 0.9 m. At a section 1, the pressure is 0.6 bar, temperature is 22°C and mass velocity is 150 kg/s-m². The pressure in space in which tube exhausts is so low that choking condition prevails.

Determine,

(i) Mach no. at section 1.

(ii) Mach no., temperature and pressure at the exit of tube.

(iii) Total force exerted in axial direction which must be exerted to hold the tube stationary.

Given, R = 287 J/kg-K $\gamma = 1.4$

Fanno line (Adiabatic constant area flow with friction)

table $\gamma = 1.4$

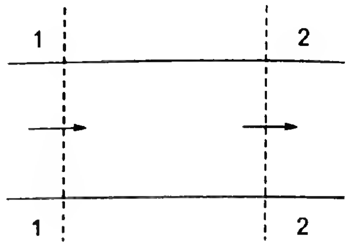
M	T/T^*	e/e^*	p/p^*	p_0/p_0^*	I/I^*	$4fL_{max}/D$
0.58	1.124	0.615	1.828	1.213	1.121	0.576
0.60	1.119	0.635	1.763	1.188	1.105	0.491
0.62	1.114	0.654	1.703	1.166	1.091	0.471
0.64	1.109	0.674	1.646	1.145	1.079	0.353

[CSE (Mains) 2012 : 20 Marks]

Solution:

Given: Area = 0.9 m^2.

At section 1: Pressure (P_1) = 0.6 bar, T_1=22°C or 295 K, (ρv) mass velocity = 150 kg/s-m^2



$$P = \rho RT$$

As, $$P_1 = \rho_1 RT_1 \quad \text{[From ideal gas equation]}$$

$\Rightarrow$ $$\rho_1 = \frac{P_1}{RT_1} = \frac{0.6\times10^5}{(287)(295)} = 0.708 \text{ kg/m}^3$$

Mass velocity, $$\rho_1 V_1 = 150$$

$\Rightarrow$ $$V_1 = \frac{150}{\rho_1} = \frac{150}{0.708} = 211.6625 \text{ m/sec}$$

As, $$M_1 = \frac{V_1}{\sqrt{\gamma RT_1}}$$

(i) $$M_1 = \frac{211.6625}{\sqrt{1.4\times287\times295}} = 0.62$$

(ii) As the choking conditions prevails at exit, $M_2 = 1$

From chart, corresponding to (M = 0.62)

$$\frac{T}{T^*} = 1.114 \Rightarrow T^* = T_2 = \frac{T}{1.114} = \frac{295}{1.114} = 264.81 \text{ k}$$

and $$\frac{P}{P^*} = 1.703 \Rightarrow P^* = P_2 = \frac{P}{1.703} = \frac{0.6}{1.703} = 0.3525 \text{ bar}$$

As, $$P^* = \rho^* RT^*$$

$\Rightarrow$ $$\rho^* = \frac{0.3523\times10^5}{(287)(264.81)} = 0.4635 \text{ kg/m}^3$$

Velocity at exit $$V^* = \frac{150}{0.4635} = 323.57 \text{ m/sec}$$ [As $V^*\rho^* = 150$]

Let f_x be the total force exerted in axial direction to hold the tube stationary.

$$(P_1 - P_2)A = \dot{m}(V_2 - V_1) + f_x$$

$$(P_1 - P_2)A = \rho AV(V_2 - V_1) + fx$$
$$(0.6 - 0.3523) \times 10^5 \times 0.9 = 150 \times 0.9(323.57 - 211.6625) + f_x$$
$$f_x = 22293 - 15107.5125 = 7185.4875 \text{ N}$$

Q.107 What is the effect of Mach number on the compressibility? Derive an expression for pressure coefficient in terms of Mach number.

[CSE (Mains) 2013 : 10 Marks]

Solution:

Mach Number is the ratio of speed of the gas to the speed of sound in the gas. The speed of sound is equal to the speed of transmission of small, isentropic disturbances in the flow.
According to conservation of momentum equation,

$$(\rho V dv = -d\rho) \quad \ldots \text{(i)}$$

where, ρ is fluid density, V is the velocity and P is the pressure. dV and dP denote differential changes in the velocity and pressure.
Also, for isentropic flow conditions,

$$\frac{P}{V^\gamma} = \text{constant}$$

$$\frac{dP}{P} = \gamma \cdot \frac{d\rho}{\rho} \qquad \left(dP = \gamma . P \frac{d\rho}{\rho}\right)$$

Also,

$$P = \rho RT$$
$$dP = (VRT)d\rho$$

∵ $$\text{Speed of sound}(a) = \sqrt{\gamma RT}$$

$$(dP = a^2 d\rho) \quad \ldots \text{(ii)}$$

From (i) & (ii),

$$\rho V dv = -a^2 d\rho$$

$$\rho V^2 \frac{dV}{V} = -a^2\, d\rho$$

$$-\left(\frac{V^2}{a^2}\right)\frac{dV}{V} = \frac{d\rho}{\rho}$$

or

$$-M^2 \frac{dV}{V} = \frac{d\rho}{\rho} \quad (M = \text{Mach number}) \quad \ldots \text{(iii)}$$

So,

(i) For low speed or subsonic conditions, the Mach number is less than one, $M < 1$ and the square of Mach number, is very small. Therefore, change in density is very small. For the low value of Mach Number ($M < 0.3$) compressibility can be ignored.

(ii) When Mach number is nearly equal to one, $M = 1$ and flow is said to be transonic. For transonic flows, change in density is nearly equal to the change in velocity.

(iii) When $M > 1$ and flow is said to be hypersonic or supersonic. For such flows, the density changes faster than the velocity changes by a factor equal to square of Mach Number. Compressibility effects become more important with higher Mach number.

Let the pressure coefficient be C_P.

$$C_P = \frac{P_A - P_\infty}{\frac{1}{2}\rho_\infty U_\infty^2}$$

$\Rightarrow$ $$C_p = \frac{2P_\infty}{\rho_\infty U_\infty^2}\left[\frac{P_A}{P_\infty} - 1\right]$$

As $$\frac{P_A}{P_\infty} = \left(\frac{T_A}{T_\infty}\right)^{\frac{\gamma}{\gamma-1}}$$

and $$\frac{T_A}{T_\infty} = \frac{1+\left(\frac{\gamma-1}{2}\right)M_\infty^2}{1+\left(\frac{\gamma-1}{2}\right)M_A^2}$$

$\Rightarrow$ $$C_p = \frac{2P_\infty}{\rho_\infty U_\infty^2}\left[\left(\frac{1+\left(\frac{\gamma-1}{2}\right)M_\infty^2}{1+\left(\frac{\gamma-1}{2}\right)m_A^2}\right)^{\frac{\gamma}{\gamma-1}} - 1\right] \quad \text{. . . (i)}$$

and $$\frac{2P_\infty}{\rho_\infty U_\infty^2} = \frac{2RT_\infty}{U_\infty^2} = \frac{2\gamma RT_\infty}{\gamma U_\infty^2}$$

$$\frac{2P_\infty}{r_\infty U_\infty^2} = \frac{2_a^2}{\gamma U_\infty^2} \Rightarrow \frac{2}{\gamma M_\infty^2} \quad \text{...(ii)}$$

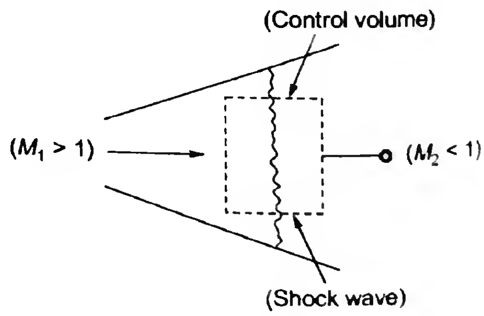
From (i) and (ii), $$C_{pA} = \frac{2}{\gamma M_\infty^2}\left[\left(\frac{1+\left(\frac{\gamma-1}{2}\right)M_\infty^2}{1+\left(\frac{\gamma-1}{2}\right)M_A^2}\right)^{\frac{\gamma}{\gamma-1}} - 1\right]$$

Q.108 **Derive an expression for entropy change across a normal shock wave occurring in a nozzle. Show the trend of this entropy change (in the form of a diagram), with respect to the Mach number value before the shock.**

[CSE (Mains) 2013 : 15 Marks]

Solution:

Entropy change across a normal shock wave occurring in a nozzle.



The normal shock waves are extremely thin, so the entrance and exit flow area for control volume are approximately equal. Assuming steady flow with no heat and work interactions and no potential energy changes. Denoting the properties upstream of the shock by subscript 1 and those downstream of shock by 2.

The intersection of the Fanno and Rayleigh lines at two points (points 1 and 2) represent the two states at which all the conservation equations are satisfied.

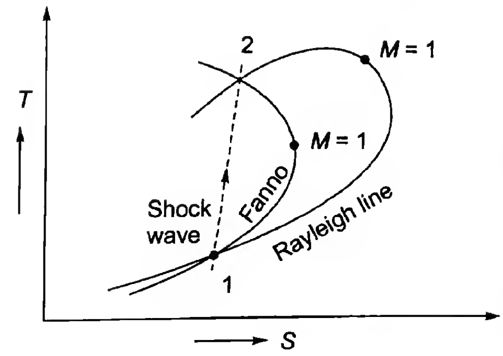
One of these (state 1) corresponds to state before the shock, and the other (state 2) corresponds to the state after the shock.

$$\text{Entropy Change} = S_2 - S_1$$

$$S_2 - S_1 = C_p ln\left(\frac{T_2}{T_1}\right) - R ln\left(\frac{P_2}{P_1}\right)$$

where, $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^2 \left(\frac{M_2}{M_1}\right)^2$

and $\frac{P_2}{P_1} = \frac{1+\gamma M_1^2}{1+\gamma M_2^2}, \left(C_P = \frac{\gamma R}{\gamma - 1}\right)$



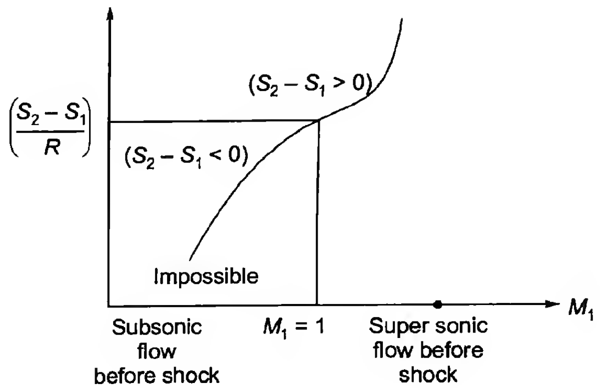
$$\therefore \quad S_2 - S_1 = \left\{\left(\frac{\gamma R}{\gamma-1}\right) ln \left\{\left(\frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right)^2 \left(\frac{M_2}{M_1}\right)^2\right\} - R ln \left(\frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right)\right\}$$

$$\Rightarrow \quad S_2 - S_1 = \left(\frac{2\gamma R}{\gamma-1}\right) ln \left(\frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right) + \left(\frac{2\gamma R}{\gamma-1}\right)\left[ln \left(\frac{M_2}{M_1}\right)\right] - R ln \left(\frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right)$$

$$\Rightarrow \quad S_2 - S_1 = \left(\frac{\gamma+1}{\gamma-1}\right) R ln \left(\frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right) + \frac{2\gamma R}{\gamma-1} ln \left(\frac{M_2}{M_1}\right)$$

where, $M_2^2 = \left(\frac{M_1^2 + \frac{2}{\gamma-1}}{\frac{2\gamma}{\gamma-1}M_1^2 - 1}\right)$

A plot of non-dimensional entropy change across the normal shock $\left(\frac{S_2 - S_1}{R}\right)$ versus M_1 (Mach Number before shock)



Note : Shock waves can exist only for supersonic flows, $M_1 > 1$, for adiabatic flows.

Q.109 Derive the following expression for normal shock in an ideal gas:

$$\frac{P_y}{p_x} = \frac{2\gamma}{\gamma+1}M_x^2 - \frac{\gamma-1}{\gamma+1}$$

where x and y are conditions before and after the shock, y is ratio of specific heats and M is Mach number.

[CSE (Mains) 2014 : 10 Marks]

Solution:

In an ideal gas, for normal shock:

To prove: $\left(\frac{P_y}{P_x} = \frac{2\gamma}{\gamma+1}M_x^2 - \frac{\gamma-1}{\gamma+1}\right)$

Here x and y are conditions before and after the shock, γ is ratio of specific heats.

For normal shock,

$$P_x + \rho_x V_x^2 = P_y + \rho_y V_y^2$$

$$\Rightarrow \quad P_x + \frac{P_x}{RT_x}V_x^2 s = P_y + \frac{P_y}{RT_y}V_y^2$$

$$\Rightarrow \quad P_x + \frac{\gamma P_x}{\gamma RT_x}V_x^2 = P_y + \frac{\gamma P_y}{\gamma RT_y}V_y^2$$

$$\Rightarrow \quad (P_x + \gamma M_x^2 P_x) = (P_y + \gamma P_y M_y^2)$$

$$\Rightarrow \quad P_x(1+\gamma M_x^2) = P_y(1+\gamma M_y^2)$$

$$\Rightarrow \quad \frac{P_y}{P_x} = \frac{1+\gamma M_x^2}{1+\gamma M_y^2} \qquad \text{...(i)}$$

Also,

$$\frac{T_y}{T_x} = \left(\frac{P_y}{P_x}\right)^2\left(\frac{M_y}{M_x}\right)^2$$

and

$$\frac{T_y}{T_x} = \frac{1+\left(\frac{\gamma-1}{2}\right)M_x^2}{1+\left(\frac{\gamma-1}{2}\right)M_y^2}$$

so,

$$\frac{P_y}{P_x} = \left(\frac{M_x}{M_y}\right)\sqrt{\frac{1+\left(\frac{\gamma-1}{2}\right)M_x^2}{1+\left(\frac{\gamma-1}{2}\right)M_y^2}} \qquad \text{...(ii)}$$

From (i) and (ii),

$$M_y^2 = \frac{M_x^2 + \frac{2}{\gamma-1}}{\frac{2\gamma}{\gamma-1}M_x^2 - 1} \qquad \text{...(iii)}$$

Putting value of M_y^2 from eq. (iii) in eq. (i),

$$\frac{P_y}{P_x} = (1+\gamma M_x^2)\times\frac{1}{1+\left\{\frac{\gamma M_x^2 + \frac{2\gamma}{\gamma-1}}{\frac{2\gamma}{\gamma-1}M_x^2 - 1}\right\}} = (1+\gamma M_x^2)\times\left[\frac{1}{\left\{\frac{\left(\frac{2\gamma}{\gamma-1}\right)M_x^2 - 1 + \gamma M_x^2 + \frac{2\gamma}{\gamma-1}}{\frac{2\gamma}{\gamma-1}M_x^2 - 1}\right\}}\right]$$

$$\Rightarrow \quad = (1+\gamma M_x^2)\times\left\{\frac{\frac{2\gamma}{\gamma-1}M_x^2 - 1}{\gamma\left(\frac{\gamma+1}{\gamma-1}\right)M_x^2 + \left(\frac{\gamma+1}{\gamma-1}\right)}\right\}$$

$$\Rightarrow \quad \frac{P_y}{P_x} = (1+\gamma M_x^2)\times\left\{\frac{\frac{2\gamma}{\gamma-1}M_x^2 - 1}{\left(\frac{\gamma+1}{\gamma-1}\right)(\gamma M_x^2+1)}\right\} = \left(\frac{2\gamma}{\gamma+1}\right)M_x^2 - \left(\frac{\gamma-1}{\gamma+1}\right)$$

$$\Rightarrow \quad \frac{P_y}{P_x} = \frac{2\gamma}{r+1}M_x^2 - \frac{(\rho-1)}{(\rho+1)}$$

8. Nuclear Power Plant

Q.110 A nuclear power plant is set up with a generating capacity of 10 MW, the capital cost being Rs. 80,000 per kW. It meets the following demands:

(i) Service sector: Total load 400 kW at 30% load factor

(ii) Cottage industries: Total load 3-6 MW at 50% load factor

(iii) Household: Total load 6 MW at 20% load factor.

The operative cost of the plant is Rs 30,00,000 per annum and annual rate of interest and depreciation is 10%. If as per promotion policy of government, a flat rate is to be charged from all categories of consumers, calculate the overall production cost. If the power is sold at the rate of production cost, estimate the loss/gain each sector consumer will get per kWh for power consumed.

[CSE (Mains) 2006 : 30 Marks]

Solution:

Given: Capacity = 10 MW, Capital cost = 80000 per KW

Energy supplied per year to all three types consumer.

$$= [400 \times 3 + 3600 \times 0.5 + 6000 \times 0.2] \times 8760 = 3120 \times 8760 \text{ kWh}$$

$$\text{Operating charges per kWh} = \frac{300000}{3120 \times 8760} = 0.10976 \text{ or } 10.976 \text{ paise}$$

As,

$$\text{Capital cost of the plant} = 80 \times 10^3 \times 10000 = ₹\, 8 \times 10^8$$

$$\text{Fixed charges per year} = 0.1 \times 8 \times 10^8 = ₹\, 8 \times 10^7$$

$$\text{Fixed charges per kW} = \frac{₹8 \times 10^7}{10^4 \text{ kW}} = ₹8000$$

(i) For service sector consumers:

$$\text{Total charges} = 400 \times 8000 + 0.10976 \times 400 \times 0.3 \times 8760 = ₹33,15,379.7$$

$$\text{Overall cost per kWh} = \frac{₹33,15,379.7}{400 \times 0.3 \times 8760} = ₹3.154$$

(ii) For cottage industries:

$$\text{Total charges} = 8000 \times 3600 + 0.10976 \times 3600 \times 0.5 \times 8760 = ₹\, 30,530,695.68$$

$$\Rightarrow \quad \text{Overall cost per kWh} = \frac{₹30,530,695.68}{1800 \times 8760} = ₹1.936$$

(iii) For household consumers:

$$\text{Total charges} = 8000 \times 6000 + 0.10976 \times 6000 \times 0.2 \times 8760 = ₹49,153,797.12$$

$$\Rightarrow \quad \text{Overall cost per kWh} = \frac{₹49,153,797.12}{1200 \times 8760} = ₹4.676$$

■■■■

6 Refrigeration and Air-Conditioning

1. Introduction and Basic Concepts

Q.1 Show that the COP of a cascade refrigeration system is

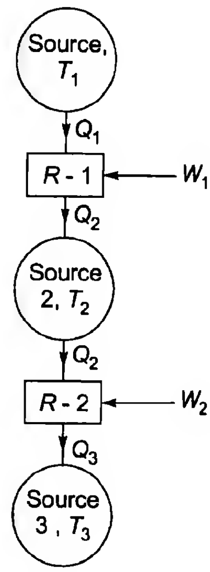
$$\text{COP} = \frac{\text{COP}_1 \times \text{COP}_2}{1 + \text{COP}_1 + \text{COP}_2}$$

where COP_1 and COP_2 are COP's of low temperature and high temperature side respectively.

[CSE (Mains) 2005 : 20 Marks]

Solution:

Consider a cascade refrigeration system as shown in figure where $T_1 < T_2 < T_3$. Two refrigerators, R-1 and R-2 are low and high temperature side refrigerators, respectively.



We have:

$$\text{COP}_1 = \frac{Q_1}{W_1} \quad \text{and} \quad \text{COP}_2 = \frac{Q_2}{W_2}$$

For 1st refrigerator, from 1st law of thermodynamics,

$$Q_2 = Q_1 + W_1$$

$$\therefore \quad \text{COP}_2 = \frac{Q_2}{W} = \frac{Q_1 + W_1}{W_2}$$

For the over all system

Net refrigeration effect between temperatures T_1 and $T_3 = Q_1$

$$\text{Net work input} = W_1 + W_2$$

$$\therefore \quad \text{Overall COP} = \frac{Q_1}{W_1 + W_2} = \frac{Q_1}{\frac{Q_1}{\text{COP}_1} + \frac{Q_1 + W_1}{\text{COP}_2}} = \frac{1}{\frac{1}{\frac{1}{\text{COP}_1} + \frac{1 + 1/\text{COP}_1}{\text{COP}_2}}} = \frac{1}{\frac{1}{\frac{1}{\text{COP}_1} + \frac{\text{COP}_1 + 1}{\text{COP}_1 \times \text{COP}_2}}}$$

$$\Rightarrow \quad \text{COP} = \frac{\text{COP}_1 \cdot \text{COP}_2}{1 + \text{COP}_1 + \text{COP}_2} \quad \text{Hence, proved.}$$

Q.2 A customer complained of poor cooling for an air-conditioning system of 100 TR capacity. The supplier carried out test on condenser which is water cooled and noted power input to the motor. The observations made are as under:

Cooling water flow rate	:	10 Litre/sec
Inlet water temperature	:	30°C
Outlet water temperature	:	41.12°C
Power input to motor	:	120 kW (94.92% efficiency)

Determine the actual refrigerating capacity and state whether the cooling capacity is lower, higher or as per specifications.

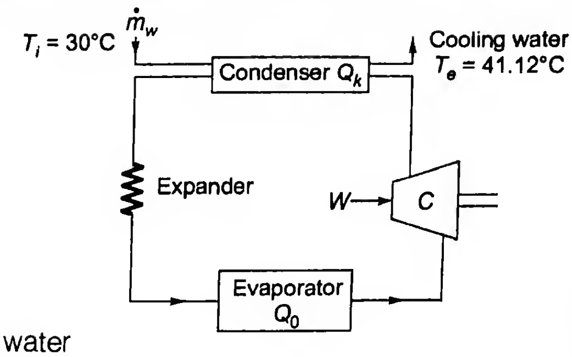
[CSE (Mains) 2008 : 20 Marks]

Solution:

Given: Capacity = 100 TR,

$\dot{m}_w$ = 10 litre/s, T_i = 30°C, T_e = 41.12°C

Consider the refrigerating/AC system as shown below



Condenser details:

$$\dot{m}_w = 10 \text{ litre/sec} = 10 \cdot 10^{-3} \text{ m}^3\text{/sec} = 10^{-2} \text{ m}^3\text{/sec}$$

Heat rejection rate in condenser = rate of heat gain by water

$$\Rightarrow \quad Q_k = \dot{m}_w C_{p,w}(T_e - T_i)$$

$$= 10^{-2} \text{ m}^3\text{/sec} \cdot 10^3 \text{ kg/m}^3 \cdot 4.168 \text{ kJ/kg°C} \cdot (41.12 - 30)\text{°C}$$

[As ρ_w = 1000 kg/m³]

$$= 463.482 \text{ kW}$$

Ideal power input to motor = 120 kW

Actual power input to motor = 12 × 0.9492 = 113.904 kW [As motor efficiency = 94.92 X]

From 1st law of thermodynamics,

$$Q_k = Q_0 + W$$

$$\Rightarrow \quad Q_0 = Q_k - W = 349.578$$

∴ Refrigeration capacity, Q_0 = 349.578 kW = 99.4 TR

This is slightly lower than ideal or specified cooling capacity.

Q.3 **An air refrigerator used for food storage provides 50 tons of refrigeration. The temperature of air entering the compressor is 7°C and the temperature of air before entering into the expander is 27°C. Assuming 30% more power is required than theoretical, find (i) actual COP of the cycle and (ii) kW capacity required to run the compressor.**
The quantity of air circulated in the system is 100 kg/min. The compression and expansion follow the law $pv^{1.3}$ = constant. Take γ = 1.4 and C_p = 1 kJ/kg-°C for air.

[CSE (Mains) 2013 : 25 Marks]

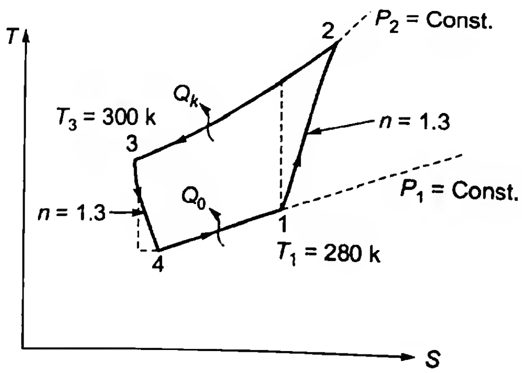
Solution:

Given: $RC = Q_0$ = 50 TR, T_1 = 7°C = 280 K,

T_3 = 273 + 27 = 300 K, $\dot{m}_a$ = 100 kg/min

Consider the air refrigerator as shown in T-S diagram here

Compression and expansion processes follow polytropic processes with n = 1.3.



Refrigeration effect, Q_0 = 50 TR = 50 × 3.5167 kW = 175.835 kW

As, Quantity of air circulated = 100 kg/min = 1.67 kg/s

Refrigeration effect = $\dot{m}_a c_p(T_1 - T_4) = Q_0$ = 175.835

$$\Rightarrow \quad T_4 = 174.71 \text{ K}$$

For process 3-4:

$$\frac{P_3}{P_4} = \left(\frac{T_3}{T_4}\right)^{n/n-1} = \left(\frac{300}{174.71}\right)^{1.3/1.3-1} = 10.411$$

Since heat addition and heat rejection are constant pressure processes,

$$\frac{P_3}{P_4} = \frac{P_2}{P_1} = 10.411$$

For process 1-2 $T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(n-1)/n} = 280 \times (10.411)^{(1.3-1)/1.3} = 480.8 \text{ K}$

∴ Actual COP of cycle, $\frac{Q_0}{W} = \frac{175.835}{163.93} = 1.07$

Ideal work input required to run compressor,

$$W_C = \dot{m}_a\left[\frac{R(T_2 - T_1)}{n-1}\cdot n\right] = \dot{m}_a \cdot \frac{1.3}{1.3-1}\cdot\frac{1.4-1}{1.4}\cdot 1\cdot(480.8-280)$$

$$= 414.43 \text{ kW}$$

Assuming 30% higher capacity is required to run compressor, actual required capacity of compressor

$$= W_c \times 1.3 = 538.76 \text{ kW}$$

Expander work, $\dot{W}_E = \frac{n}{n-1}\dot{M}\,R(T_3 - T_4) = \frac{1.3}{0.3}\times\frac{100}{60}\times\frac{8.314}{29}(300-174.71) = 258.817 \text{ KW}$

$$W_{NET} = (W_C)_{ACTUAL} - W_E$$

$$= 538.76 - 258.82 = 279.52 \text{ KW}$$

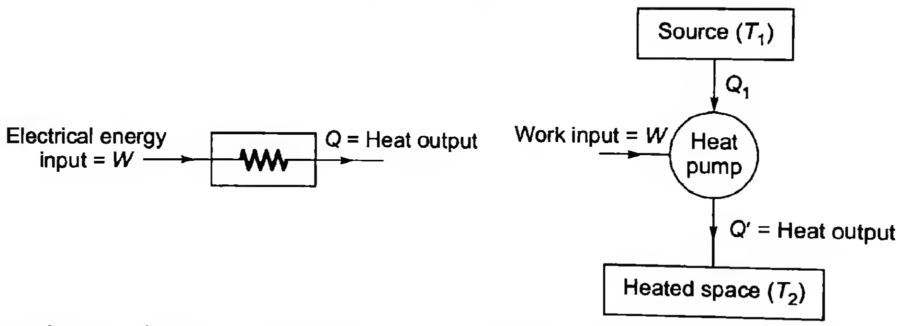
$$\text{Actual COP} = \frac{D\cdot E}{W_{NET}} = \frac{50\times 3.5}{279.52} = 0.626$$

Q.4 It is thermodynamically advantageous to employ a heat pump rather than employing a direct electrical resistance heater for a room air heating application. Explain Why.

[CSE (Mains) 2014 : 10 Marks]

Solution:

Schematics of a heat pump and an electrical resistance heater are as shown below –



Direct electrical resistance heater converts the supplied electrical energy into heat energy as shown in figure. In the limiting condition of maximum efficiency, all the electrical energy supplied can be converted into heat energy for heating a room.

∴ Maximum possible COP of resistance heater,

$$COP_{RH} = \frac{Q}{W} = 1$$

Heat pump supplied with an energy input of 'W', extracts heat from a source at temperature T_1 (lower than room to be heated) and supplies heat Q' to heated space.

From 1st law of thermodynamics,

$$Q' = Q_1 + W$$

$$\text{COP of heat pump, } COP_{HP} = \frac{Q'}{W} = \frac{Q_1 + W}{W} = 1 + \frac{Q_1}{W}$$

$$\Rightarrow \quad COP_{HP} > 1$$

Hence, $COP_{HP} > COP_{RH}$ and heat pump can supply more heat to room than resistance heater and is more efficient. COP of heat pump will also be higher if it approaches reversibility and the limiting case will be of a heat pump working on carnot cycle.

Hence, it is thermodynamically more advantageous to use a heat pump than a direct electrical resistance heater.

2. Vapour Absorptions Systems

Q.5 How, in a refrigeration system using ammonia, the compressor could be replaced by another suitable set of equipments?

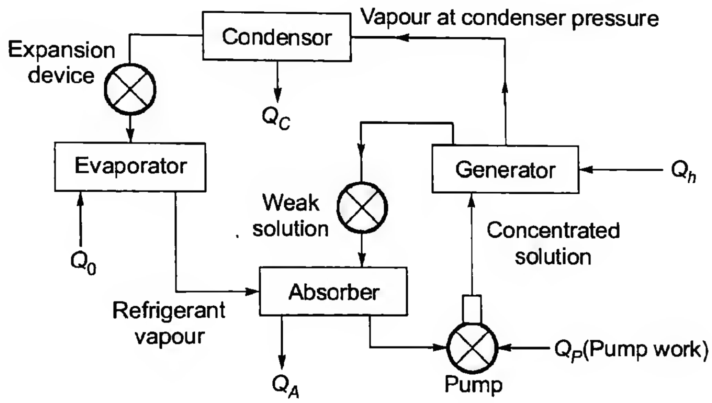
[CSE (Mains) 2001 : 20 Marks]

Solution:

If a refrigeration system using ammonia is operated on a vapour absorption cycle, compressor work can be replaced by following additional equipments –

- **Absorber:** Absorbs saturated vapour from evaporator and forms a concentrated solution of refrigerant vapour using a work solution and rejection of latent heat due to condensation.
- **Pump:** Used to pump concentrated solution formed as above to condenser pressure.
- **Generator or disorder:** Used to generate refrigerant vapours by absorbing heat from a heat source leaving weak solution for recycling.

Compressor work in a vapour compression refrigeration system is used to convert saturated vapour from evaporator to vapour at condenser temperature and pressure to ensure heat rejection is possible and refrigerant can be recycled. Consider the schematic of vapour absorption system as a replacement of this process of compression –



As shown in the schematic above, absorber converts saturated vapour using a weak solution and by rejecting latent heat, into concentrated solution. This is pumped to condensor pressure to generator which distills it into vapour to be sent to compressor.

Q.6 Draw a neat sketch of aqua-ammonia vapour absorption system. On this sketch:

(i) Indicate thermodynamic state points with (1) at the inlet of pump.

(ii) Show the direction of the following energy transfers

e_A – energy transfer to absorber e_p – energy transfer to pump

e_g – energy transfer to generator e_D – energy transfer to dephlegmator

e_c – energy transfer to condenser e_e – energy transfer to evaporator

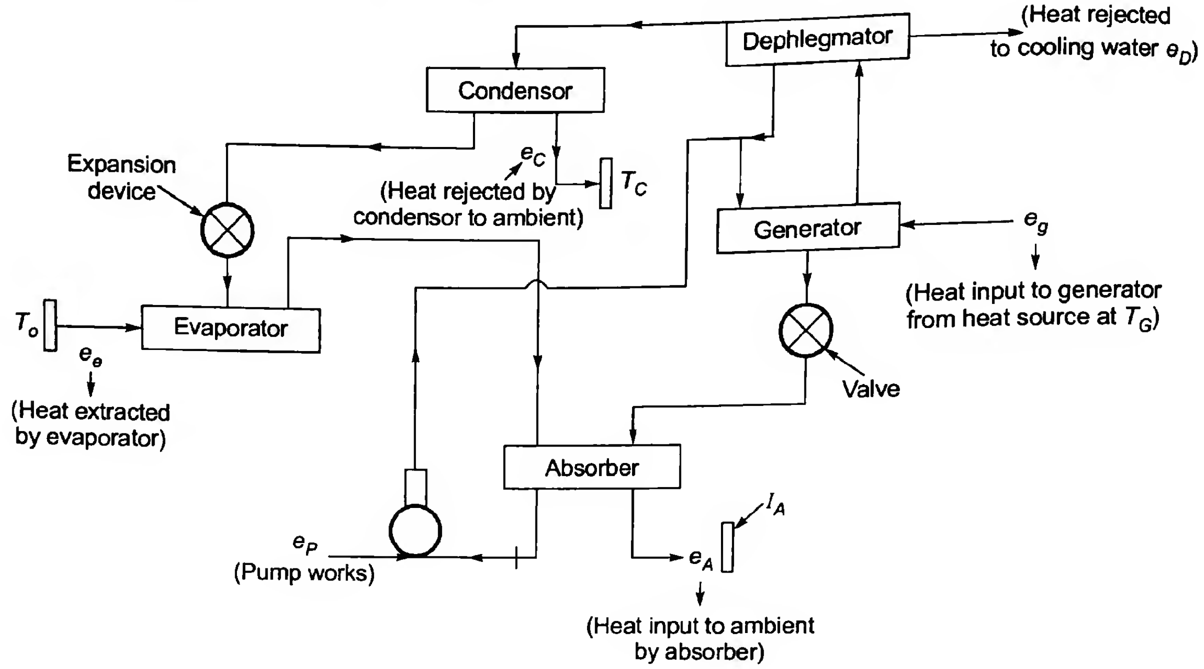
(iii) Mention for each of the energy transfer in (ii) above whether it is in the form of work or heat.

(iv) With heat sink temperature of 27°C, heat source temperature of 127°C and refrigeration temperature of –13°C, find max COP of the vapour absorption system and mention the assumptions made.

[CSE (Mains) 2010 : 20 Marks]

Solution:

Vapour absorption system with aqua-ammonia as refrigerant is used to utilize available heat energy as an alternative to compressor work. General schematic is as shown below:



As compared to a vapour compression system differences are:

- **Absorber:** Refrigerant vapours of ammonia are absorbed in a weak solution with water.
- **Pump:** Used to pump aqua-ammonia solution to condensor pressure.
- **Generator:** Distillation of ammonia vapour from the rich solution leaving weak solution for recirculation.
- **Dephlegmator:** Further concentration of vapour to separate water from ammonia vapour.

Nature of heat/work interactions as required are mentioned in the figure.

Refrigerant vapour of NH_3 formed as a result of heat extraction in evaporator goes to absorber where it is dissolved in a weak aqua-ammonia solution. Ammonia vapour loses heat e_A which is rejected by the absorber and forms a concentrated solution which is pumped to condenser pressure by pump. Pump work = $e_P(-vdp)$, is usually negligible since specific volume-v of liquid is very loss. In generator distillation of vapour of NH_3 takes place by supplying heat from source- e_g at T_h. Weak solution is recirculated through a valve to absorber. Further concentration of ammonia vapours takes place in dephlegmator along with rejection of heat e_D to cooling water.

Maximum COP of cycle

Aqua-ammonia cycle is a heat operated refrigeration machine. It can be approximated as a combination of a heat engine operated on solution circuit and a refrigerator operated on ammonia cycle.

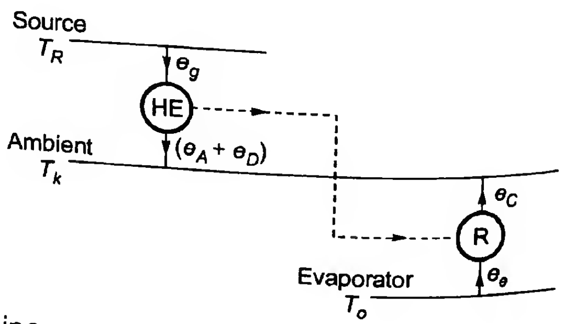
For heat source temperature T_h, ambient temperature ($T_A = T_C = T_K$) and evaporator temperature T_0, above system can be shown as:

Work output of heat engine is used to power the refrigerator.

Energy balance of above system can be shown as below

$$e_e + e_P + e_g = e_C + e_D + e_A = e_K$$

(net heat rejected)



$$\text{COP of cycle} = \frac{e_e}{e_g} = \frac{e_e}{W} + \frac{W}{e_g}$$

$$= \text{COP}_R \times \eta_{HE}$$

COP_R = COP of refrigerator

η_{HE} = efficiency of heat engine

For max COP - Assumptions

(a) Heat engine and refrigerator operate on reversible cycle as Carnot engine and refrigerator.

(b) No pressure drop or heat loss during any of the process.

$$COP_{max} = COP_R \times COP_{HE} = \left(\frac{T_0}{T_K - T_0}\right) \times \left(1 - \frac{T_K}{T_R}\right)$$

From given data, we get,

$$T_K = 27°C = 300\text{ K}$$
$$T_h = 127°C = 400\text{ K}$$
$$T_0 = -13°C = 260\text{ K}$$

$$\therefore \quad COP_{max} = \left(\frac{260}{300-260}\right) \times \left(1 - \frac{300}{400}\right) = 1.625$$

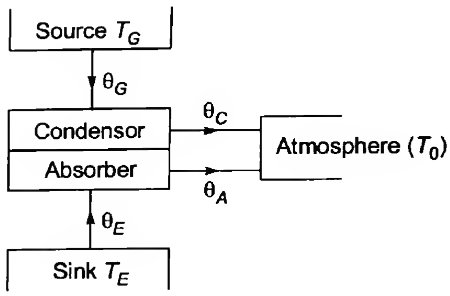
Q.7 Explain, with the help of neat sketches, vapour absorption cycle for refrigeration and also derive an expression to calculate ideal COP of it.

[CSE (Mains) 2016 : 20 Marks]

Solution:

We know that in VARC, heat is supplied in generator (acting as source) and heat is extracted in refrigerated space and heat is rejected to surrounding in absorber and condensor.

Writing heat balance of above system,



$$Q_G + Q_E = Q_A + Q_C \quad ...(i)$$

Also we know from second law of thermodynamics, (for minimum work input entropy remain constant)

$$\frac{Q_G}{T_G} + \frac{Q_E}{T_E} = \frac{Q_A}{T_0} + \frac{Q_C}{T_0}$$

$$\Rightarrow \quad \frac{Q_G}{T_G} + \frac{Q_E}{T_E} = \frac{Q_A + Q_C}{T_0} = \frac{Q_G + Q_E}{T_0} \quad \text{(from equation (i))}$$

$$\Rightarrow \quad \frac{Q_G}{T_G} + \frac{Q_E}{T_E} = \frac{Q_G}{T_0} + \frac{Q_E}{T_0}$$

$$\Rightarrow \quad \frac{Q_E}{T_E} - \frac{Q_E}{T_0} = \frac{Q_G}{T_0} - \frac{Q_G}{T_G}$$

$$\Rightarrow \quad Q_E\left(\frac{T_0 - T_E}{T_E \cdot T_0}\right) = Q_G\left(\frac{T_G - T_0}{T_G T_0}\right)$$

$$\Rightarrow \quad \frac{Q_E}{Q_G} = \left(\frac{T_G - T_0}{T_0 - T_E}\right) \times \frac{T_E}{T_G}$$

We know,

$$COP = \frac{DE}{W_{IN}} = \frac{Q_E}{Q_G + W_{P/P}} \qquad \{W_{P/P} \simeq 0\}$$

Thus,

$$COP = \frac{Q_E}{Q_G} = \left(\frac{T_G - T_0}{T_0 - T_E}\right) \times \frac{T_E}{T_G}$$

3. Refrigeration Equipments

Q.8 Discuss the importance of liquid-vapour heat exchanger (LVHE) in the vapour compression refrigeration system. Clearly mention advantages and disadvantages of using such heat exchangers. Represent the cycle on T-s and P-h diagrams and illustrate the influence of LVHE.

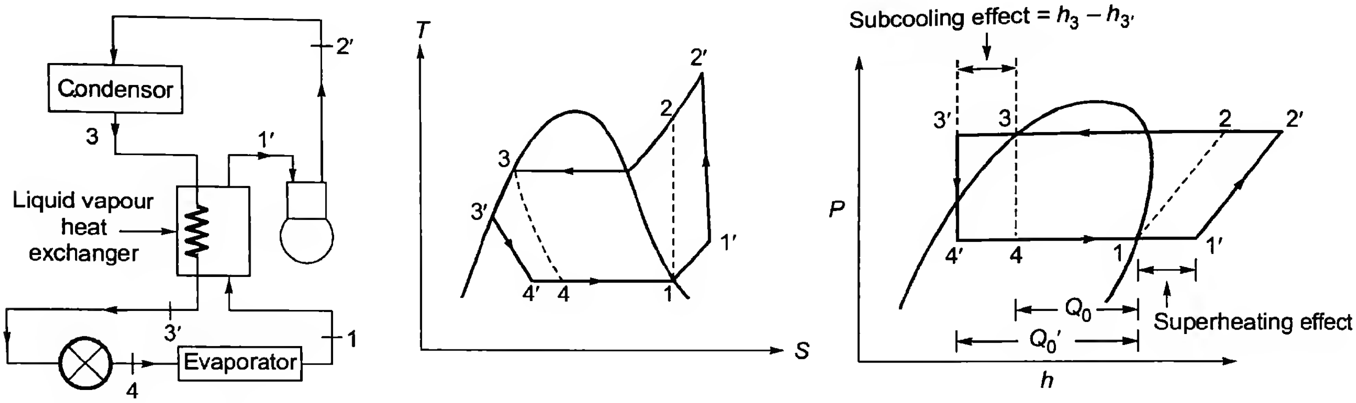
[CSE (Mains) 2008 : 15 Marks]

Solution:

Importance of liquid-vapour heat exchanger (LVHE)

LVHE is vapour compression refrigeration systems is used to ensure subcooling of condensor liquid and superheating of saturated evaporated vapour to the same time.

Consider the schematic of LVHE and corresponding indicator diagrams as shown below



As shown in figure above—

1. Cycle 1-2-3-4 — without LVHE
2. Cycle 1′-2′-3′-4′ — using LVHE
3. Subcooling effect on liquid is shown by $h_3 - h_3'$
4. Superheat effect on saturated vapour is shown by $h_1' - h_1$
5. Increased refrigeration effect is shown by $h_4 - h_4'$

Advantages of LVHE

- Subcooling of liquid ensures less flashing of liquid and less amount of vapour in expansion process.
- Superheating of vapour by LVHE ensures no liquid in compressor which could have proved harmful for compressor.
- Increased refrigeration effect in evaporator.
- System optimization and improved performance — Due to subcooling happening outside condensor, condensor capacity can be more effectively utilized for heat rejection.
- If LVHE is not used and superheating takes place inside evaporator heat extraction will take place at a higher temperature $(t_1' - t_1)$. While by using LVHE, heat extraction can be done at a lower temperature t_1, and increasing refrigeration effect at the same time.

Disadvantages of LVHE

- Increased compressor work $(h_{1'} - h_{2'})$ as compared to $(h_1 - h_2)$ due to vapour superheat.
- Increased isentropic discharge temperature t_2' as compared to t_2.
- COP of refrigeration might actually decrease due to LVHE.
- Suction vapour volume for compressor increases, hence capacity of compressor required is higher.

Q.9 Comment on the following (Be brief) with the help of schematic diagram, if required:

(i) In the reciprocating compressors used in vapour compression system, the mass of refrigerant discharged by the compressor reduces as the pressure ratio is increased.

(ii) Thermostatic expansion valve is preferred over automatic expansion valve as throttling device.

(iii) COP of refrigeration system increases when water cooled condenser is used in place of air cooled condenser.

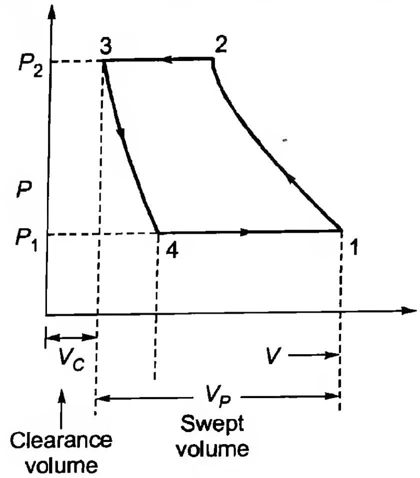
(iv) Vapour at suction to the hermetically sealed compressor is always superheated vapour.

(v) For low sensible heat factor applications, reheat is necessary.

[CSE (Mains) 2010 : 20 Marks]

Solution:

(i) Consider a reciprocating compressor with swept volume V_P and volumetric efficiency η_V operating on cycle as shown in P-V diagram.



$$\text{Mass flow rate, } \dot{m} = \frac{\dot{V}_P}{v_1} \cdot \eta_V$$

Hence, we see for a given compressor swept volume V_P and suction vapour specific volume v_1,

$$\dot{m} \propto \eta_V$$

Also,

$$\eta_V = 1 + C - C\left(\frac{P_2}{P_1}\right)^{1/\gamma}$$

Hence, as pressure ratio $\left(\frac{P_2}{P_1}\right)$ is increased, volumetric efficiency decreases.

$\therefore$ So, $\left(\frac{P_2}{P_1}\right)$ increases, η_V decreases.

$\therefore$ And, $\left(\frac{P_2}{P_1}\right)$ increases, mass flow rate also decreases.

(ii) A comparison between automatic expansion valve and thermostatic expansion valve can be drawn as follows:

Automatic expansion valve

1. Maintains constant evaporator pressure (and temperature)
2. When load increases, evaporator pressure rises to control due to which mass flow rate decreases.
3. When load decreases, mass flow rate increases.

Thermostatic expansion valve

1. Maintains constant degree of superheat in the evaporator
2. When load increases, degree of superheat increases and hence mass flow rate increase.
3. When load decreases, mass flow decreases.

Hence, thermostatic expansion valve is a better choice for a throttling device since it regulates mass flow rate as per varying load and hence ensure effective utilization of evaporator.

(iii)

$$\text{COP of regulation system} = \frac{\text{Heat extracted in evaporator}}{\text{Work input}}$$

$$= \frac{\text{Heat extracted in evaporator}}{\text{Heat rejected in condensor} - \text{Heat extracted in evaporator}}$$

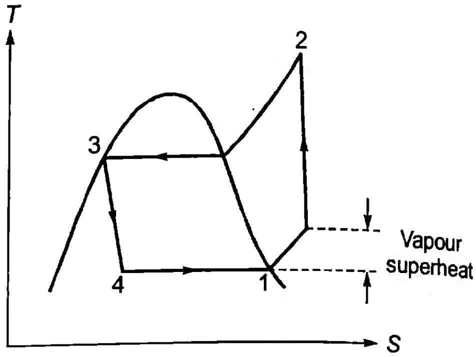
$$\text{COP}_C = \frac{T_0}{T_C - T_0}$$

For a water cooled condensor, coefficient of heat transfer is higher due to larger specific heat of water and higher thermal conductivity than air. Hence heat rejection takes place at a lower temperature for water as compared to air since —

$$Q = UA\Delta T_{lm}$$

Since U is higher ΔT_{lm} is lower.

Since heat rejection for water takes place at a lower temperature COP of refrigeration system is higher for a water condensor, as compared to an air condensor.

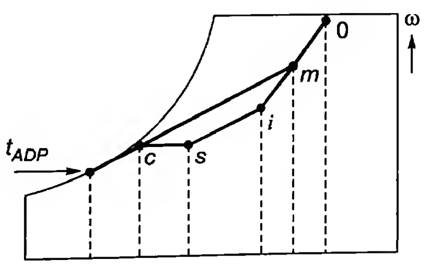


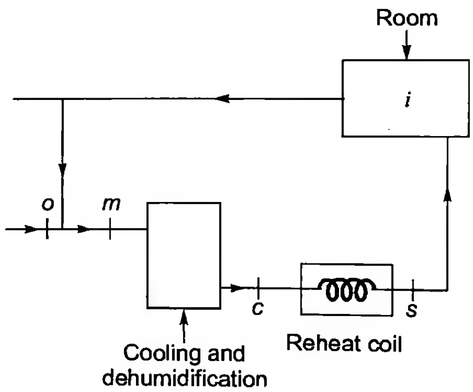
(iv) In a hermetically sealed compressor, both motor and compressor are housed in the same unit and have the same shaft.

In this case heat from motor winding is continuously transferred to suction vapour which passes over it in the refrigeration circuit. This raises the temperature of suction vapour as motor cooling load also becomes a part of the overall cooling load of refrigeration system: Hence, suction vapour which enters at a saturated condition to the compressor gets heated by motor winding and always becomes super heated.

(v) When room sensible heating factor is low or latent heat factor is high, e.g. in cases of high humidity/high dehumidification needs or for high internal latent heat loads, then simple air conditioning system has very low oil ADP. This leads to very low evaporator pressure and reduces COP of the system and increases costs.

Hence, to increase ADP of cooling coil to manageable temperatures a reheat coil is introduced. Consider the setup as shown below and corresponding psychrometric chart.





Room air at state i mixes with ventilation air from surroundings at 0 and then enters cooling and dehumidification coil at state m. Exit happens at C post which it enters re-heat coil to achieve delivery temperature t_s.

Q.10 Thermostatic expansion valve using R-134a in the power assembly is designed to produce 10°C superheat at an evaporator temperature of 0°C. What will be the superheat that it will maintain when the evaporator temperature is –30°C? Given for R134a, the saturation pressure temperature data:

Temp. (°C)	–30	–25	–20	–15	–10	–5	0	5	10
Pressure (bar)	0.8438	1.065	1.327	1.64	2.006	2.43	2.928	3.5	4.146

[CSE (Mains) 2011 : 20 Marks]

Solution:

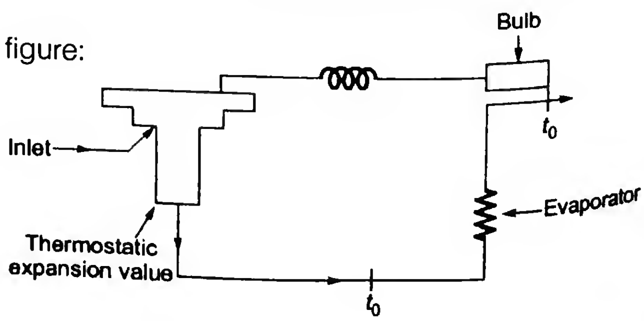
Consider the thermostatic expansion valve as shown in figure:

Evaporator inlet temperature = t_0

Evaporator outlet temperature = t_0'

Degree of superheat = $t_0' - t_0$

This degree of superheat result into a pressure difference $(P_0' - P_0)$ which actuates the valve, where



P_0' and P_0 are respective saturation pressure.

Given: **Case I:**

$$t_0 = 0°\text{C},\ t_0' = 10°\text{C}$$

$$P_0 = 2.928 \text{ bar } P_0' = 4.146 \text{ bar}$$

we have,
$$P_0' - P_0 = 1.218 \text{ bar}$$

Case II:
$$t_0 = -30°\text{C},\ P_0 = 0.8438 \text{ bar}$$

Since valve follow up spring setting remains the same, $(P_0' - P_0)$ will remain the same.

$$P_0' - P_0 = 1.21 \text{ bar}$$

$\Rightarrow$
$$P_0' = 0.8438 + 1.218 = 2.0618 \text{ bar}$$

Hence approximate refrigerant outlet temperature corresponding to this saturation pressure will be –10°C as per the given table.

Hence, degree of superheat required to maintain evaporator temperature of

$$-30°\text{C} = -10°\text{C} - (-30°\text{C}) = 20°\text{C}$$

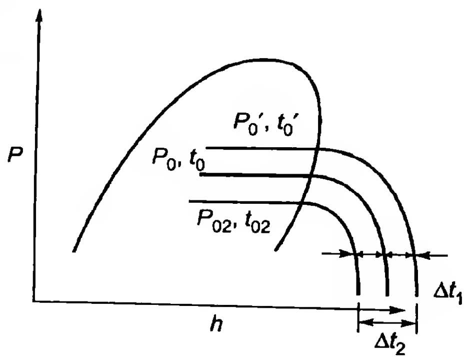
Q.11 Why do some thermostatic expansion valves have an 'external equalizer'? How does it improve its performance.

[CSE (Mains) 2011 : 20 Marks]

Solution:

During operation of a thermostatic expansion valve constant superheat is maintained by the valve between evaporator inlet and outlet.

However, during actual operation there is a drop in evaporator pressure. For example, for inlet pressure P_0 and temperature t_0, outlet pressure can drop to P_{0_2}. For a given setting of follow up spring of the valve, resultant superheat will be higher, equal to $(t_0' - t_{0_2})$ where t_{0_2} is saturation temperature at P_{0_2} and t_0' for pressure $t_0^{\propto}$.



$\therefore$
$$\text{Actual superheat} = \Delta t_2 = t_0' - t_0$$

$$\text{Ideal superheat} = \Delta t_1 = t_0' - t_{02}$$

This problem increase further at part load applications of the valve.

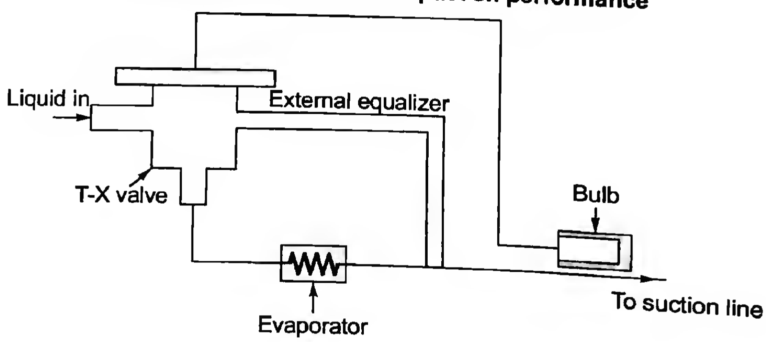
This is the reason to employ external equalizer in an expansion device to control the degree of superheat due to drop in evaporator pressure.

External equalizer involves providing a tapping to inside of bellows of *T-X* valve with the outlet of the evaporator which is at a lower pressure than inlet pressure.

Also there is no connection between evaporator inlet pressure and inside of bellows.

Hence, pressure acting inside the bellows in always equal to the pressure at the outlet of evaporator, while the pressure at the outside of bellows is equivalent to superheated vapour.

External equalizer and its impact on performance



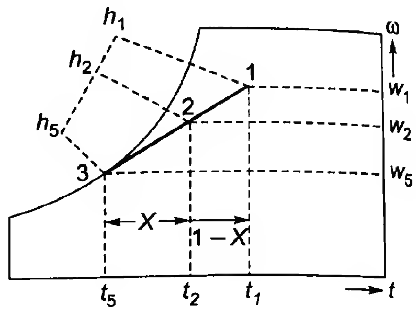
Hence, the degree of superheat resulted from *T-X* valve despite drop in evaporator pressure is equal to Δt_1, which is the initial set value by follow up spring setting.

Q.12 What is the significance of by-pass factor? For a heating coil, derive an expression of by-pass factor. Find the expression for efficiency also for heating coil.

[CSE (Mains) 2013 : 10 Marks]

Solution:

By-pass factor represents the fraction of "uncontacted air" when a heating coil is used to treat a given mass of air. When the heating coil surface comes in contact with a given mass of air, air particles which directly come in contact with the coil achieve temperature of the coil and become saturated at the coil temperature. At the same time, few air particles do not come in direct contact with coil surface. Resultant state of air is a due to complex mixing of contacted and uncontacted air. By pass factor signifies the proportion of air which does not come in contact with the coil.



By pass factor represents the proportion of air particles which "by pass" the effect of heating or cooling coil.

∴ By definition,

By-pass factor, $X = \dfrac{t_2 - t_s}{t_1 - t_s}$

where, 1 - initial state

s - state of heating coil

2 - final state

By-pass factor can also be expressed in terms of specific humidity and enthalpy as follows:

$$X = \frac{t_2 - t_s}{t_1 - t_s} = \frac{\omega_2 - \omega_s}{\omega_1 - \omega_s} = \frac{h_2 - h_s}{h_1 - h_s}$$

This is the required expression for by-pass factor.

Efficiency of heating coil is expressed by contact factor which represents the proportion of air particles which come in contact with the heating coil.

Hence, efficiency or contact factor, $\eta_H = \dfrac{t_2 - t_s}{t_1 - t_s} = \dfrac{\omega_2 - \omega_s}{\omega_1 - \omega_s} = \dfrac{h_2 - h_s}{h_1 - h_s}$ or, efficiency, $\eta_H = 1 - X$

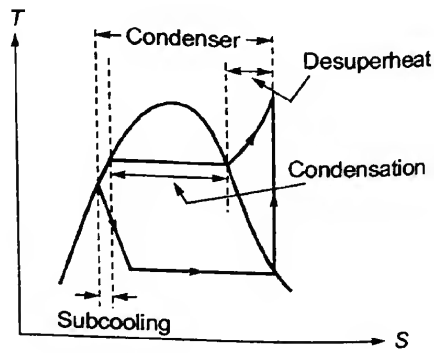
This is the required expression for efficiency of heating coil.

Q.13 What are the functions of condenser in a refrigerating machine? Name different types of condensers. Describe with neat sketch the evaporative condenser.

[CSE (Mains) 2015 : 10 Marks]

Solution:

Functions of condensor in a refrigerating machine are as follows:

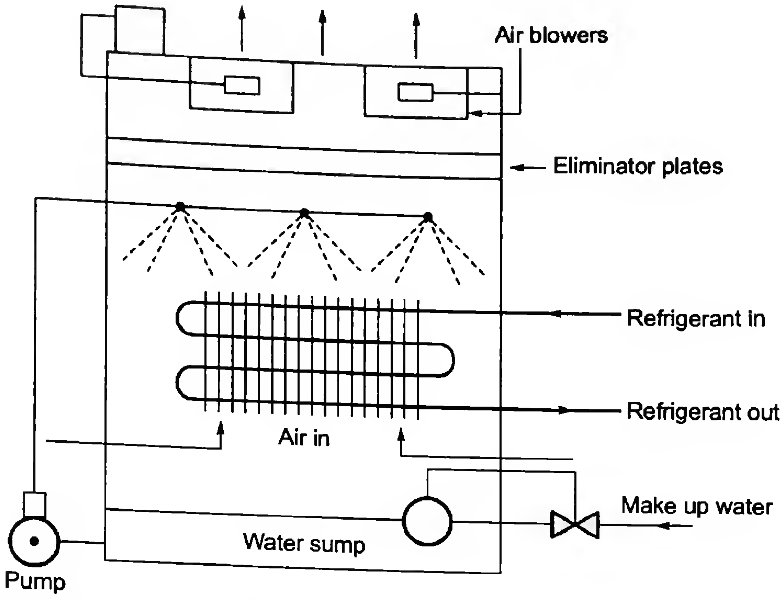


- To enable rejection of heat from the refrigerant in a refrigeration cycle.
- Enables desuperheat, condensation and subcooling of refrigerant to get vapour to condense to subcooled liquid stage which can then be fed to expansion device.

Different types of condensers are :

- Air-cooled condensers-for small cooling load operation like domestic refrigerators and air conditioners. This can be further classified as:
  - Natural convection type
  - Forced convection type
- **Water-cooled condensor:** Water is used to extract heat from the refrigerant. Further classified as:
  - Double tube type
  - Shell and tube type
  - Shell and coil type

- **Evaporative condensers:** Combines the features of both a cooling tower and a water-cooled condenser in a single unit.



Consider and evaporative condenser as shown above. Water is pumped from the sump and sprayed on tubes carrying refrigerant. Water droplets on tube get evaporated by extracting latent heat of vaporization from refrigerant. Water is continuously recirculated and make up water is added as required.

Air is continuously recirculated using blowers and used to cool the refrigerant tubes which are connected to a plate to enhance area of heat exchange as shown above. The role or air primarily to enhance evaporation of water.

Evaporative condensers are used in medium to large capacity systems. These are normally cheaper than water cooled condensers which require a separate cooling tower.

Q.14 Justify the suitability of thermostatic expansion valve in comparison to automatic expansion valve.

[CSE (Mains) 2015 : 10 Marks]

Solution:

Both thermostatic expansion valve and automatic expansion valve are expansion devices used in refrigeration systems to reduce pressure of refrigerant from condensor to evaporator pressure by throttling and to control rate of evaporation based on the load of evaporator.

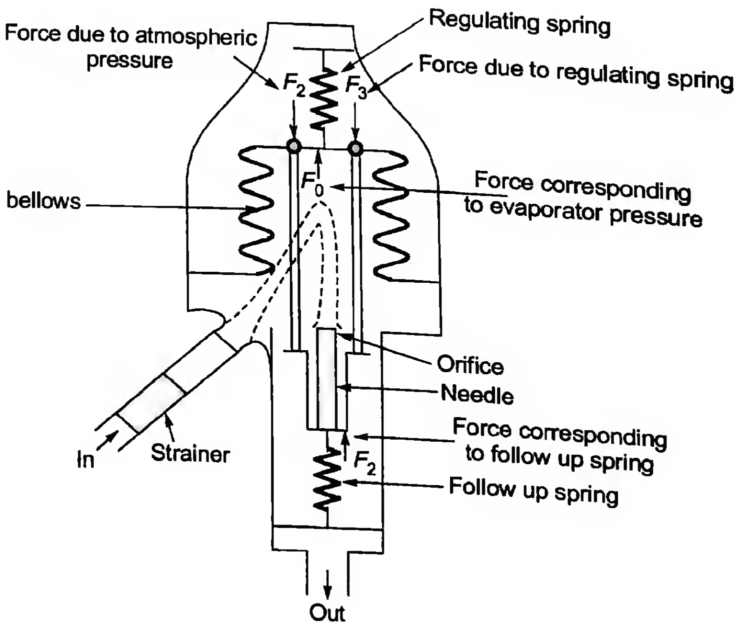
Automatic expansion valve and unsuitability of its application

Consider automatic expansion valve as shown in figure. Refrigerant enters from inlet, passes through strainer and fills up bellows chamber. Follow up spring and regulating spring control the opening of orifice along with pressure force of refrigerant and atmospheric pressure.

During off cycle when the valve is closed,

$$F_0 + F_1 > F_2 + F_3$$

when compressor starts, evaporator pressure starts decreasing and hence force corresponding to evaporator pressure (P_0) i.e. F_0 starts decreasing.



Due to this compression of follow up spring increases and elongation of regulating spring takes place. Hence F_1 increases and F_2 decreases. Under equilibrium,

$$F_0 + F_1 = F_2 + F_3$$

and the orifice opens to allow refrigerant flow through the valve.

Hence, we note an automatic expansion valve maintains same evaporator pressure under all circumstances based on initial settings or regulator and follow up springs.

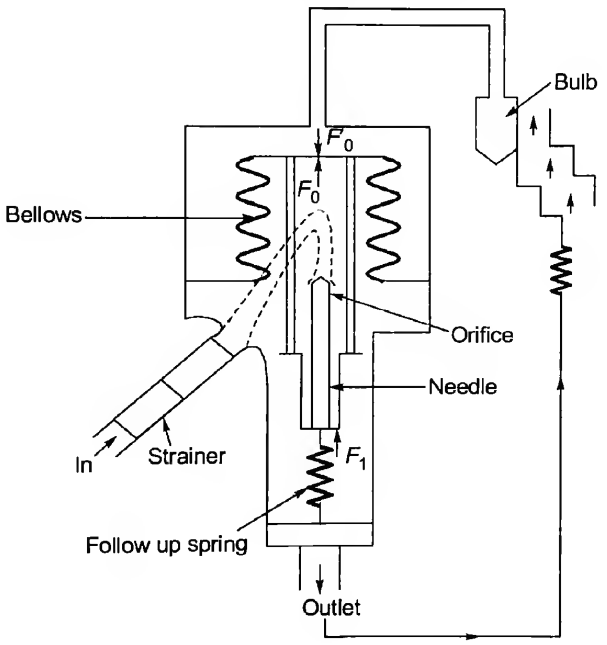
Effect of load variation:

- When load decreases, evaporator pressure decreases hence valve opens more to allow more refrigerant to flood evaporator and raise F_0 to original equilibrium valve.
- When load increases, vapour formation increases and evaporator pressure increases. This allows less refrigerant to pass since orifice remains closed due to increase in F_0.

Hence for varying load applications, automatic expansion valve is not suitable, since it increases refrigerant flow F_r reduced load and reduces refrigerant flow for increased load.

However, it is suitable for constant cooling load milk chillers which require precise temperature control.

Suitable of Thermostatic expansion value



F_0' = Force corresponding to P_0', T_0'
F_0 = Force corresponding to P_0, T_0
F_1 = Follow up spring force

Consider the thermostatic expansion valve as shown above. Refrigerant after passing through inlet and strainer fills up the bellows chamber. Bulb as shown in maintained at the temperature equivalent to evaporator exit and hence represents degree of superheat in the evaporator.

Forces acting upwards and closing the orifice are:

F_0 - corresponding to inlet evaporator temperature T_0 and saturation pressure P_0

F_1 - follow up spring force

Forces acting downwards and opening the orifice are:

F_0' - corresponding to evaporator outlet temperature T_0' and saturation pressure P_0'.

During off cycle, evaporator inlet and outlet temperatures are same and $F_0 = F_0'$

$$\therefore \quad F_0 + F_1 > F_0'$$

When compressor starts, a degree of superheat ($T_0' - T_0$) results in evaporator and corresponding pressure difference ($P_0' - P_0$) creates a downward force ($F_0' - F_0$) on the bellows and the valve opens.

Opening of the valve is determined by equilibrium when:

$$F_0' - F_0 = F_1$$

Hence, by adjusting initial compression of spring, degree of super heat can be controlled.

Also, thermostatic expansion value maintains a constant degree of superheat.

Varying load applications

(a) When evaporator load increases, evaporator produces vapour at a faster pace than compressor can suck and this results in higher degree of superheat. $\therefore$ $(F_0' - F_0)$ increases and valve opens up more to allow more refrigerant to pass through.

(b) Conversely, for lower cooling load, evaporator's degree of superheat reduces and less refrigerant is allowed to pass.

Hence thermostatic expansion valve is more suitable for varying load applications because—

- Mass flow rate increases for higher cooling load
- Mass flow rate reduces for lower cooling load
- Keeps evaporator filled as per load requirement hence ensuring effective utilization of evaporator.

4. Vapour Compression System

Q.15 Calculate the (i) c.o.p (ii) power requirement and (iii) cooling capacity of a CO_2 compressor working between 22.68 bar and 64.32 bar pressure. The liquid in the condenser gets cooled, by another system, to 15°C before entering the expansion valve. The system is assumed to work on wet-compression with $x = 0.9$. The compressor data: stroke volume = 500 c.c.; r.p.m. = 500 and $\eta_{vol} = 0.85$. Property values are: sp. heat of CO_2 vapour = 2.4 kJ/kg °C and,

Pressure	Enthalpy (kJ/kg)		V (m²/kg)		S (kJ/kg°C)		t
(bar)	Liquid	Vapour	Liquid	Vapour	Liquid	Vapour	(°C)
22.68	49.62	322.86	0.00101	0.0166	0.1976	1.2567	–15
50.92	127.75	308.08	0.00130	0.0066	0.4697	1.0959	15
64.32	164.17	283.63	0.00147	0.0042	0.5903	0.9912	25

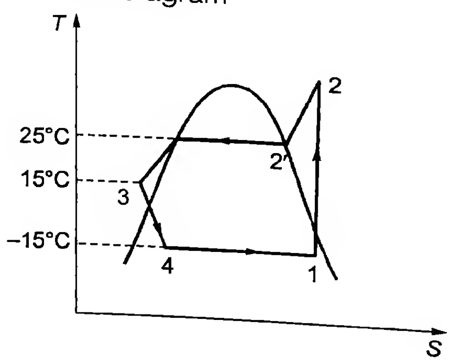
[CSE (Mains) 2001 : 30 Marks]

Solution:

Given: $x = 0.9$, stroke volume, $V_f = 500$ cc, $N = 500$ rpm, $\eta_v = 0.85$

Consider CO_2 vapour compression system as represented by cycle 1-2-3-4 in T-s diagram

We have



For wet compression, $x = 0.9$

$\therefore$ $h_1 = h_{f_{-15°C}} + x \cdot h_{fg_{-15°C}}$

$= 49.62 + 0.9 \times (322.86 - 49.62)$

$\Rightarrow$ $S_1 = 295.536$ kJ/kg

Similarly, $S_1 = S_{f_{-15°C}} + x \cdot S_{fg_{-15°C}}$

$= 0.1976 + 0.9 \cdot (1.2567 - 0.1976)$

$= 1.15079$ kJ/kg K

For isentropic process 1-2,

$S_1 = S_2 = 1.151$ kJ/kg K

$S_2' = S_{g_{25°C}} = 0.9912$ kJ/kg K

For isobaric process 2-2′,

$$\text{Entropy change} = S_2 - S_2' = C_p \ln\frac{T_2}{T_2'} = 2.4\ln\frac{T_2}{298}$$

$$\Rightarrow \quad T_2 = 318.49\text{ K}$$

$$\therefore \quad h_2 = h_2' + C_p(T_2 - T_2') = 283.63 + 2.4\,(318.49 - 298) = 332.81\text{ kJ/kg}$$

Ideal volumetric flow rate in compressor,

$$\dot{V}_f = 500\text{ cm}^3 \times \frac{500}{60s} = 4166.67 \times 10^{-6}\text{ m}^3/\text{s}$$

$$\text{Actual volumetric flow rate} = \dot{V}_{f,\text{act}} = \dot{V}_f \times \eta_v = 4.167 \times 10^{-3} \times 0.85 = 3.542 \times 10^{-3}\text{ m}^3/\text{s}$$

$$\text{Mass flow rate of refrigerant, } \dot{m} = \frac{\dot{V}_{f,\text{act}}}{v_1} = \frac{\dot{V}_{f,\text{act}}}{v_f + x\,v_{fg}} = \frac{3.542 \times 10^{-3}}{(0.00101 + 0.9 \times 0.0166)} \qquad \{V_{fg} \simeq V_g\}$$

$$= 0.222\text{ kg/s}$$

For condensation, $h_3 \equiv h_{f_{15°C}} = 127.75$ kJ/kg

For isenthalpic process 3-4, $h_4 = h_3 = 127.75$ kJ/kg

$$\therefore \quad \text{Refrigerant capacity of system} = \dot{m}(h_1 - h_4) = 0.222 \times (295.536 - 127.75)$$

$$\Rightarrow \quad Q_0 = 37.26\text{ kW}$$

$$\text{Power requirement of compressor} = W = \dot{m}(h_2 - h_1) = 0.222 \times (332.81 - 295.536) = 8.27\text{ kW}$$

$$\therefore \quad \text{COP of system} = \frac{Q_0}{W} = \frac{37.26}{8.27} = 4.51$$

Q.16 A vapour compression refrigerator works between pressure limits of 10 bar and 3 bar. The working fluid is dry at the end of compression and there is no undercooling before the expansion valve. If refrigerant flow rate is 10 kg/min, determine (i) COP and (ii) the capacity of the refrigerant. Table for properties of the refrigerant is as under:

Pressure bar	Saturation Temp. °C	Liquid heat (kJ/kg)	Latent heat (kJ/kg)	Liquid entropy (kJ/kg K)
10	25	298.90	1166.94	1.1242
3	–10	135.37	1297.68	0.5443

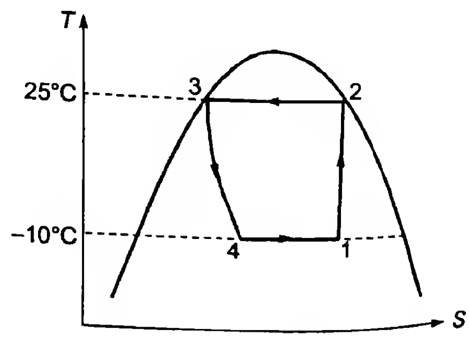
[CSE (Mains) 2002 : 30 Marks]

Solution:

Consider the vapour compression refrigeration cycle as shown in T-s diagram here

Mass flow rate of refrigerant = 10 kg/min

$$\Rightarrow \quad \dot{m} = 0.167\text{ kg/s}$$



For isentropic process 1-2, $S_1 = S_2 = S_{g_{25°C}}$

At constant pressure and temperature

$$T_{sat} = 25°\text{C}, P_{sat} = 10\text{ bar}$$

As,

$$S_{fg} = \frac{h_{fg}}{T_{sat}}$$

$$\Rightarrow \quad (S_g - S_f)_{25°C} = \frac{h_{fg_{25°C}}}{298}$$

$$\Rightarrow \quad S_{g_{25^\circ C}} = S_{g_{25^\circ C}} + \frac{h_{fg_{25^\circ C}}}{298} = 1.1242 + \frac{1166.94}{298} = 5.04 \text{ kJ/kg K}$$

$$\therefore \quad S_1 = S_2 = S_{g_{25^\circ C}} = 5.04 \text{ kJ/kg K}$$

$$\Rightarrow \quad S_f + x\, S_{fg} = 5.04$$

$$\Rightarrow \quad S_{f_{-10^\circ C}} + x \cdot \frac{h_{fg_{-10^\circ C}}}{263} = 5.04 \Rightarrow x = 0.91$$

$$\therefore \quad h_1 = h_{f_{-10^\circ C}} + x \cdot h_{fg_{-10^\circ C}} = 1316.26 \text{ kJ/kg}$$

$$h_2 = h_{g_{25^\circ C}} = h_{fg_{25^\circ C}} + h_{f_{25^\circ C}} = 1465.84 \text{ kJ/kg}$$

For isenthalpic process 3-4,

$$h_4 = h_3 = h_{f_{25^\circ C}} = 298.9 \text{ kJ/kg}$$

$$\text{Refrigeration capacity} = \dot{m}(h_1 - h_4) = 0.167 \times (1316.26 - 298.9) = 169.9 \text{ kW}$$

$$\Rightarrow \quad Q_0 = \frac{169.9}{3.5167} \text{ TR} = 48.38 \text{ TR}$$

$$\text{Compressor work, } W = \dot{m}(h_2 - h_1) = 0.167 \times (1465.84 - 1316.26) = 24.98 \text{ kW}$$

$$\text{COP of system} = \frac{Q_0}{W} = 6.8$$

Q.17 In a refrigeration system of 10 TR cooling capacity using $CHClF_2$, the evaporator and condenser temperatures are –10°C and 45°C respectively. Properties of $CHClF_2$ at saturation are:

Temperature °C	Sp. volume m³/kg v_g	Enthalpy (kJ/kg)		Entropy (kJ/kg-K)	
		h_f	h_g	s_f	s_g
–10	0.0654	34.25	247.37	0.1374	0.9473
45	0.0133	101.76	261.95	0.3662	0.8697

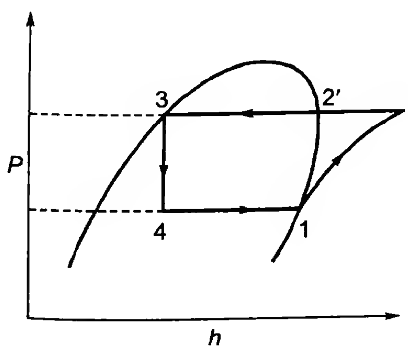
Consider standard vapour compression cycle with inlet to compressor as saturated vapour and inlet to expansion valve as saturated liquid. Assume that vapour may be treated as perfect gas in desuperheating process with average specific heat of 0.9335 kJ/kg-K show the cycle on *T-s* and *P-h* diagrams. Find (i) compressor outlet temperature and enthalpy, (ii) mass flow rate of refrigerant, (ii) work requirement, (iv) condenser heat rejection (v) COP and (vi) swept volume of compressor in m³/s assuming 100% volumetric efficiency.

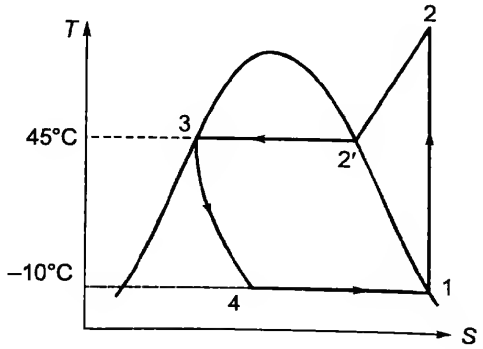
[CSE (Mains) 2003 : 30 Marks]

Solution:

Given: R.C. = Q_0 = 10 TR, C_p = 0.9335 kJ/kg-K

Consider the refrigeration system as shown in *P-h* and *T-s* diagrams





Compressor process 1-2 is isentropic in nature

$$\therefore \quad S_2 = S_1 = S_{g_{-10°C}} = 0.9473 \text{ kJ/kg K}$$

$$S_2' = S_{g_{45°C}} = 0.8697 \text{ kJ/kg K}$$

for isobaric process 2-2′,

$$\text{entropy change} = S_2 - S_2' = C_p \ln\frac{T_2}{T_2'} = 0.9335 \cdot \ln\frac{T_2}{318}$$

$$\Rightarrow \quad \ln\frac{T_2}{318} = \frac{0.9473 - 0.8697}{0.9335}$$

$$\Rightarrow \quad T_2 = 345.56$$

∴ Compressor outlet temperature is 345.56 K

Compressor outlet specific enthalpy, $h_2 = C_pT_2 = h_2' + C_p(T_2 - T_2') = 261.95 + 0.9335(345.56 - 318)$

$$= 287.68 \text{ kJ/kg}$$

$$h_1 = h_{g_{-10°C}} = 247.37 \text{ kJ/kg}$$

For isenthalpic process 3-4, $h_4 = h_3 = h_{f_{45°C}} = 101.76$ kJ/kg

∴ Refrigeration effect, $h_1 - h_4 = 145.61$ kJ/kg

∴ Mass flow rate of refrigerant $= \dfrac{Q_0}{h_1 - h_4} = \dfrac{10 \times 3.5167}{145.61}$

$$\Rightarrow \quad \dot{m} = 0.2415 \text{ kg/s}$$

∴ Compressor outlet enthalpy, $\dot{m} \cdot h_2 = 69.47$ kJ/s

Work requirement in compressor $= \dot{m} \cdot (h_2 - h_1) = 9.73$ kW

Condensor that rejection $= \dot{m} \cdot (h_2 - h_3) = 44.9$ kW

$$\text{COP of system} = \frac{Q_0}{W} = \frac{10 \times 3.5167}{9.73} = 3.61$$

Swept volume of compressor assuming 100% volumetric efficiency,

$$\dot{V}_s = \dot{m} \cdot v_1 = \dot{m} \cdot v_{g_{-10°C}} = 0.2415 \times 0.0654$$
$$= 0.0158 \text{ m}^3\text{/s}$$

Q.18 A vapour compression refrigerator operates between the pressure limits of 462.47 kN/m² and 1785.90 kN/m². At entry to the compressor the refrigerant is dry saturated and after compression it has a temperature of 59°C. The compressor has a bore and stroke of 75 mm and runs at 8 rev/s with a volumetric efficiency of 80 percent. The temperature of the liquid refrigerant as it leaves the condenser is 32°C and the specific heat capacity of the superheated vapour may be assumed constant. Determine :

(i) The coefficient of performance of the refrigerator

(ii) The mass flow of the refrigerant in kg/h.

(iii) The cooling water required by the condenser in kg/h if the cooling water temperature rise is limited to 12°C.

Take the specific heat capacity of water as 4.187 kJ/kg K and the specific heat of liquid refrigerant as 1.32 kJ/kg K. The relevance properties of the refrigerant are given in the table below:

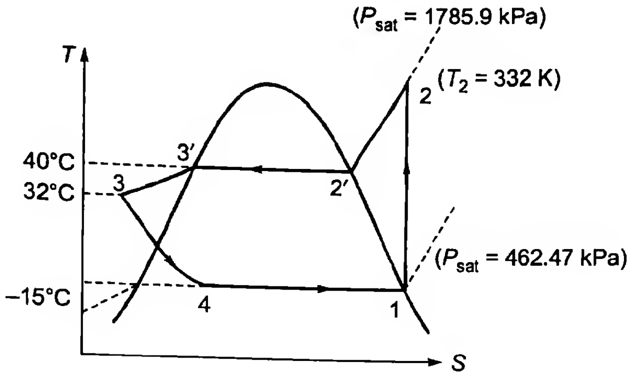
Pressure (kN/m²)	Sat. Temp. (t_s°C)	Sp. Enthalpy		Sp. Vol. (m³/kg)		Sp. Entropy	
		h_f	h_g	v_f	v_g	s_f	s_g
462.47	−10	35.732	231.40	0.0008079	0.04573	0.1418	0.8614
1785.90	40	99.270	246.40	0.0009487	0.01105	0.3537	0.8093

[CSE (Mains) 2004 : 20 Marks]

Solution:

Given: $T_2 = 59°C = 332$ K, $D = 75$ mm $= L$, $N = 8$ rev/s, $\eta_v = 80\%$

Consider vapour compression refrigeration system as shown in T-s diagram.



$$\text{Volume of compressor} = \frac{\pi D^2}{4}\cdot L = \frac{\pi}{4}\cdot(0.075)^2\cdot(0.075) = 3.3134\times10^{-4}\ \text{m}^3$$

$$\text{Ideal volumetric flow rate} = 3.134\times10^{-4}\times 8\ \text{m}^3/\text{s} = 2.651\times10^{-3}\ \text{m}^3/\text{s}$$

$$\text{Actual volumetric flow rate} = \dot{V}\times\eta_V = 2.651\times10^{-3}\times0.8$$

$$\Rightarrow \quad \dot{V}_{act} = 2.12\times10^{-3}\ \text{m}^3/\text{s}$$

$\therefore$ Mass flow rate of refrigerant

$$\dot{m} = \frac{\dot{V}_{act}}{v_1} = \frac{\dot{V}_{act}}{v_{g-10°C}} = \frac{2.12\times10^{-3}}{0.04573}\ \text{kg/s}$$

$$= 0.0464\ \text{kg/s} = 167.04\ \text{kg/hour}$$

For isentropic process 1-2,

$$S_2 = S_1 = S_{g-10°C} = 0.8614\ \text{kJ/kg K}$$

$$S_2' = S_{g40°C} = 0.8093\ \text{kJ/kg K}$$

For isobaric process 2-2′,

$$S_2 - S_2' = C_p \ln\frac{T_2}{T_2'} = C_p \ln\frac{332}{313}$$

$$\Rightarrow \quad C_p = 0.8841\ \text{kJ/kg K}$$

$$\therefore \quad h_2 = h_2' + C_p(T_2 - T_2') = 246.4 + 0.8841\,(59-40) = 263.2\ \text{kJ/kg}$$

$$h_1 = h_{g-10°C} = 231.4\ \text{kJ/kg}$$

$$h_3' = h_{g40°C} = 99.27\ \text{kJ/kg}$$

$$\Rightarrow \quad h_3 - h_3' = C_{p,\,iq}\cdot(T_3 - T_3') = 1.32\times(32-40) = -10.56$$

$$\Rightarrow \quad h_3 = h_3' - 10.56 = 88.71\ \text{kJ/kg}$$

For isentropic process 3-4, $\quad h_4 = h_3 = 88.71$ kJ/kg

Refrigeration effect, $\quad Q_0 = \dot{m}(h_1 - h_4) = 6.621$ kW

Compressor input, $\quad W = \dot{m}(h_2 - h_1) = 1.476$ kW

$$\therefore \quad \text{COP of system} = \frac{Q_0}{W} = \frac{6.621}{1.476} = 4.49$$

Let the mass flow rate of cooling water required be $\dot{m}_w$.

∴ Heat rejected in condensor $= \dot{m}(h_2 - h_3) = \dot{m}_w C_{p,w} \Delta T$

$$\Rightarrow \quad \dot{m}_w = \frac{0.0464 \times (263.2 - 88.71)}{4.187 \times 12} = 0.161 \text{ kg/s} = 580.11 \text{ kg/hr}$$

∴ Mass flow rate of water required is 580.11 kg/hour.

Q.19 **A VCC refrigerating machine using R-12 refrigerant produces 10 tonnes of refrigeration at 10°C when the ambient is at 35°C. A temperature difference of minimum 5°C is required at the evaporator and condenser for spontaneous heat transfer. The refrigerant is dry saturated at the outlet and to the inlet of compressor. The adiabatic efficiency of the compressor is 90%. The enthalpy at the end of isentropic compression is estimated to be 370 kJ/kg. Determine:**

(i) COP

(ii) Power of the compressor

(iii) Capacity of the condenser

Represent the cycle on hand drawn *T-S* plane and show the refrigerating effect, compressor work and condenser capacity on the same.

Temperature (°C)	Pressure (Bar)	Specific Enthalpy, kJ/kg	
		Sensible	Evaporation
5	3.60	204.64	148.97
10	4.23	209.32	146.37
15	4.91	214.10	143.69
30	7.45	228.54	135.04
35	8.47	233.50	131.90
40	9.60	238.53	128.62

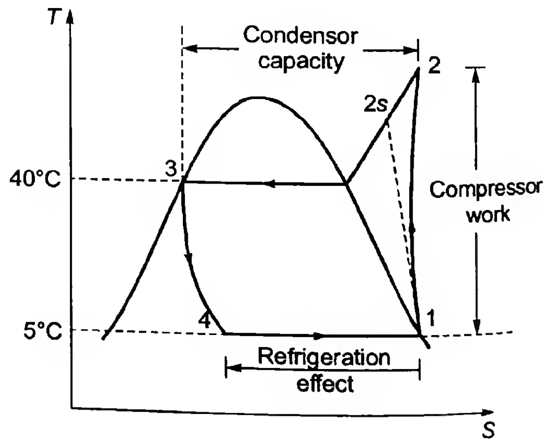
[CSE (Mains) 2006 : 30 Marks]

Solution:

Consider vapour compression refrigeration cycle as represented in figure.

Cycle operates between surroundings at 35°C and refrigerated space at 10°C. Since a minimum 5°C temperature difference is required for operation, evaporator and condensor temperatures are 5°C and 40°C, respectively.

Refrigeration effect, compressor work and condensor capacity are as depicted in figure.



$$h_1 = h_{g_{5°C}} = h_{f_{5°C}} + h_{fg_{5°C}}$$
$$= 353.61 \text{ kJ/kg}$$
$$h_{2s} = 370 \text{ kJ/kg}$$

$$\text{Adiabatic efficiency of compressor, } \eta = \frac{h_1 - h_{2s}}{h_1 - h_2} = 0.9$$

$$\Rightarrow \quad \frac{353.61 - 370}{353.61 - h_2} = 0.9$$

$$\Rightarrow \quad h_2 = 371.82 \text{ kJ/kg}$$

$$h_4 = h_3 = h_{f_{40°C}} = 238.53 \text{ kJ/kg} \quad \text{(Process 3-4 is isentropic)}$$

$$\text{Refrigeration effect} = h_1 - h_4 = 115.08 \text{ kJ/kg}$$

$$\therefore \quad \text{Mass flow rate of refrigerant} = \frac{Q_0}{h_1 - h_4} = \frac{10 \times 3.5167}{115.08} = 0.3056 \text{ kg/s}$$

Power of compressor, $W = \dot{m}(h_2 - h_1) = 0.3056\,(371.82 - 352.61) = 5.56$ kW

$$\text{COP} = \frac{Q_0}{W} = \frac{10 \times 3.5167}{5.56} = 6.32$$

Capacity of condensor $= \dot{m}(h_2 - h_3) = 0.3056 \times (371.82 - 238.53) = 40.73$ kW

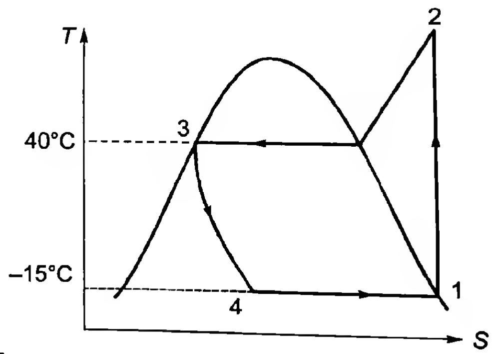
Q.20 An ammonia ice plant operates on simple saturation cycle at the following temperatures:
Condensing temperature = 40°C
Evaporation temperature = –15°C
It produces 10 tonnes of ice per day at –5°C from water at 30°C. If the COP of the system is 5, determine:
(i) Capacity of the refrigeration plant in tonnes of refrigeration
(ii) Mass flow rate of the refrigerant, kg/min
(iii) Isentropic discharge enthalpy, kJ/kg
Take:
C_p = 4.187 kJ/kg-K for water = 2.000 kJ/kg-K for ice
Latent heat of fusion of ice = 335 kJ/kg
Sensible enthalpy at 40°C = 600 kJ/kg
Enthalpy of sat. vapour at –15 = 1675 kJ/kg

[CSE (Mains) 2006 : 20 Marks]

Solution:

Given: Ice produced 'm' = 10 tonnes/day, COP = 5

Consider ammonia samples saturation cycle as represented in T-s diagram here.



Cycle operates between –15°C and 4°C.

Amount of heat removed to form ice in given condition

$$= m[C_{p'w}(T_f - 273) + h_{\text{fusion}} + C_{p,\text{ice}}(T_f - 273)]$$
$$= 10 \times 10^3\,[4.187 \times 30 + 335 + 2 \times 5]$$
$$= 470.61 \times 10^4 \text{ kJ} = 4.7061 \times 10^6 \text{ kJ}$$

$$\text{Refrigeration capacity of plant} = \frac{4.7061 \times 10^6 \text{ kJ}}{24 \times 3600 \text{ sec}} = 54.47 \text{ kW} = \frac{54.47}{3.5167} \text{ tonnes of refrigeration}$$
$$= 15.5 \text{ TR}$$

We have, $h_1 = 1675$ kJ/kg

$$h_4 = h_3 = h_{f_{40°C}} = 600 \text{ kJ/kg}$$

$$\therefore \quad \text{Mass flow rate of refrigerant} = \frac{Q_0}{h_1 - h_4} = \frac{54.47}{1675 - 600} = 0.051 \text{ kg/s}$$

$$\text{COP of system} = \frac{Q_0}{W} = \frac{Q_0}{(h_2 - h_1)\dot{m}} = 5$$

$$\Rightarrow \quad h_2 - h_1 = \frac{Q_0}{m \cdot 5} = \frac{54.47}{0.051 \times 5}$$

$$\Rightarrow \quad h_2 - 1675 = 215.004$$

$$\Rightarrow \quad h_2 = 1890 \text{ kJ/kg}$$

Hence, isentropic discharge enthalpy is 1890 kJ/kg.

Q.21 A refrigeration system operates using simple saturated cycle with a certain refrigerant. The condensing and evaporating temperatures for the refrigerant are 35°C and –15°C respectively. Determine the COP of the system. If a liquid vapour heat exchanger is installed in the system, with the temperature of the vapour leaving the heat exchanger at 15°C, what will be the change in the COP?

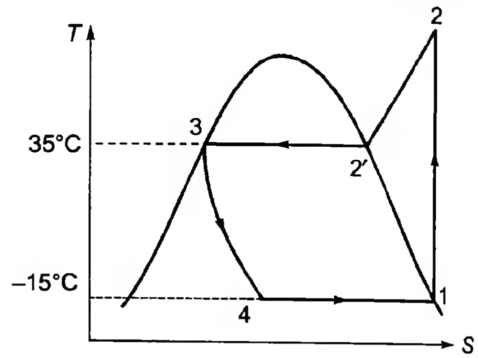
Use the following data for the refrigerant used:

				Superheated			
				20 K		40 K	
h_g	t	h_f	s_g	h	s	h	s
kJ/kg	(°C)	(kJ/kg)	(kJ/kg-L)	(kJ/kg)	(kJ/kg K)	(kJ/kg)	(kJ/kg K)
201.5	35	69.5	0.6839	216.4	0.731	231.0	0.7741
181.5	–15	-	0.7052	193.2	0.751	205.7	0.7942

[CSE (Mains) 2007 : 30 Marks]

Solution:

Consider the simple saturated refrigeration cycle as represented by T-s diagram here.



For isentropic process 1-2,

$$S_1 = S_2 = S_{g_{-15°C}}$$
$$= 0.70520 \text{ kJ/kg K}$$
$$S_2 < 0.731 \text{ kJ/kg K}$$

∴ For superheat table

Similarly, $$\frac{T_2 - T_2'}{S_2 - S_2'} = \frac{20}{0.731 - 0.6839}$$

$$\Rightarrow \quad T_2 = (35+273) + \frac{20}{0.731-0.6839} \times (0.7052 - 0.6839)$$
$$= 317.04 \text{ K}$$

$$\frac{h_2 - h_2'}{T_2 - T_2'} = \frac{216.4 - 201.5}{20}$$

$$\Rightarrow \quad h_2 = h_2' + \frac{216.4 - 201.5}{20} \times (317.04 - 308)$$

$$\Rightarrow \quad h_2 = 208.23 \text{ kJ/kg}$$

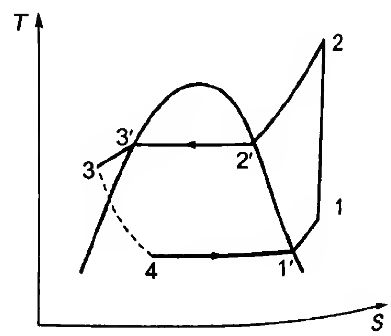
$$\therefore \quad \text{Compressor power} = h_2 - h_1 = h_2 - h_{g_{-15°C}} = 208.23 - 181$$

$$\Rightarrow \quad W = 27.23 \text{ kW}$$

$$\text{Refrigeration capacity, } q_0 = h_1 - h_4 = h_1 - h_3 = h_1 - h_{f_{35°C}} = 181 - 69.5 = 111.5 \text{ kW}$$

$$\therefore \quad \text{COP of the system} = \frac{q_0}{W} = \frac{111.5}{27.23} = 4.09$$

(ii) Now when H.E is added



- Given temperature of vapour leaving H.E is 15°C thus 30 K superheat.
- Calculating value of s & h for evaporator at 30 K superheat by taking average.

Thus, $$h_1 = 199.45 \text{ kJ/kg}$$
$$s_1 = 0.7726 \text{ kJ/kg-K}$$

We know that $$s_1 = s_2 = 0.7726 \text{ kJ/kg-K}$$

$\Rightarrow$ Now interpolating s_2 to get h_2 $$= \frac{0.7726-0.6839}{0.7741-0.6839} = \frac{h_2-201.5}{231-201.5}$$

$$h_2 = 230.51 \text{ kJ/kg}$$

Also in H.E,

$$h_1 - h_1' = h_3' - h_3$$

$$199.45 - 181 = 69.5 - h_3$$

$\Rightarrow$

$$h_3 = 51.05 \text{ kJ/kg}$$

$$W_{I.P} = h_2 - h_1 = 230.51 - 199.45 = 31.06 \text{ kJ/kg}$$

Q.22 A saturated vapour compression refrigeration system is extracting heat from a thermal reservoir at –10°C and rejecting heat to another thermal reservoir at 36°C. The saturation temperature of evaporator is –20°C and that of condenser is 46°C. The mass flow rate of refrigerant (R – 134 a) is 0.1 kg/s. Assume environment temperature equal to 36°C. Find :

(i) Refrigerating capacity in Tons
(ii) Power input in kW
(iii) COP
(iv) COP of Carnot refrigeration cycle
(v) Second law efficiency of the cycle.

Compare with the help of T-s diagram, the vapour compression cycle and Carnot refrigeration cycle and show the deviation between the two cycles by shaded areas.

Properties of refrigerant (Saturated)

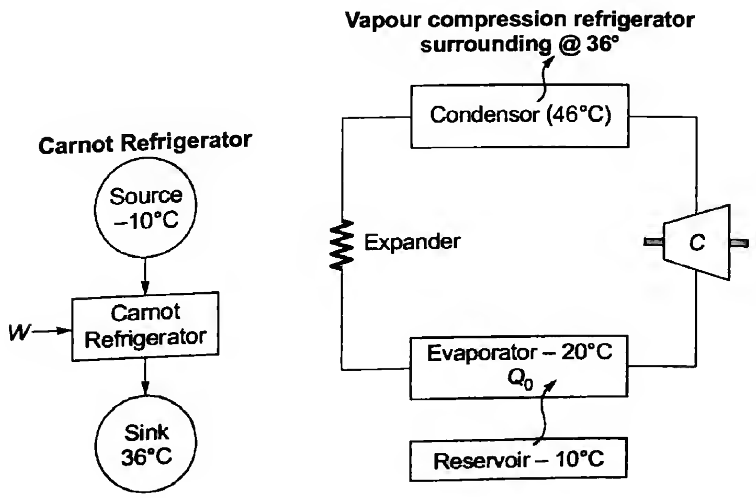
Temp (°C)	Saturation pressure (MPa)	Enthalpy sat liquid (h_f)	(kJ/kg) sat vapour (h_g)	Entropy sat liquid (s_f)	(kJ/kg-K) sat vapour (s_g)
–20	0.13273	173.64	386.55	0.9002	1.7413
–10	0.20060	186.70	392.66	0.9506	1.7334
36	0.91185	250.48	417.65	1.1717	1.7124
46	1.1903	265.47	421.92	1.2186	1.7089

Super heated

Pressure (MPa)	h_g (kJ/kg)	s_g (kJ/kg-K)
1	428.91	1.7413
1.2	436.12	1.7413

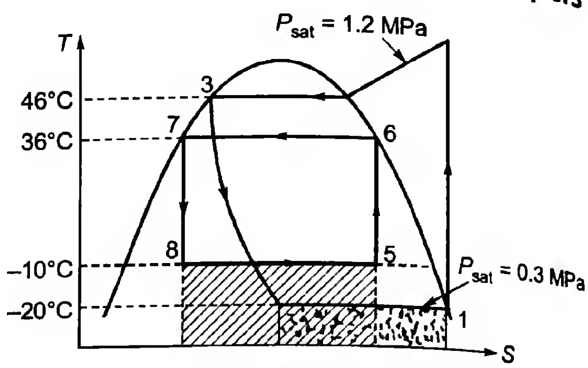
[CSE (Mains) 2010 : 20 Marks]

Solution:



Consider a Carnot refrigerator and a vapour compression refrigerator - both extracting heat from reservoir @ −10°C and rejecting heat to surrounding at 36°C.

Carnot cycle in this case is represented by process 5-6-7-8 and shaded area ▨ represents the refrigeration capacity while vapour compression cycle is shown by process 1-2-3-4 and shaded area ▦ represents its refrigeration capacity.



Mass flow rate of refrigerant = 0.1 kg/s

Since process 1-2 is isentropic

$$S_2 = S_1 = S_{g\text{-}20°C} = 1.7413 \text{ kJ/kg K}$$

Since $P_2 = P_3 = P_{sat} = 1.2$ MPa, from superheat table, we get

$$h_2 = 436.12 \text{ kJ/kg}$$

$\Rightarrow$ $$\text{Refrigerating capacity} = \dot{m}(h_1 - h_4) = \dot{m}(h_{g_{-20°C}} - h_{f_{46°C}}) \qquad (h_4 = h_4 = h_{f_{46°C}})$$

$$= 0.1 \times (386.55 - 265.47) = 12.108 \text{ kW} = 3.44 \text{ TR}$$

$$\text{Power input to compressor, } \dot{m}(h_2 - h_1) = 0.1 \times (436.12 - 386.55) = 4.957 \text{ kW}$$

$\Rightarrow$ $$\text{COP} = \frac{Q_0}{W} = \frac{12.108}{4.957} = 2.443$$

$$\text{COP of Carnot refrigerator} = \frac{T_0}{T_K - T_0} = \frac{253}{319 - 253} = 3.833$$

$$\text{Second law efficiency of the cycle, } \eta_{II} = \frac{\text{COP}}{\text{COP}_{\text{Carnot}}} = \frac{2.443}{3.833} = 0.6373 = 63.73\%$$

Q.23 A commercial refrigerator with refrigerant 134a as the working fluid keeps a space cooled at −30°C. It rejects heat to cooling water that enters the condenser at 18°C and at the rate of 0.25 kg/s and it leaves at 26°C. The refrigerant enters the condensor at 1.2 MPa and 65°C and it leaves at 42°C. The inlet state of compressor is 60 kPa abd −34°C. It gains a net heat of 450 W from the surroundings. Sketch *T-S* diagram and determine,

(i) refrigeration load (ii) COP

(iii) theoretical maximum refrigerant load for the same power input to the compressor.

Given properties:

$h^{60\text{ kPa}}_{-34°C}$ **= 230.03 kJ/kg** $\qquad$ $h^{1200\text{ kPa}}_{65°C}$ **= 295.16 kJ/kg**

$h^{1200\text{ kPa}}_{42°C}$ **= 111.23 kJ/kg** $\qquad$ $x^{60\text{ kPa}}_{111.23\text{ kJ/kg}}$ **= 0.47 kJ/kg**

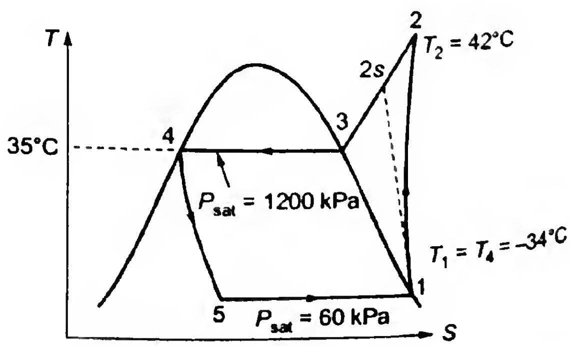
$h_{\text{water, }18°C}$ **= 75.4 kJ/kg** $\qquad$ $h_{\text{water, }26°C}$ **= 108.9 kJ/kg**

[CSE (Mains) 2012 : 20 Marks]

Solution:

Consider vapour compression refrigeration cycle as represented by *T-s* diagram.

Process 1-2 of compression involves absorption of heat from surroundings.



We have,

$$h_1 = h^{60\text{ kPa}}_{-34°C} = 230.03 \text{ kJ/kg}$$

$$h_2 = h^{1200\text{ kPa}}_{65°C} = 295.16 \text{ kJ/kg}$$

$$h_3 = h^{1200\text{ kPa}}_{42°C} = 111.23 \text{ kJ/kg}$$

Cooling water enters at 18°C at 0.25 kg/s and leaves at 26°C.

$\therefore$ Heat rejected in condensor $= \dot{m}_w \cdot (h_{w,26°C} - h_{w,18°C}) = 0.25 \times (108.9 - 75.4) = 8.375$ kW

For refrigerant:

Heat rejected in condensor $= \dot{m}(h_2 - h_3) = 8.375$ kW

$\Rightarrow$ $\dot{m} = 0.0455$ kg/s

$\therefore$ Refrigeration load $= \dot{m}(h_1 - h_4) = 0.0455 \times (230.03 - 111.23)$

$\Rightarrow$ $Q_0 = 5.409$ kW

Applying SFEE for operation of compressor,

$$h_1 + q = h_2 + w$$

$\Rightarrow$ $w = -[(h_2 - h_1) - q] = -(2.963 - 0.45) - 2.513$ kW

Since this is work done on the compressor, it is expressed as negative.

$$\text{COP of refrigeration} = \frac{Q_0}{W} = \frac{5.409}{2.513} = 2.15$$

For theoretical maximum refrigerant load, heat gain from surroundings is neglected.

Hence work input to compressor $= W = (h_2 - h_1)\,\dot{m} = 2.963$ kW

Condensor load $= 8.375$ kW

$\therefore$ Max. refrigeration load $= 8.375 - 2.963 = 5.412$ kW

Q.24 In a standard vapour compression, refrigeration cycle, the specific enthalpies of refrigerant at the end states of different processes in ascending order are 74.6 kJ/kg, 185.4 kJ/kg and 208.0 kJ/kg. If the mass flow rate of refrigerant is 30 kg/min, calculate power consumption and COP of the cycle.

[CSE (Mains) 2013 : 10 Marks]

Solution:

Consider a standard vapour compression refrigeration cycle as shown in figure.

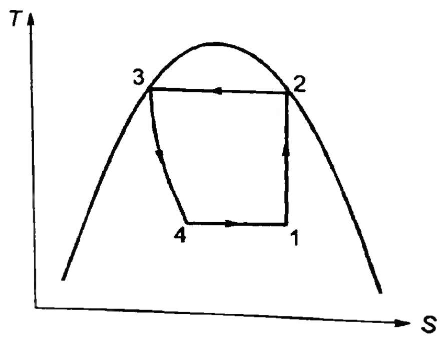
As mentioned, enthalpies at various states are

$h_3 = h_4 = 74.6$ kJ/kg

(isenthalpic process)

$h_1 = 185.4$ kJ/kg

$h_2 = 208.0$ kJ/kg



Mass flow rate of refrigerant, $\dot{m} = 30$ kg/min $= 0.5$ kg/s

Power consumption by compressor,

$$W = \dot{m}(h_2 - h_1)$$
$$= 0.5 \times (208 - 185.4) = 11.3 \text{ kW}$$

Refrigeration capacity, $q = \dot{m}(h_1 - h_4) = 0.5 \times (185.4 - 74.6) = 55.4$ kW

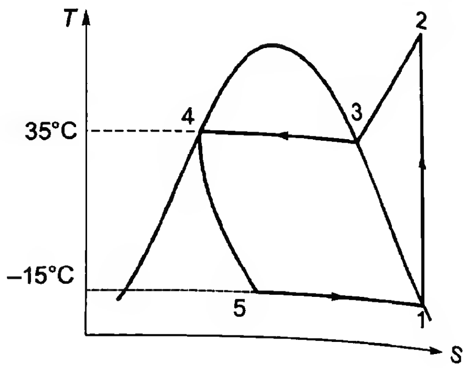
$$\text{COP of the cycle, COP} = \frac{q}{W} = \frac{55.4}{11.3} = 4.9$$

Q.25 An R_{12} simple saturation cycle operates at temperatures of 35°C and –15°C. Determine the COP and HP per ton of refrigeration of the system.

				Superheated			
				20 K		40 K	
t(°C)	h_f(kJ/kg)	h_g(kJ/kg)	s_g(kJ/kg K)	h(kJ/kg)	s(kJ/kg K)	h(kJ/kg)	s(kJ/kg K)
35	69.5	201.5	0.6839	216.4	0.731	231.0	0.7741
–15		181.0	0.7052	193.2	0.751	205.7	0.7942

[CSE (Mains) 2014 : 10 Marks]

Solution:



Consider a simple saturation vapour compression cycle as represented in T-s diagram here.

Cycle operates between 35°C (308 K) and –150° (258 K).

For isentropic process 1-2,

$$S_2 = S_1 = S_{g_{-15°C}}$$
$$= 0.7052 \text{ kJ/kg K}$$
$$< 0.731 \text{ kJ/kg K}$$

$$\therefore \quad T_2 = T_3 + \frac{0.7052 - 0.6839}{0.731 - 0.6839} \times 20 = 317.04 \text{ K}$$

Hence actual degree of superheat is $T_2 - T_3 = 9.04$ K

$$\therefore \quad h_2 = h_3 + \frac{216.4 - 201.5}{20} \times (T_2 - T_3)$$

$$\Rightarrow \quad h_2 = h_{g_{35°C}} + 6.738 = 201.5 + 6.738 = 208.24 \text{ kJ/kg}$$

$$\therefore \quad \text{compressor capacity} = h_2 - h_1 = h_2 - h_{g_{-15°C}} = 27.24 \text{ kJ/kg}$$

Since process 4-5 is isenthalpic,

$$h_5 = h_4 = h_{f_{35°C}} = 69.5 \text{ kJ/kg}$$

$$\therefore \quad \text{Refrigeration capacity, } q_0 = h_1 - h_5 = 181 - 69.5 = 111.5 \text{ kJ/kg}$$

$$\therefore \quad \text{COP of refrigeration} = \frac{q_0}{W} = 4.09$$

$$H \cdot P/\text{ton of refrigeration COP} = \frac{R \cdot E}{W_{I/P}} \Rightarrow 4.09 = \frac{3.5 \text{ kW}}{W_{I/P}}$$

$$W_{I/P} = 0.8557 \text{ kW or } 1.147 \text{ HP/tonn refrigeration}$$

Q.26 A refrigerating machine, rated to produce 40 tons of refrigeration, is used for air conditioning between the operative temperatures of 42°C and 6°C of condenser and evaporator respectively. The refrigerant is dry saturated at the end of compressor. Find the capacity of the plant, power and capacities of compressor and condenser.

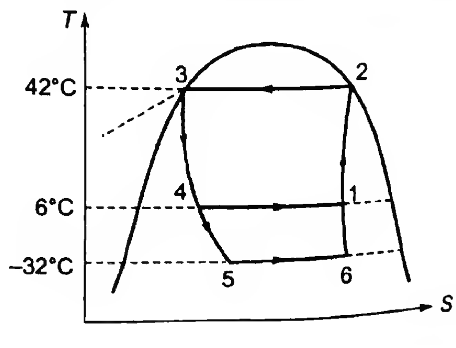
Properties of refrigerant

Temp. (°C)	Pressure bar	Enthalpy (kJ/kg)		Entropy (kJ/kg-K)		Volume (m^3/kg)
		h_f	h_g	s_f	s_g	
42	1.957	249.7	410.7	1.125	1.6712	0.6975
6	0.5160	-	407.15	1.018	1.687	0.04035
–32	0.0875	-	390.85	0.9178	1.715	0.1665

If the evaporator's temperature is reduced to –32°C, what will be effect on capacity of plant, power and capacities of compressor and condenser?

[CSE (Mains) 2015 : 20 Marks]

Solution:



Consider the simple vapour compression cycle as represented in T-s diagram.

Process 1-2-3-4 represents the refrigeration cycle working between 42°C and 6°C.

$$\text{Refrigeration capacity, } Q_0 = 40 \text{ TR} = 40 \times 3.5167$$
$$= 140.668 \text{ kW}$$

$$\text{Refrigeration effect} = h_1 - h_4 = h_1 - h_3 \quad \dots (1)$$

Since 1-2 is an isentropic process, $S_2 = S_1 = S_{f_1} + xS_{f_g}$

$\Rightarrow$ $S_2 = S_{g2} = 1.6712 = 1.018 + x \cdot (1.687 - 1.018)$

$\Rightarrow$ $x = 0.976$

$$h_{f_{42°C}} = 249.7 \text{ kJ/kg}$$

$$h_2 = h_{g_{42°C}} = 410.7 \text{ kJ/kg}$$

For phase change at 6°C or 279 K:

Entropy change, $$(S_g - S_f) = \frac{h_g - h_f}{T_{sat}}$$

$\Rightarrow$ $$h_{f_{6°C}} = -(S_{fg_{6°C}} \times 279) + h_{g_{6°C}} = 220.5 \text{ kJ/kg}$$

Similarly, $$h_{f_{-32°C}} = h_{g_{-32°C}} - s_{fg_{-32°C}} \times 241 = 198.73 \text{ kJ/kg}$$

$\therefore$ From (1), Refrigeration effect, $q_0 = h_1 - h_3 = (h_{f_{6°C}} + x\,h_{g_{6°C}}) - h_3$

$$= 402.6704 - 249.7 = 152.97 \text{ kJ/kg}$$

$\therefore$ Mass flow rate of refrigerant $= \dfrac{Q_0}{q_0} = \dfrac{140.668 \text{ kW}}{152.97 \text{ kJ/kg}}$

$\Rightarrow$ $\dot{m} = 0.92$ kg/s

Refrigeration capacity of plant = 40TR = 140.67 kW

Power of compressor $= \dot{m}(h_2 - h_1) = 0.92 \cdot (410.7 - 402.67) = 7.38$ kW

Volumetric capacity of compressor, $\dot{m}v_1 = 0.92 \times 0.04035 = 0.037 \text{ m}^3/\text{s}$

Power of condensor $= \dot{m}(h_2 - h_3) = 0.92 \cdot (410.7 - 249.7) = 148.12$ kW

Volumetric capacity of condensor, $\dot{m}v_2 = 0.92 \times 0.6975 = 0.642 \text{ m}^3/\text{s}$

Now evaporator temperature is reduced to –32°C.
For isentropic compression 6 – 2 —

$$S_2 = S_{g_{42°C}} = S_6 = S_{f_{-32°C}} + x\,S_{fg_{-32°C}}$$

$\Rightarrow$ $x = 0.945$

$$h_6 = h_{fg_{-32°C}} = 380.29 \text{ kJ/kg}$$

Refrigeration capacity of plant now, $Q_0' = \dot{m}(h_6 - h_5)$ (since 3-5 is an isenthalpic process)

$$= \dot{m}(h_6 - h_3) = 0.92 \times (380.29 - 249.7) = 120.15 \text{ kW}$$

Power of compressor $= \dot{m}(h_2 - h_6) = 0.92 \times (410.7 - 380.29) = 27.98 \approx 28$ kW

Volumetric capacity of compressor $= \dot{m}v_6 = 0.92 \times 0.1665 = 0.153 \text{ m}^3/\text{s}$

Condensor operation remains same in this case and hence condensor capacity and power remain same as evaporator temperature of 6°C.

Q.27 A vapour compression refrigeration cycle works between pressure limits 10 bar and 3 bar. The working fluid is dry at the end of compression and there is no undercooling before the expansion valve. If refrigerant flow rate is 10 kg/min, determine:

(i) the COP;

(ii) the capacity of the refrigerator.

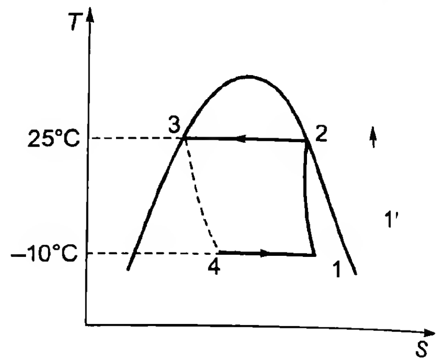
Table for properties of the refrigerant is as under:

Pressure (bar)	Saturation temperature (°C)	Liquid heat (kJ / kg)	Latent heat (kJ / kg)	Liquid entropy (kJ / kg K)	Vapour entropy (kJ / kg K)
10	25	298.9	1166.94	1.1242	5.0391
3	–10	135.37	1297.68	0.5443	5.4770

[CSE (Mains) 2016 : 20 Marks]

Solution:

Given: $\dot{m}_r$ = 10 kg/min



We know,

$$S_1 = S_2$$
$$S_2 = 5.0391 \text{ kJ/kg-K}$$
$$S_1 = 0.5443x(5.4770 - 0.5443)$$
$$\Rightarrow \quad 0.5443 + x(5.4770 - 0.5443) = 5.0391$$
$$x = 0.91$$

Now for calculating h_1:

$$h_1 = 135.37 + 0.91 \times 1297.68$$
$$= 1316.26 \text{ kJ/kg}$$
$$h_2 = 298.9 + 1166.94 = 1465.84 \text{ kJ/kg}$$
$$h_3 = h_4 = 298.9 \text{ kJ/kg}$$

(i)
$$\text{COP} = \frac{R \cdot E}{W_{in}} = \frac{h_1 - h_4}{h_2 - h_1} = \frac{1316.26 - 298.9}{1465.84 - 1316.26} = 6.8$$

(ii)
$$\text{Capacity of refrigerator} = \dot{m}_r \times \text{Ref. effect} = \dot{m}_r\ (h_1 - h_4)$$
$$= \frac{10}{60} \times (1316.26 - 298.9) = 169.56 \text{ kW or } 48.45 \text{ tonn}$$
$$\text{Capacity} = 48.45 \text{ TR}$$

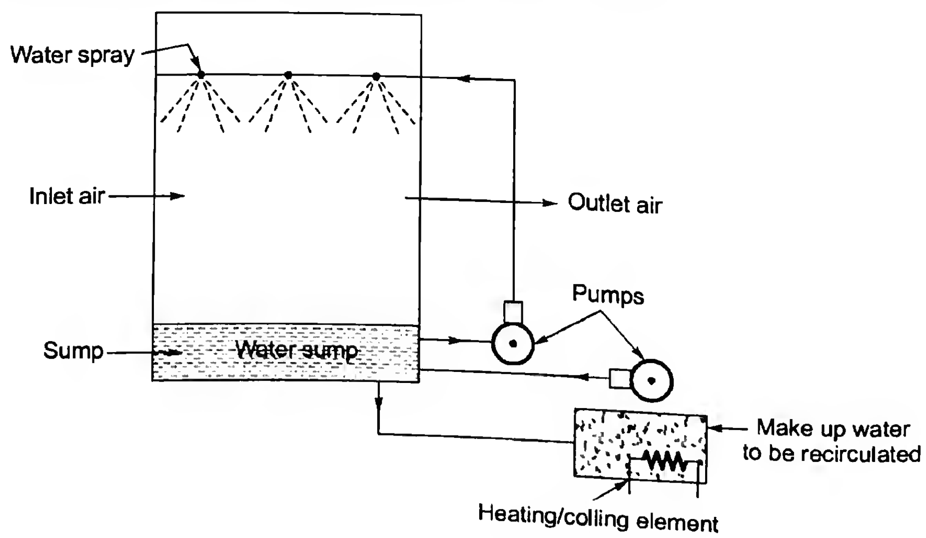
5. Psychrometry and Air Conditioning Processes

Q.28 Discuss and shown on the skeleton psychrometric chart, the process for the air passing through air washers with three types of spray (i) recirculated (ii) chilled and (iii) heated water.

[CSE (Mains) 2001 : 10 Marks]

Solution:

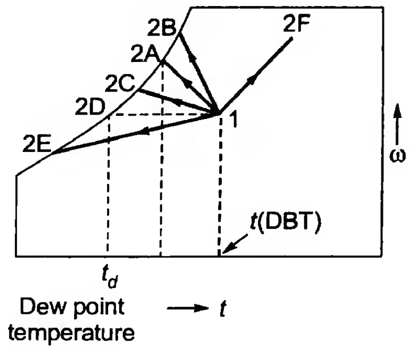
Schematic of an air-washer for given applications is as shown below:



Inlet air passes through the air washer and water is sprayed on it during the process which is recirculated using a pump. Make up water is heated or cooled (as required) and supplied.

Different processes depending on condition of water are as shown in psychrometric chart below.

Various processes can be as follows



(i) Recirculated water (no heating or cooling) 1-2A:

In this case adiabatic saturation of inlet air takes place in this process, recirculated water and outlet air both achieve wet bulb temperature and outlet air gets saturated in the process:

(ii) Chilled water

(a) If $t' < t_s < t$ (water temperature t_s is greater than WBT).
In this case cooling and humidification of inlet air takes place as shown by process 1-2B. Enthalpy of air increases despite cooling due to humidification. Make up water needs to be externally heated.

(b) $t_d < t_s < t'$ - Cooling and humidification takes place.
But enthalpy of air decreases, so water needs to be externally cooled. This process is shown by 1-2C.

(c) $t_s = t_D$ - Sensible cooling of air takes places as shown by 1-2D and water needs to be cooled.

(d) $t_s < t_D$ - Cooling and dehumidification takes place as shown by 1-2E.

(iii) Heated water - $t_S > t$

In this case heating and humidification of inlet air takes place as shown by process 1-2F. Enthalpy of air increases and supply water needs to be continuously externally heated.

Q.29 In an air-conditioning plant equipment exists for reheating the air before supply to a room to be maintained at 25° dbt with 55% R.H. The RSHL = 150,000 W and RLHL = 122,000 W. The discharge from cooling coil is 12°C dbt, 95% R.H. Calculate :

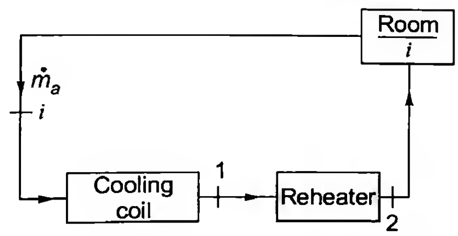
(i) the supply air state and rate (ii) reheating, if any.

[CSE (Mains) 2001 : 30 Marks]

Solution:

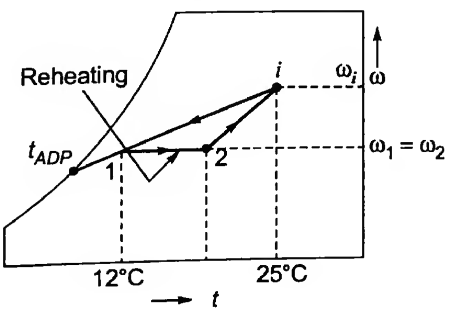
Given: RSHL = 150000 W, RLHL = 122000 W

Schematic for the air conditioning plant can be shown in this figure



Recirculated air post discharge from cooling coil passes through a reheater before being finally supplied to the room.

Various processes can be represented on a psychrometric chart as below



from psychrometric chart, we get —

$$\omega_i = 10.5 \times 10^{-3} \text{ kg/kg d.a.}$$
$$\omega_1 = \omega_2 = 8.4 \times 10^{-3} \text{ kg/kg d.a.}$$

Assume supply air rate is $\dot{m}_a$

∴ Room latent heat load, $\quad RLHL = \dot{m}_a(\omega_i - \omega_1) \times 2500 = 122 \text{ kW}$

$$\Rightarrow \quad \dot{m}_a = \frac{122}{2500 \times (10.5 - 8.4) \times 10^{-3}} = 23.24 \text{ kg/s}$$

Room sensible heat load, $\quad RSHL = \dot{m}_a C_p(t_i - t_s) = 150 \text{ kW}$

$$\Rightarrow \quad t_i - t_2 = \frac{150}{23.24 \times 1.0216} = 7.823$$

$$\Rightarrow \quad t_2 = 25 - 6.318 = 18.7°C$$

∴

Supply air state $\quad$ Supply air rate $= \dot{m}_a = 23.24$ kg/s

DBT $= 18.7°C$

Specific humidity, $\quad \omega_2 = 8.4 \times 10^{-3}$ kg/kg d.a.

$$\text{Reheating done} = \dot{m}_a C_{p,m}(t_s - t_1) = 23.24 \times 1.0216 \times (18.7 - 12) = 158.65 \text{ kW}$$

Q.30 (a) Show the following processes on the psychrometric chart:

(i) Heating and humidification **(ii) Cooling and dehumidification**

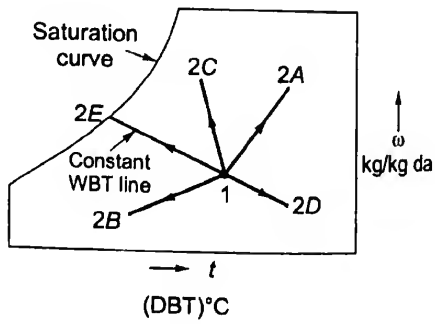
(iii) Cooling and humidification **(iv) Heating and dehumidification**

(v) Adiabatic saturation

[CSE (Mains) 2002 : 20 Marks]

Solution:

Processes on psychrometric chart are shown:



1-2A – Heating and humidification
- Process heating and textile plants
- Achieved by steam injection in industries

1-2B – Cooling and dehumidification
- Most commonly employed in air conditioning during high humidity conditions

1-2C – Cooling and humidification
- Employed in summer air conditioning applications in low humidity conditions - example evaporative cooling using desert air coolers.

1-2D – Heating and dehumidification
- Used in industrial applications and laundry

1-2E – Adiabatic saturation
- Achieved by passing air through an air washer and continuously pumping water so that air becomes completely saturated and achieves wet bulb temperature.

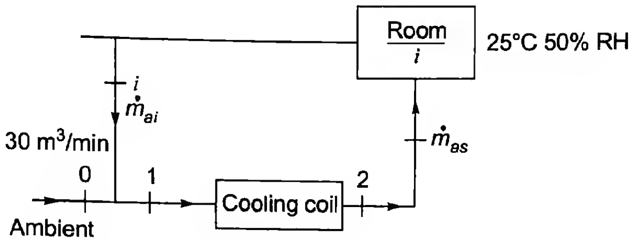
Q.31 In an air-conditioning system 30 m³/min of fresh out-door air is introduced at 43°C dry-bulb temperature and 30% relative humidity. The remaining air is re-circulated from the room maintained at 25°C dry-bulb temperature and 50% RH. The by-pass factor of the cooling coil is 0.15 an apparatus dew point is 11.8°C. RSH = 100 kW and RLH = 15 kW. Determine (i) humidity ratios for outdoor and room condition, (ii) OASH and OALH, (iii) ERSH and ERLH, (iv) supply air temperature, (v) supply air volume flow rate using ERSH and (vi) temperature at inlet to cooling coil by assuming the density of outdoor air and recirculated air to be the same.

Given that saturation pressures of water at 25°C and 43°C are 3.1693 kPa and 8.6495 kPa respectively. Show the process on psychrometric chart schematically. Standard atmospheric pressure = 1.01325 bar

[CSE (Mains) 2003 : 30 Marks]

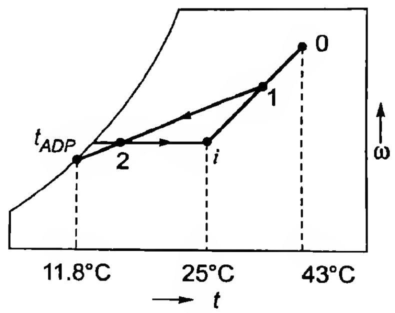
Solution:

Schematic of air conditioning system can be shown in this figure



Various processes in the system can be represented on psychrometric chart as below:

Given:



$$\text{Saturation pressure for 25°C, } P_{S_{@25°C}} = 3.1693 \text{ kPa}$$

$$\text{RH for room air} = 50\%$$

$$\therefore \quad \text{Vapour pressure for room air} = P_s \times 0.5$$

$$\Rightarrow \quad P_v = 1.58465 \text{ kPa}$$

Humidity ratio or specific humidity for room air,

$$\omega_i = 0.622\frac{P_v}{P - P_v}$$

$$= 9.88 \times 10^{-3} \text{ kg/kg d.a.}$$

Similarly for outdoor air:

$$P_{sat@43°C} = 8.6495 \text{ kPa}$$

$$\phi = RH = 30\%$$

$$\therefore \quad P_v = P_s \times \phi = 2.59485$$

$$\therefore \quad \text{Humidity ratio for outdoor air, } \omega_0 = 0.622 \cdot \frac{2.59485}{101.325 - 2.59485} = 16.35 \times 10^{-3} \text{ kg/kg d.a.}$$

Density of outdoor air and recirculated air can be taken as 1.2 kg d.a./m³.

∴ Mass flow rate of dry air in outdoor air taken in

$$\dot{m}_{a_0} = \dot{Q}_f \times \rho = \frac{30}{60} \times 1.2 = 0.6 \text{ kg/s}$$

Assume supply air rate as $\dot{m}_{s_a}$

Ventilation load:

$$\text{Outside air sensible heat, (OASH)} = \dot{m}_{s_0} \cdot C_{p,m}(t_0 - t_i) = 0.6 \times 1.0216 \times (43 - 25) = 11.03 \text{ kW}$$

$$\text{Outside air latent heat, (OALH)} = \dot{m}_a \cdot 2500(\omega_o - \omega_i) = 0.6 \times 2500 \times (16.35 - 9.88) \times 10^{-3}$$

$$\Rightarrow \quad \text{OALH} = 9.705 \text{ kW}$$

$$\text{Effective room sensible heat, (ERSH)} = \text{RSH} + \text{BPF} \times \text{OASH} = 100 + 0.15 \times 11.03 = 101.66 \text{ kW}$$

$$\text{Effective room latent heat} = \text{RLH} + \text{BPF} \times \text{OALH} = 15 + 0.15 \times 9.705 = 16.46 \text{ kW}$$

We know,

$$\text{ERSH} = \dot{m}_{a_s} \cdot C_{p,m}(t_i - t_{ADP})$$

$$\Rightarrow \qquad \dot{m}_{a_s} = \frac{101.66}{1.0216 \times (25 - 11.8)} = 7.54 \text{ kg/s}$$

∴ Volume flow rate of supply air, $\frac{\dot{m}_{a_s}}{\rho} = \frac{7.54}{1.2} = 6.283 \text{ m}^3\text{/s} = 377 \text{ m}^3\text{/min}$

∴ Temperature at the inlet of cooling coil,

$$t_1 = \dot{m}_{a_0} t_0 + (\dot{m}_{a_s} - \dot{m}_{a_0}) \cdot t_i = 26.57°\text{C}$$

$$\text{By pass factor} = \frac{t_2 - t_{ADP}}{t_1 - t_{ADP}} = 0.15$$

$$\Rightarrow \qquad t_2 = 11.8 + 0.15\,(26.57 - 11.8) = 14.016 \simeq 14°\text{C}$$

Q.32 The air handling unit of an air conditioning plant supplies a total 4500 cubic metre/min of dry air which comprises 20 percent fresh air at 40°C DBT and 27°C WBT and 80 percent of recirculated air at 25°C DBT and 50% RH. The air leaves the cooling coil at 13°C saturated state. Determine the total cooling load and the room heat gain. Take specific volume of air entering the cooling coil as 0.869 m³/kg d.a.

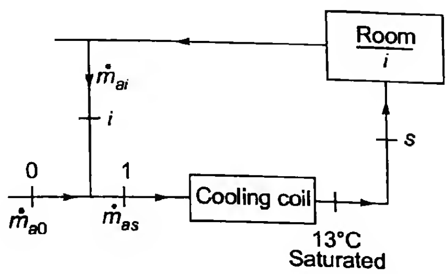
Condition	DBT (°C)	WBT (°C)	RH (%)	Sp. Humidity (gwv/kg) da	Enthalpy (kJ/kg) da
Outside	40	27	-	17.2	85.0
Inside	25	-	50	10.0	50.8
ADP	13	-	100	9.4	37.0

Note: ADP – Apparatus dew point
wv – water vapour da – dry air

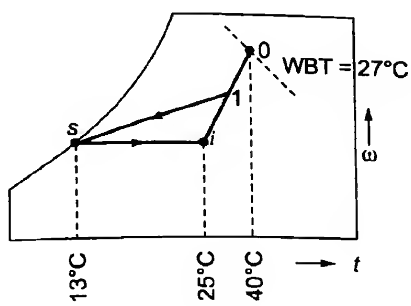
[CSE (Mains) 2004 : 20 Marks]

Solution:

Schematic of the air-conditioning plant is as shown:



All the states and AC processes can be represented on a psychrometric chart as shown below:



Air at state 'S' –13°C, saturated is supplied to room for air conditioning.

Enthalpy of air at state 1, $h_1 = 0.2 \cdot h_o + 0.8\, h_i \;\Rightarrow h_1 = 57.64$ kJ/kg

Mass flow rate of dry air for supply, $\dot{m}_{a_s} = \dfrac{4500}{60 \times 0.869} = 86.31$ kg/s

∴ Room heat gain for the process,

$$\dot{m}_{a_s}(h_i - h_s) = 86.31 \times (50.8 - 37) = 1191.02 \text{ kW}$$

Total cooling load of the coil

$$\dot{m}_{a_s}(h_1 - h_s) = 86.31 \times (57.648 - 37) = 1781.44 \text{ kW}$$

Q.33 An air conditioning plant is designed to maintain a room at a condition of 20°C dry bulb temperature and specific humidity of 0.0079 kg/kg dry air when the outside condition is 30°C dry bulb temperature and 40% saturation. The corresponding heat gains are 18000 W (sensible) and 3600 W (latent). The supply air contains one-third outside air by mass and the supply temperature is to be 15°C dry bulb temperature.

The plant consists of mixing chamber for fresh and recirculated air, an air washer with chilled spray water with an efficiency of 80%, an after heater battery and a supply fan.

Neglecting temperature changes in fan and ducting, calculate :

(i) the mass flow rate of supply air necessary;

(ii) the specific humidity of the supply air;

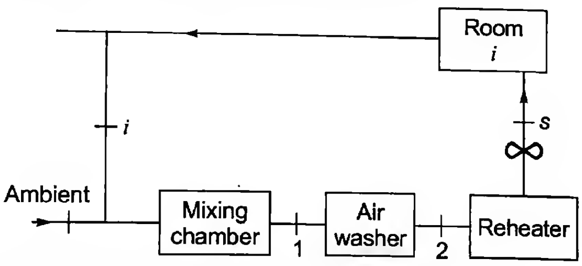
(iii) the cooling load on the washer;

(iv) the heating load on the after heater.

[CSE (Mains) 2005 : 40 Marks]

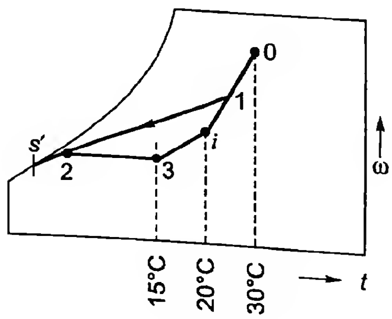
Solution:

Overall setup of the plant can be shown in the sketch below:



Various processes taking place in this setup can be shown as below:

We have,



Supply air temperature, $t_s = 15°C$

Room sensible head gain (RSH) $= \dot{m}_{a_s} C_{p,m}(t_i - t_s) = 18$ kW

$$\Rightarrow \quad \dot{m}_{a_s} = \frac{18}{1.0216 \times (20 - 15)} = 3.524 \text{ kg/s}$$

This is the required mass flow rate of supply air.

Room latent heat load (RLH) $= \dot{m}_{a_s} \cdot 2500\,(\omega_i - \omega_s) = 3.6$ kW

$$\Rightarrow \quad \omega_s = 0.0079 - \frac{3.6}{3.524 \times 2500} = 0.0075 \text{ kg/kg d.a.}$$

Inlet air temperature to air washer, $t_1 = \dfrac{1}{3} \cdot t_o + \dfrac{2}{3} \cdot t_i = \dfrac{30}{3} + \dfrac{2}{3} \cdot 20 = 23.33°C$

Specific humidity post air washer = specific humidity of supply air

$$\Rightarrow \quad \omega_2 = \omega_s = 0.0075$$

specific humidity of outside air, $\omega_o = 10.6 \times 10^{-3}$ kg/kg d.a.

specific humidity post mixing chamber, $\omega_1 = \frac{1}{3}\cdot\omega_o + \frac{2}{3}\omega_i = 8.8 \times 10^{-3}$ kg/kg d.a.

$$\text{Efficiency of air washer, } \eta = \frac{\omega_2 - \omega_1}{\omega_s - \omega_1} = 0.8$$

$$\Rightarrow \qquad \omega_s = 0.0088 + \frac{(0.0075 - 0.0088)}{0.8} = 7.715 \times 10^{-3} \text{ kg/kg d.a.}$$

From psychrometric chart, for given value of ω_s and ϕ = 100% we get,

we get, $t_s \simeq 9°C$

$$\eta = \frac{t_2 - t_1}{t_{s'} - t_1} = 0.8$$

$$\Rightarrow \qquad t_2 = t_1 + 0.8\,(t_{s'} - t_1) = 23.33 + 0.8\,(9 - 23.33) = 11.87°C$$

$$\text{Cooling load on air washer} = \dot{m}_{a_s}\cdot(h_1 - h_2)$$

$$= 3.524 \times [1.0216\,(23.33 - 11.87) + 2500\,(0.0088 - 0.0075)]$$

$$\Rightarrow \qquad \text{Cooling load on air washer} = 52.71 \text{ kW}$$

$$\text{Heat load on after heater} = \dot{m}_{a_s} C_{p,m}(t_s - t_2) = 3.524 \times 1.0216 \times (15 - 11.87) = 11.27 \text{ kW}$$

Q.34 Explain the concept of effective temperature (ET) used in airconditioning practice. Discuss the parameters on which it depends.

[CSE (Mains) 2006 : 20 Marks]

Solution:

In Air conditioning comfort depends on temperature, humidity and velocity of air, hence as a result it is difficult to define comfort in terms of single parameter. Thus, concept of Effective Temperature (ET) is used to define index of comfort.

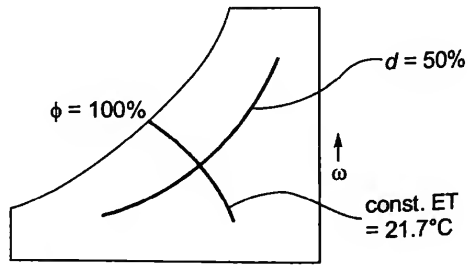
ET is defined as that temperature of saturated air at which subject would experience same feeling of comfort as experienced in actual unsaturated environment based on concept of ET some comfort charts have been developed.

These are referred to when a compromise in the inside design conditions is to be achieved.

In general practice following are the condition for comfort in summer air conditioning.

ET – 21.7°C DBT – 25 ± 1°C

RH – 50 ± 5% Air velocity – 0.4 m/s



Effective Temperature depends on:

1. **Climate and seasonal different:** People living in colder climate are feeling comfortable at lower ET than people living in warm region.
2. **Age and gender:** Children and old aged person's need 2° – 3°C higher ET in comparision to adults and similar is case with women.
3. **Kind of activity:** Persons who are involved in activities like dancing forging shop etc. need lower ET in comparison to person who are in rest condition.
4. **Density of occupant:** Density occupied place need lower ET than the less dense places.

Q.35 10 cu m of atmospheric air at 25°C DBT and 12°C WBT is flowing per minute through a duct. Dry saturated steam at 100°C is injected into the air steam with a rate of 1.2 kg/min.

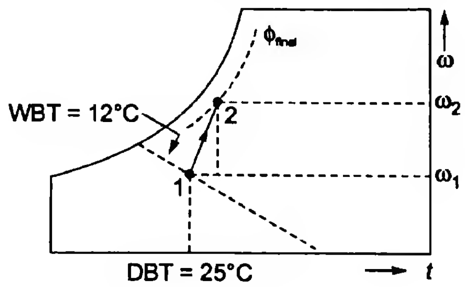
(i) What is the temperature of air after mixing the steam?

(ii) What is the relative humidity of air after mixing the steam?

[CSE (Mains) 2007 : 30 Marks]

Solution:

The process in this can be represented between inlet state (1) and outlet state (2) as shown on psychrometric chart below



$$\text{Mass balance, } \dot{m}_a(\omega_2 - \omega_1) = \dot{m}_s \quad \text{... (1)}$$

where, $\dot{m}_a$ = mass flow rate of steam

$$\text{Energy balance, } \dot{m}_a(h_2 - h_1) = \dot{m}_s \cdot h_{g@100°C} \quad \text{... (2)}$$

Assuming $h_{g@100°C}$ = 2600 kJ/kg,

From psychrometric chart, we get,

$$v_1 = 0.85 \text{ m}^3\text{/kg d.a.}$$
$$h_1 = 34 \text{ kJ/kg d.a.}$$
$$\omega_1 = 3.5 \times 10^{-3} \text{ kg/kg d.a.}$$

$\therefore$ $$\text{Mass flow rate, } \dot{m}_a = \frac{\dot{Q}_f}{v_1} = \frac{10}{0.85} = 11.765 \text{ kg/min d.a.}$$

From (1), we get,

$$11.765\,(\omega_2 - 4 \times 10^{-3}) = 1.2$$

$\Rightarrow$ $$\omega_2 = 0.106 \text{ kg/kg d.a.}$$

From (2), we get,

$$11.765\,[h_2 - 34] = 1.2 \times 2600$$

$\Rightarrow$ $$h_2 = 299.2 \text{ kJ/kg d.a.}$$

we know, $$h_2 = (1.005 + 1.88\,\omega_2)\,t_2 + 2500\,\omega_2 = 299.2$$

$\Rightarrow$ $$t_2 = 29.46°C$$

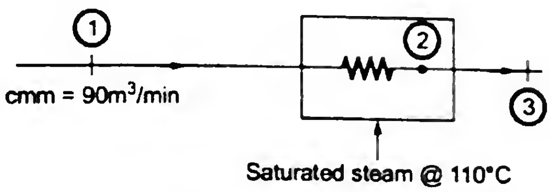
Q.36 **Moist air enter a chamber at 5°C DBT and 2.5°C WBT at a rate of 90 m³/min. The barometric pressure is 1.01325 bar. While passing through the chamber, the air absorbs sensible heat at the rate of 40.7 kW and picks up 40 kg/hr of saturated steam at 110°C. Determine the dry and wet bulb temperatures of leaving air.**

Properties of steam are given below:

Enthalpy of saturated steam at 110°C is 2691.3 kJ/kg.

[CSE (Mains) 2008 : 30 Marks]

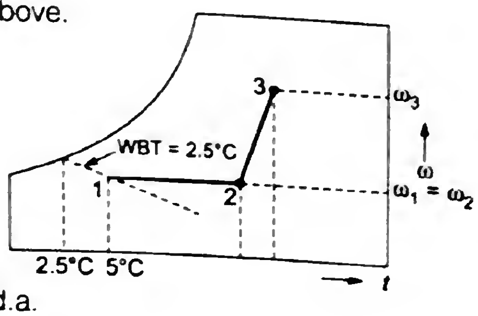
Solution:



Consider mixing of air and steam leading to outlet state (3) as shown above.

$$\text{Volumetric flow rate of air, } \dot{V}_f = 90 \text{ m}^3\text{/min}$$

Representing the processes here in a psychrometric chart as —



From psychrometric chart, we get:

$$v_1 = 0.792 \text{ m}^3\text{/kg d.a.}$$

$\therefore$ $$\text{Mass flow rate} = \frac{90 \times 60}{0.792} = 6818.2 \text{ kg/hr of d.a.}$$

$$\omega_1 = \omega_2 = 3.6 \times 10^{-3} \text{ kg/kg d.a.}$$

Mass balance between states (1) and (3),

$$\dot{m}_a(\omega_3 - \omega_1) = 40 \text{ kg/hr}$$

$$\Rightarrow \qquad \omega_3 = \frac{40}{6818.2} + 3.6 \times 10^{-3} = 9.47 \times 10^{-3} \text{ kg/kg d.a.}$$

$$h_1 = (1.005 + 1.88\,\omega_1)\,t_1 + 2500\,\omega_1 = 14.06 \text{ kJ/kg d.a.}$$

Energy balance between states (1) and (3),

$$\dot{m}_a(h_3 - h_1) = 40.7 \times 3600 + 40 \times 2691.3$$

$$\Rightarrow \qquad h_3 = 14.06 + \frac{1}{6818.2}(40.7 \times 3600 + 40 \times 2691.3) = 51.337 \text{ kJ/kg d.a.}$$

Also, $$h_3 = (1.005 + \omega_3 \cdot 1.88)\,t_3 + 2500\,\omega_3 = 51.337$$

$$\Rightarrow \qquad t_3 = \frac{51.337 - 2500 \times 9.47 \times 0^{-3}}{1.005 + 9.47 \times 10^{-3} \times 1.88} = 27.05°\text{C}$$

∴ DBT at final state = 27.05°C

from psychrometric chart, WBT at state (3) = 18°C

Q.37 Develop an expression for enthalpy of moist air per unit mass of dry air as under:
$h = 1.005\,t + \omega\,(2500 + 1.88\,t)$ kJ/kg where t is DBT and ω is humidity ratio.

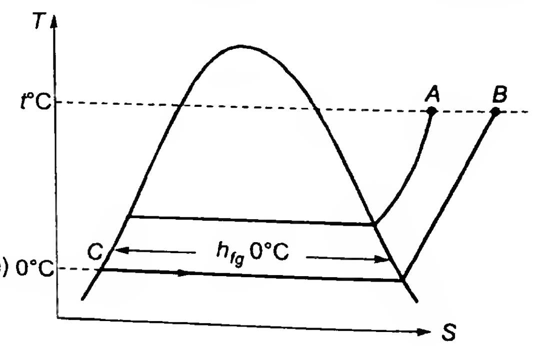
[CSE (Mains) 2008 : 15 Marks]

or

Show the enthalpy of humid air per kg of dry air is given by
$h = C_{pm} \times \text{DBT} + 2500\,\omega$
where, C_{pm} = humid air specific heat = $(1.005 + 1.88\,\omega)$, ω = specific humidity kg/kg of dry air, h_{fg} = 2500 kJ/kg at 0°C for water and DBT = dry-bulb temperature.

[CSE (Mains) 2014 : 10 Marks]

Solution:



For low temperature and pressure, both water vapour and air can be considered to be ideal gases

By Gibb's law, enthalpy of mixture of ideal gases is equal to sum of individual enthalpies of its component gases.

∴ Enthalpy of moist air per unit mass of dry air

$$h = h_a + \omega\,h_v$$

where, h_v = enthalpy of water vapour at given state (A)

ω = specific humidity

Water vapour at state A is present in superheated state at temperature T°C. Assume t = 0°C to be the reference state which is at zero enthalpy.

If water vapour is heated at constant pressure along saturation curve from C to B,

$$h_B = h_C + h_{fg_{0°C}} + C_{p,v}(t_B - 0) = h_{fg_{0°C}} + C_{p,v}t$$

where, $h_{fg_{0°C}}$ = latent heat of vaporization at 0°C = 2500 kJ/kg

$C_{p,v}$ = specific heat of water vapour = 1.88 kJ/kg K

At low pressure, enthalpy can be assumed to be a function of temperature. Hence enthalpy of water vapour at state A is nearly equal to enthalpy at B.

∴ $$h_v = h_A \simeq h_B$$

∴ Enthalpy of moist air, $$h = h_a + \omega\,h_v = C_{p,a}\,t + \omega\,(1.88\,t + 2500)$$

$$\Rightarrow \qquad h = 1.005\,t + \omega\,(1.88\,t + 2500)$$

Q.38 What are the components to be considered for estimating (i) cooling load, (ii) heating load for an air-conditioning system?

How do you calculate heat gain through ducts for an air-conditioning system?

[CSE (Mains) 2009 : 20 Marks]

Solution:

Cooling load estimate:

Room Load:

A. Room sensible heat (RSH)

(i) solar and transmission heat transfer through wall, roof, glass.

(ii) infiltration

(iii) internal heat gains, (people, lighting, appliance etc.)

(iv) additional heat gains.

(v) Supply duct heat gain supply duct leakage loss, fan horsepower.

Sum of above give RSH load.

(vi) By passed outside air load

$$\text{RSH} + \text{(vi)} = \text{ERSH}$$

B. Room latent heat (RLH)

(i) infiltration
(ii) internal heat gain
(iii) vapour transmission
(iv) additional heat gain
(v) supply duct leakage loss

Sum of above give RLH load

RLH + by passed outside air load = ERLH

ATB = RTH (Room total heat load)

Grand total heat (GTH) = TSH + TLH

TSH = ERSH + Sensible heat of outside air that is not by passed + return duct heat gain

TLH = ERLH + Latent heat of outside air not bypassed + return duct leakage gain.

Heating load estimate:

1. Transmission heat loss calculated from wall, roof etc. due to difference between outside air and inside air temperature.
2. Solar radiation
3. Internal heat gain

Duct heat gain:

Normally supply air is at temperature of 10–15°C. The duct may pass through an unconditioned space having ambient temperature of 40°C. This results in significant heat gain by the time air reaches conditioned space even though when duct is insulated.

$$\text{Heat gain } (Q) = UA(t_a - t_s)$$

where,

U = Heat transfer coefficient

A = Surface area

t_a and t_s = ambient and supply temperature

As rough estimate, a maximum of 5% of room sensible heat may be added to total heat load.

Q.39 The outdoor summer condition for a Bank for one hundred persons is T_{db} = 310 K and T_{wb} = 300 K. The required inside conditions are T_{db} = 295 K and ϕ = 60%. The room sensible heat is 4,00,000 kJ/hr. The room latent heat is 2,00,000 kJ/hr. Ventilation requirement per person is 0.0047 m³/hr. The By-pass factor is 0.15.

Evaluate:

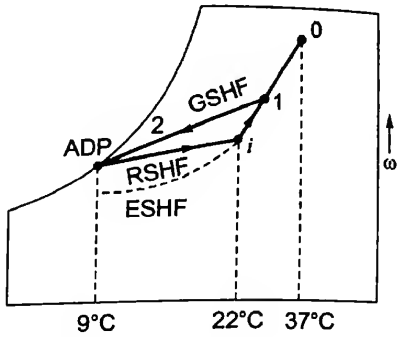
(i) Grand total heat
(ii) Effective sensible heat factor (ESHF)
(iii) Apparatus dew point
(iv) Volume flow rate of dehumidified air

[CSE (Mains) 2009 : 40 Marks]

Solution:

First observing various condition from chart:

Condition	DBT	WBT	RH	ω	h	v
Outside	37°C	27°C		0.0186	85	0.905
Inside	22°C		60%	0.0102	48	0.85



Ventilation requirement per person = 0.0047 m³/hr

∴ For 100 person = 0.0047 × 100 = 0.47 m³/hr

$$\text{CMM (m}^3\text{/min)} = \frac{47}{60} = 7.83\times10^{-3}\ \text{m}^3\text{/min}$$

(i) Outside air sensible heat, OASH $= 0.0204\ \text{CMM}\ (t_0 - t_i)$

$= 0.0204 \times 7.83 \times 10^{-3} \times (37.22) = 2.396 \times 10^{-3}$ kW

Outside air latent heat, OALH $= 50\ \text{CMM}\ (\omega_0 - \omega_i)$

$= 50 \times 7.83 \times 10^{-3} \times (0.0186 - 0.0102) = 3.367 \times 10^{-3}$ kW

Outside air total heat or ventilation load

OATH = OASH + OALH

$= 2.396 \times 10^{-3} + 3.367 \times 10^{-3} = 5.763 \times 10^{-3}$ kW

Total sensible heat, TSH = RSH + OASH

$$= \frac{4{,}00{,}000}{3600} + 2.396\times10^{-3} = 111.11\ \text{kW}$$

Total latent heat, TLH = RLH + OALH

$$= \frac{2{,}00{,}000}{3600} + 3.367\times10^{-3} = 55.56\ \text{kW}$$

Grand total heat, GTH = TSH + TLH = 166.67 kW

(ii) With by-pass factor of 0.15:

Effective room sensible heat,

ERSH = RSH + by-pass factor × OASH

$= 111.11 + 0.15 \times 2.396 \times 10^{-3} = 111.11$ kW

Effective room latent heat, ERLH = RLH + by-pass factor × OALH

$= 55.55 + 0.15\ 3\times 3.367 \times 10^{-3} = 55.55$ kW

Effective sensible heat factor,

$$\text{ESHF} = \frac{\text{ESH}}{\text{ESH} + \text{ELH}} = \frac{111.11}{111.11 + 55.55} = 0.666$$

(iii) Drawing 0.666 SHF line from inside condition. The intersection with the saturation curve give ADP as 9°C.

(iv) Dehumidified air quantity

$$(\text{CMM})_d = \frac{\text{ERSH}}{0.0204(t_i - t_{\text{ADP}})(1 - \text{BPF})}$$

$$= \frac{111.11}{0.0204(22-9)(1-0.15)} = 492.9\ \text{m}^3\text{/min}$$

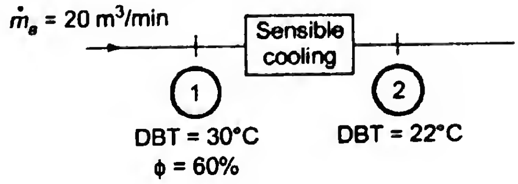
Q.40 20 m³ of air per minute at 30°C DBT and 60% RH is sensibly cooled to 20°C DBT. Take saturation pressure of water vapour at 30°C and 22°C temperatures to be 0.425 bar and 0.0265 bar respectively. Find heat removed from air.

Take atmospheric air pressure p_b = 1 bar.

[CSE (Mains) 2009 : 20 Marks]

Solution:

Consider sensible cooling of air between states (1) and (2) as shown in sketch here



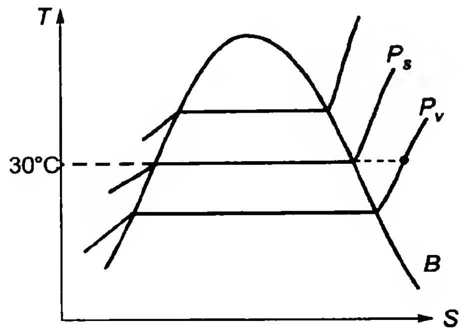
$$\dot{m}_a = 20\,\text{m}^3/\text{min}$$

Saturation pressure of water vapour,

$$P_{S@30°C} = 0.425\ \text{bar}$$

∴ Vapour pressure of water vapour at this temp. (30°C)

$$P_V = P_S \times \phi = 0.425 \times 0.6$$
$$= 0.255\ \text{bar}$$



∴ Humidity of air at this state, $\omega_1 = 0.622\dfrac{P_v}{P-P_v}$

⇒ $$\omega_1 = 0.622 \times \frac{0.255}{1-0.255}$$
$$= 0.213\ \text{kg/kg d.a.}$$

Specific heat of air, $C_{p,m} = C_{p,a} + \omega\, C_{p,v} = 1.005 + 0.213 \times 1.88$
$$= 1.4053\ \text{kJ/kg d.a.}$$

Density of air can be assumed to be 1.2 kg/m³ d.a.

∴ Mass flow rate of dry air $= \dot{V}_f \times \rho = 20\ \text{m}^3/\text{min} \times 1.2\ \text{kg d.a./m}^3$
$$= 24\ \text{kg/min} = 0.4\ \text{kg/s}$$

For sensible cooling, rate of heat removed from air,

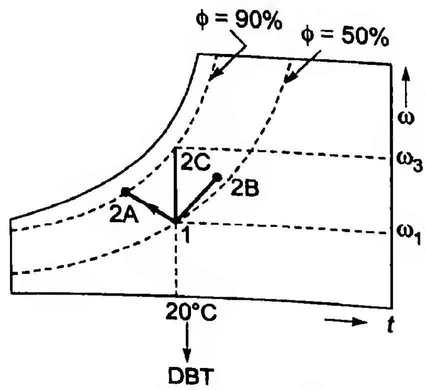
$$= \dot{m}_a C_{p,m}(t_2 - t_1) = 0.4 \times 1.4053 \times (22 - 30) = 4.5\ \text{kW}$$

Q.41 Illustrate the following processes on psychrometric chart with initial state of moist air as dry bulb temperature equal to 20°C and relative humidity of 50% :
(i) Cooling and humidification (ii) Heating and humidification
(iii) Humidification at constant dry bulb temperature.
Mention one application each for above humidification process.
If the moist air leaves the system for the case (iii) above, at 90% relative humidity, determine per unit mass of dry air:
- **increase in humidity ratio, Δω**
- **increase in enthalpy, Δh**
- **increase in dry bulb temperature, Δt**
- **Determine Sensible Heat Factor (SHF) for this process.**

[CSE (Mains) 2010 : 20 Marks]

Solution:

Consider a sketch of psychrometric chart below for the description of given processes



1. 2A - cooling and humidification: Used in air cooling applications in summer in dry areas of low humidity e.g. desert coolers.

1 - 2B - heating and humidification: Used in winter air conditioning processes

1 - 2C - humidification at constant DBT: Used in textile industries and in laundries to maintain required moisture at a given temperature.

Initial state in this case -1

$$\text{DBT} = t_1 = 20°\text{C}, \quad \phi = 50\%$$

From psychrometry chart, we get —

$$\omega_1 = 7.0 \times 10^{-3} \text{ kg/kg d.a.}$$

Final state - 2:

$$\text{DBT} = t_2 = 20°\text{C}, \quad \phi = 90\%$$

$$\omega_2 = 12.7 \times 10^{-3} \text{ kg/kg d.a.}$$

$\therefore$ Increase in humidity ratio, $\Delta\omega = \omega_2 - \omega_1 = 5.7 \times 10^{-3}$ kg/kg d.a.

Increase in enthalpy, from char, $\Delta h = h_2 - h_1 = 37.5 = 13.5$ kJ/kg d.a.

Increase in dry bulb temperature, $\Delta t = t_2 - t_1 = 0$

$$\text{Sensible heat factor} = \frac{Q_s}{\Delta h} = \frac{C_p(T_2 - T_1)}{\Delta h} = 0$$

Q.42 It is desired to maintain a room at temperature of 20°C when outside temperature is 30°C. The volume of this room is 300 m³. The pressure in the room and outside is 1 bar.

(i) The air in the room is renewed completely in 1 hour. Calculate the mass of air that the air conditioning system pumps into the room.

(ii) Compute maximum possible C.O.P. of this A/C system.

(iii) When this A/C system is switched off, the temperature inside the room reaches 21°C in 20 minutes. Calculate the amount of heat transferred from the surroundings to the room.

(iv) Calculate power required by this system.

[CSE (Mains) 2011 : 15 Marks]

Solution:

We have, Ambient temperature, $T_0 = 30°\text{C} = 303$ K, Room temperature, $T_K = 20°\text{C} = 293$ K, Pressure =1 bar

Gas constant for air can be taken as 0.287 kJ/kg K.

$\therefore$ Mass of air contained in the room, $m = \dfrac{PV}{RT_K} = \dfrac{100 \times 300}{0.287 \times 293} = 356.76$ kg

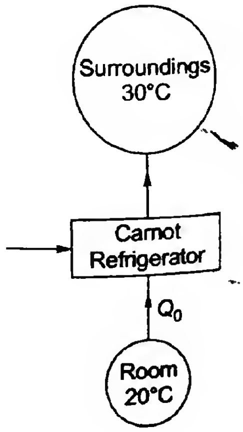
This mass of air needs to be renewed within an hour.

$\therefore$ Mass flow rate of air required to be pumped in the room

$$\dot{m}_a = \frac{356.76}{60} = 5.95 \text{ kg per min}$$

Maximum possible COP will be obtained for the refrigeration machine when it is equivalent to a reversible Carnot generator operating between surroundings and room.

$$\therefore \quad \text{COP}_{max} = \frac{T_K}{T_O - T_K} = \frac{293}{303 - 293} = 29.3$$



This can be illustrated by the sketch:

Since heat loss from the room takes place at constant pressure, rate of heat loss from the room

$$\dot{Q}_0 = \frac{Q}{\Delta t} = \frac{mC_p\Delta T}{\Delta t} = \frac{356.76 \times 1.005 \times (21 - 20)}{20 \times 60}$$

[where Δt = time = 20 minutes]

$$\Rightarrow \quad \dot{Q}_0 = 298.8 \text{ Watt}$$

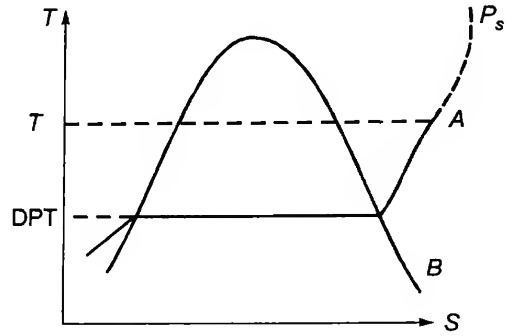
$\therefore$ Power required by the system $= \dfrac{\dot{Q}_0}{\text{COP}} = \dfrac{298.8}{29.3} = 10.2$ Watt

Q.43 Cold water bottles kept in a room often start condensing atmospheric moisture. What is the minimum temperature to which water bottle can be cooled without any dripping of moisture from its surface when kept in a room at 25°C. DBT and 60% RH? If it is desired that a water bottle at 10°C should also not condense moisture on its surface, what should be the RH in room keeping the DBT the same?

[CSE (Mains) 2011 : 15 Marks]

Solution:

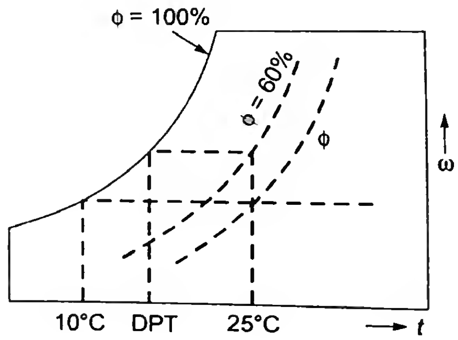
When air is cooled at constant pressure, water vapour also cools along saturation pressure line. This can be represented by T-s diagram below. Water vapour in air is present in the from of superheated vapour at a state *A*.



When air is cooled at atmospheric pressure (in which vapour pressure of water vapour is P-v), water vapour's temperature starts reducing and at state *B*, when temperature becomes equal to saturation temperature equivalent to P-v or dew point temperature condensation starts taking place.

This is the minimum temperature-dew point temperature to which cold water bottle should be cooled to enable any dripping of moisture. Above this temperature no water will be formed on bottle surface.

Consider psychrometric chart as shown in figure.



Saturation curve on the graph corresponds to 100% relative humidity.

For DBT = 25°C and 60% RH,

Horizontal line from this state intersects saturation curve at 16.5°C. This is dew point temperature above which no moisture will from on the bottle.

For minimum temperature for moisture formation to be 10°C, consider horizontal drawn from saturation curve at 10°C. This intersects the vertical from DBT = 25°C at ϕ = 38%.

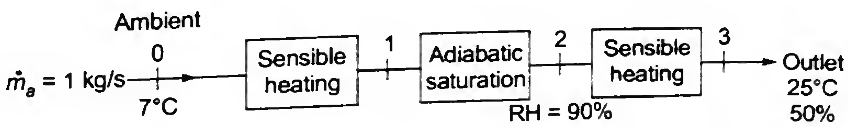
Hence if air is at DBT = 25°C and ϕ less than 38%, moisture formation will not happen even if the bottle is cooled up to 10°C.

Q.44 In a water heating air conditioning system 1 kg/s of ambient air at 7°C DBT, 80% RH is sensibly heated, then adiabatically saturated to a high humidity of RH - 90%, and then further sensibly heated to the desired outlet state of 25°C DBT and 50% RH.

Determine the temperature of air at various state points and show all the processes on a psychrometric chart. Also determine the amount of moisture added per hour in the adiabatic saturator.

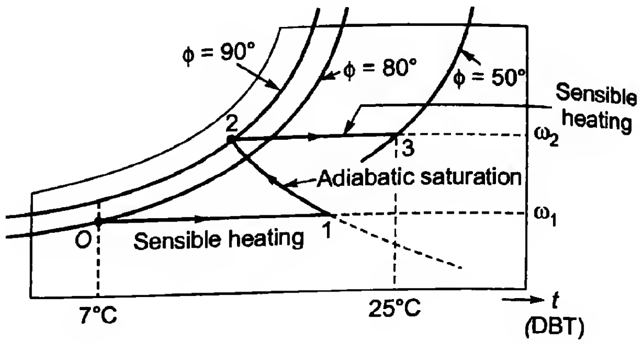
[CSE (Mains) 2011 : 20 Marks]

Solution:



Consider the series of processes which ambient air goes through from state 'o' to supply state 's', as shown in figure above.

These processes can be shown in a psychrometric chart as below:



Point *o* and *s* are plotted on psychrometric graph. Horizontal line extended backwards representing sensible cooling is drawn at 's' to intersect ϕ = 90% line at 2. Similarly, horizontal line representing sensible heating is drawn from *o* and allowed to intersect constant WBT line from 2 at 1-since for adiabatic saturation, WBT remains unchanged.

Reading temperatures and specific humidity at state 1 and 2 from Psychrometric graph, we get,

$$t_1 = 25.5°C,\ t_2 = 15°C$$
$$\omega_1 = 5 \times 10^{-3} \text{ kg/kg d.a.}$$
$$\omega_2 = 9.4 \times 10^{-3} \text{ kg/kg d.a.}$$

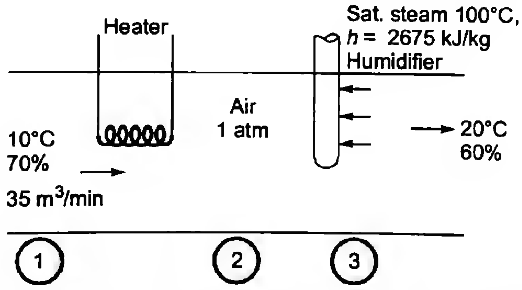
$$\text{Amount of moisture added} = \dot{m}_a(\omega_2 - \omega_1) = 1 \text{ kg/s} \times (9.4 \times 10^{-3} - 5 \times 10^{-3}) \text{ kg/kg d.a.}$$
$$= 4.4 \times 10^{-3} \text{ kg/s} = 15.84 \text{ kg/hour}$$

Q.45 An air-conditioning system (see figure) operates at a total pressure of 1 atm. It consists of a heating section and a humidifier that supplies wet steam (saturated) at 100°C. Air enters the heating section at 10°C and 70 percent relative humidity at the rate of 35 m³/min. It leaves at 20°C and 60% relative humidity, Determine

(i) temperature and relative humidity of air when it leaves the heating section

(ii) the rate of heat transfer to the heating section, and

(iii) the rate at which water is added to the air in the humidifying section.



Also draw skeleton Psychrometric chart representation showing the process.

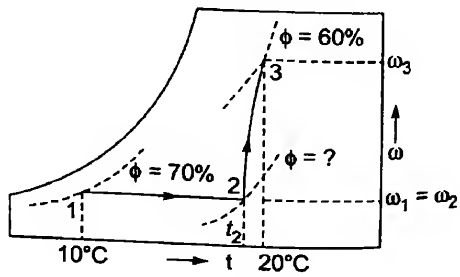
[CSE (Mains) 2012 : 20 Marks]

Solution:

Consider the psychrometric chart below to represent processes from inlet state (1) to outlet state (3)

From psychrometric chart

Specific volume at inlet conditions,



$$v_1 = 0.81 \text{ m}^3\text{/kg d.a.}$$
$$\omega_1 = \omega_2 = 5.4 \times 10^{-3} \text{ kg/kg d.a.}$$
$$\omega_3 = 8.75 \times 10^{-3} \text{ kg/kg d.a.}$$

$$\text{Mass flow rate of dry air, } \dot{m}_a = \frac{\dot{Q}_f}{v_1} = \frac{35}{60 \times 0.81}$$
$$= 0.7202 \text{ kg/s}$$

Assume mass flow rate of steam to be $\dot{m}_s$.

$$\Rightarrow \quad h_1 = (1.005 + 1.88\,\omega_1)\,t_1 + 2500\,\omega_1 = 23.65 \text{ kJ/kg d.a.}$$

and, $h_3 = (1.005 + 1.88\,\omega_3)\,t_3 + 2500\,\omega_3 = 42.304$ kJ/kg d.a.

Assume rate of heat addition in heater section is $\dot{Q}$ kW.

Mass balance in humidifier section,

$$\dot{m}_a(\omega_3 - \omega_2) = \dot{m}_s$$

$$\Rightarrow \quad \dot{m}_s = 0.7202 \times (8.75 - 5.4) \times 10^{-3}\text{ kg/s} = 2.413 \times 10^{-3}\text{ kg/s}$$

Energy balance, $\dot{m}_a(h_3 - h_1) = \dot{Q} + \dot{m}_s \times h_{g@100°C}$

$$\Rightarrow \quad 0.7202\,(42.304 - 23.65) = \dot{Q} + 2.413 \times 10^{-3} \times 2675$$

$$\Rightarrow \quad \dot{Q} = 6.98\text{ kW}$$

As, $\dot{Q} = \dot{m}_a(h_2 - h_1) = 6.98$

$$\Rightarrow \quad h_2 = 33.34\text{ kJ/kg d.a.}$$

We know, $h_2 = (1.005 + 1.88\,\omega_2)\,t_2 + 2500\,\omega_2 = 33.34$

$$\Rightarrow \quad t_2 = 19.55°C$$

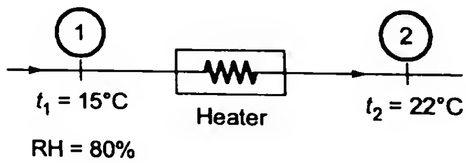
From psychrometric chart, we see for state 2, relative humidity, $\phi = 38\%$

Q.46 100 m³ of air per minute at 15°C DBT and 80% relative humidity is sensibly heated until its temperature becomes 22°C. Saturation pressures of water vapour at 15°C and 22°C are 0.017 bar and 0.02645 bar respectively. Find heat added to air per minute. Take atmospheric pressure = 1.013 bar.

[CSE (Mains) 2013 : 25 Marks]

Solution:

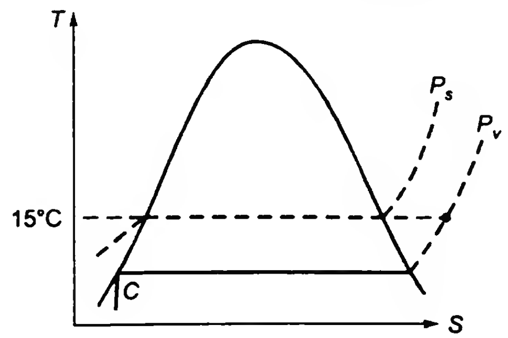
Consider heating of air as shown in figure between states (1) and (2)



Saturation pressure at 15°C DBT = $P_s = 0.017$ bar

∴ Vapour pressure @ 80% relative humidity

$$P_v = P_s = 0.8 = P_s \times \phi$$
$$= 0.8 \times 0.017 = 0.0136\text{ bar}$$



∴ Specific humidity of air $= \omega_1 = \omega_2 = 0.622 \cdot \dfrac{P_v}{P - P_v}$ kg/kg d.a.

$$= 0.622 \cdot \frac{0.0136}{1.013 - 0.0136} = 8.464 \times 10^{-3}\text{ kg/kg d.a.}$$

Specific heat of air, $C_{p,m} = C_{p,a} + \omega\,C_{p,v} = 1.005 + 1.88\,\omega$

$$= 1.005 + 1.88 \times 8.464 \times 10^{-3} = 1.0209\text{ kJ/kg d.a. K}$$

Volume flow rate of air, cmm = 100 m³/min

$$\text{Mass flow rate of air} = \frac{\text{cmm}}{60} \cdot \rho = \frac{100}{60} \times 1.2\text{ kg/s} = 2\text{ kg/s}$$

$$\therefore \quad \text{Heat added in sensible heating} = \dot{m}_a C_{p,m}(t_2 - t_1) = 2 \cdot 1.0209 \times (22 - 15)\text{ K}$$
$$= 14.2926\text{ kW} \simeq 14.29\text{ kW}$$

Q.48 Differentiate between summer and winter air-conditioning processes.

[CSE (Mains) 2014 : 10 Marks]

Solution:

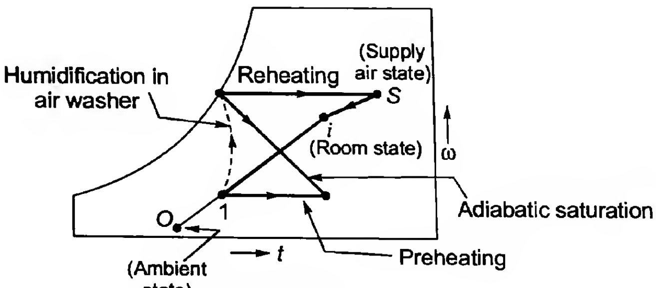
Summer air conditioning requires reduction of ambient air temperature and humidity to comfortable levels due to very high dry bulb temperature and high relative humidity in summers. This involves treatment of air in a cooling coil which carries out "cooling and dehumidification" process in the air-conditioning setup. Recirculated air and ambient air interact with a low Apparatus dew point coil and are supplied to the room.

On the other hand, winter air conditioning requires heating and humidification of ambient air owing to low temperature and humidify conditions at the time. This can be achieved by one of the following two methods:

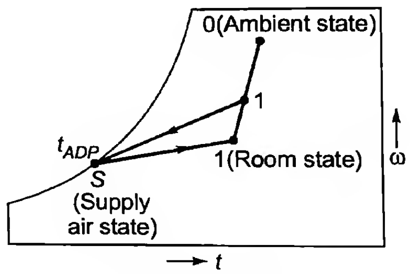
(a) Preheating room air with steam or hot water followed by adiabatic saturation and reheat.

(b) Humidification in an air washer using externally heated water followed by reheat.

Processes followed in both summer and winter air conditioning can be shown in psychrometric chart below



Winter AC



Summer AC

Q.49 Mechanical air-conditioning can be used in all geographical locations, whereas desert air-coolers can be used only in some geographical locations. Explain why. Show the processes involved in both these equipments.

[CSE (Mains) 2014 : 10 Marks]

Solution:

Air has capacity to hold moisture upto saturation point. Air cooler in its operation uses this principle. In an air cooler when air is passed over water, dry air picks up water, which get converted to vapour using heat from the air itself, thus the air gets moist and its temperature reduces. But this process is only possible till saturation point. Thus for the cooler to work efficiently it is important that the air is dry having less moisture otherwise, there will not be much difference in before and after condition of air.

While in air conditioning direct cooling of air take place by bringing the air in contact with a surface which is cooler than air, thus this system will work anywhere, irrespective of surrounding condition.

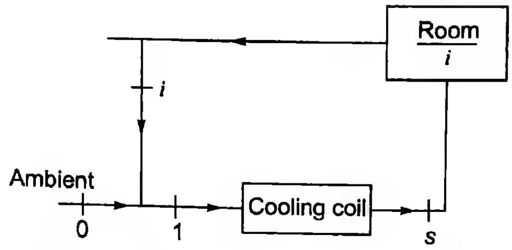
Q.50 The air handling unit in an AC plant supplies a total of 4500 m³/sec of dry air which comprises by weight 20% fresh air at DBT and 50% RH. Air leaves the cooling at 13°C saturated. Calculate total cooling load and room heat gain:

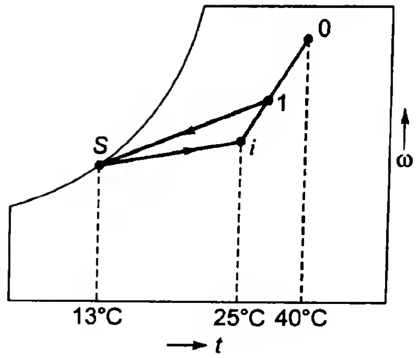
Condition	DBT (°C)	WBT (°C)	RH (%)	Sp. humidity (kg/kg of dry air)	Enthalpy (kg/kg of dry air)
Outside air.	40	27		17.2	85
Room air	25		50	10.0	50.8
ADP	13		100	9.4	37.0

[CSE (Mains) 2014 : 20 Marks]

Solution:

Schematic of AC plant and room can be shown below





Processes taking place in the plant can be shown in a psychrometric chart as given: Room recirculated air at state 'i' and ambient air at state 'o' combine before cooling coil at state 1.

Total cooling load in the AC plant is represented by state change of moist air between state 1 and supply air state 's'.

Density of air at 50% RH and 25°C can be taken as 1.2 kg/m³ d.a.

∴ Mass flow rate of dry air, $\dot{m}_a = \dot{Q}_f \cdot \rho = \frac{4500}{60} \times 1.2 = 90$ kg/s

Enthalpy at state (1), $h_1 = 0.2 \cdot h_0 + 0.8 \cdot h_i$

$= 0.2 \times 85 + 0.8 \times 50.8 = 57.64$ kJ/kg d.a.

∴ Total cooling load of coil, $\dot{m}_a(h_1 - h_s) = 90 \times (57.64 - 37) = 1857.6$ kW

Room heat gain $= \dot{m}_a(h_i - h_s) = 90 \times (50.8 - 37)$ 1242 kW

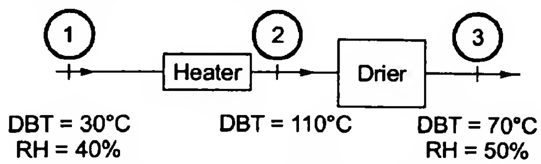
Q.51 In a drier operating at steady state atmospheric air at 30°C and 101.325 kPa with a relative humidity of 40% is first heated to 110°C at constant pressure. The heated air is then allowed to pass over the material to be dried and the air leaves the drier at 70°C with a relative humidity of 0.5. If it is required to remove 60 kg/min of moisture from the material, determine the mass flow rate of dry air required in kg/min and the rate of heat transfer to the air as it passes through the heating unit.

Use $h_{g110°C}^{1\,bar} = 2696.12$ kJ/kg

[CSE (Mains) 2015 : 20 Marks]

Solution:

Consider schematic for the drier as shown in figure:



Post heating in the heater at constant pressure at 110°C is used for drying.

From Psychrometric chart, for state (1),

$$\omega_1 = 10.2 \times 10^{-3} \text{ kg/kg d.a.}$$

$$\omega_2 = w_1 = 10.2 \times 10^{-3} \text{ kg/kg d.a.}$$

Note- P_{sat} for 70°C should be mentioned in this equation. 70°C limit is outside psychrometry graph and hence no other way of calculating w_3.

Taking P_{sat} at 70°C = 31.2 kPa

∴ P_v at 70°C $= R{\cdot}H \times P_s = 0.5 \times 31.2 = 15.614$ kPa

∴ $$\omega_3 = 0.622 \cdot \frac{P_v}{P - P_v} = 0.622 \frac{15.614}{101.325 - 15.614}$$

$$= 113.31 \times 10^{-3} \text{ kg/kg d.a.}$$

Moisture removal rate in the drier = 60 kg/min

Assume mass flow rate of dry air in the drier is $\dot{m}_a$ kg/min.

Mass balance in drier for moisture,

$$\dot{m}_a(\omega_3 - \omega_2) = 60 \text{ kg/min}$$

$$\Rightarrow \quad \dot{m}_a = \frac{60}{(113.31 - 10.2) \times 10^{-3}} = 583.6 \text{ kg/min}$$

$$\Rightarrow \quad h_2 = h_a + \omega_2 \cdot h_v = C_{p,a}\, t_2 + \omega_2 \cdot h_{g_{110°C}}$$
$$= 1.005 \times 110 + 0.0102 \times 2696.\ 12 = 138.86 \text{ kJ/kg}$$

Similarly,
$$h_1 = h_a + \omega_1 \cdot h_v = C_{p,a}\, t_1 + \omega_1\,(C_{p,v} t_1 + h_{fg_0})$$

$$\Rightarrow \quad h_1 = 1.005 \times 30 + 0.0102\,(1.88 \times 30 + 2501) = 56 \text{ kJ/kg}$$

∴ Rate of heat transfer to the air in reheater

$$= \dot{m}_a(h_2 - h_1) = \frac{581.9}{60} \times (138.86 - 56.2) = 801.66 \text{ kW}$$

Q.52 Air at dry-bulb temperature of 30°C and 60% relative humidity enters a cooling coil at the rate of 250 m³/min.

(i) Determine the refrigeration in ton needed to bring the temperature of the air to the coil temperature of 23°C and also the relative humidity at that condition.

(ii) If the effective surface temperature of the cooling coil or ADP is 12°C and the by-pass factor is 0.1, determine the refrigeration in ton needed and the mass of water condensed out at the cooling coil per minute. Determine also sensible heat factor for die process through the coil.

[CSE (Mains) 2016 : 20 Marks]

Solution:

Inlet condition:

DBT = 30°C, d = 60%, V_f = 250 m³/min Corresponding DPT = 20.5°C

Since cooling above DPT, thus sensible cooling will take place with constant ω.

$$\omega_1 \text{ at } 30°C = 0.152 \text{ kg/kg of dry air} = \omega_2$$
$$h_1 = 66.5 \text{ Kj/KG OF DRY AIR}$$
$$h_2 = 60 \text{ kJ/kg}$$

from chart

$$\text{Thus cooling load (refrigeration)} = \dot{m}_a(h_1 - h_2) = \frac{250}{60} \times 1.2 \times (66.5 - 60) = 32.5 \text{ kW or } 9.29 \text{ tonn}$$

(assume density of air = 1.2 kg d.a/m³)

Relative humidity @ state 2 is 90% (from chart)

(ii) When ADP is 12°C, then along with cooling, condensation will also take place. Thus, calculating inlet temperature, specific humidity and enthalpy with help of bypass factor.

At ADP temperature of 12°C

$$\omega_s = 0.009 \text{ kg/kg of d.a}$$
$$h_s = 34.5 \text{ kJ/kg of d.a}$$

from chart.

Now,
$$X = \frac{t_2 - t_s}{t_1 - t_s}$$

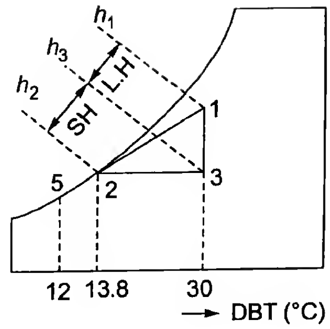
$$\Rightarrow \quad 0.1 = \frac{t_2 - 12°}{30° - 12°} \Rightarrow t_2 = 13.8°C$$

Similarly,
$$h_2 = 37.7 \text{ kJ/kg of d.a}$$

$$\omega_2 = 0.00962 \text{ kg/kg of d.a}$$

$$\text{Refrigeration} = \dot{V}_f \times \rho \times (h_1 - h_2) = \frac{250}{60} \times 1.2 \times (66.5 - 37.7)$$

$$= 144 \text{ kW} = 41.14 \text{ ton}$$



$$\text{Mass of water condensed out per minute} = \dot{V}_f \times \rho \times (\omega_1 - \omega_2)$$

$$= 250 \times 1.2 \times (0.152 - 0.00962)$$

$$= 1.674 \text{ kg/min}$$

We know that,

$$\text{SHF} = \frac{SH}{SH + LH} = \frac{h_3 - h_2}{(h_3 - h_2) + (h_1 - h_3)}$$

(from chart $h_3 = 53$ kJ/kg)

$$\Rightarrow \quad \text{SHF} = \frac{h_3 - h_2}{h_1 - h_2} = \frac{53 - 37.7}{66.5 - 37.7} = 0.531$$

■■■■

CSE Mains 2017
(Mechanical Engineering Paper-II)

Unit-1 Thermodynamics

1. Basic Concepts, Heat and Work

Q.1 The pressure in an automobile tyre depends on the temperature of the air in the tyre. When the air temperature is 25°C the pressure gauge reads 210 kPa. If the volume of the tyre is 0.025m^3, determine the pressure rise in the tyre when the air temperature in the tyre rises to 50°C. Also, determine the amount of air that must be bled off to restore pressure to its original value at this temperature. Assume the atmospheric pressure is 100 kPa and gas constant of air, R = 0.287 kPa m^3/kgK.

[CSE (Mains) 2017 : 10 Marks]

Solution:

The absolute pressure in the tyre is

$$P_1 = P_{gauge} + P_{atm} = 210 + 100 = 310 \text{ kPa}$$

$$\text{Gas constant of air, } R = 0.287 \text{ kPa m}^3/\text{kgK}$$

$$= 0.287 \frac{\text{kN}}{\text{m}^2} \times \text{m}^3 \times \frac{1}{\text{kgK}} = 0.287 \text{ kJ/kgK}$$

As the air is an ideal gas, the final pressure in the tyre can be determined using the ideal equation as follows:

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

Since the volume in the tyre remains constant, $V_1 = V_2$.

$$P_2 = \frac{T_2}{T_1} P_1 = \frac{(273+50)}{(273+25)} \times 310$$

$$= \frac{323}{298} \times 310 = 336 \text{ kPa}$$

$$\text{Pressure rise in the tyre, } \Delta P = P_2 - P_1 = 336 - 310 = 26 \text{ kPa}$$

$$\text{Initial mass, } m_1 = \frac{P_1 V}{R T_1}$$

$$= \frac{310 \times 0.025}{0.287 \times 298} = 0.0906 \text{ kg}$$

$$\text{Final mass, } m_2 = \frac{P_2 V}{R T_2}$$

$$= \frac{310 \times 0.025}{0.287 \times 323} = 0.0836 \text{ kg}$$

$$\text{Amount of air to be bled off, } \Delta m = m_1 - m_2$$

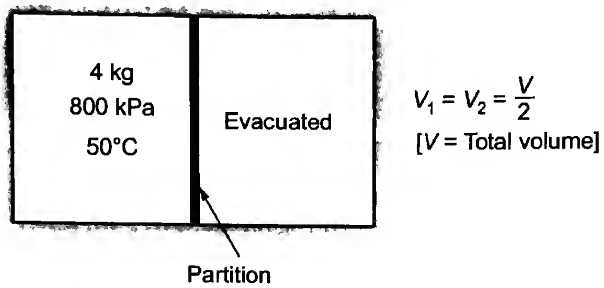
$$= 0.0906 - 0.0836 = 0.0070 \text{ kg}$$

2. First Law of Thermodynamics

Q.2 An insulated rigid tank is divided into two equal parts by a partition. Initially, one part contains 4 kg of an ideal gas at 800 kPa and 50°C, while the other part is evacuated. The partition is now removed and the gas expands into the entire tank. Determine the final temperature and pressure in the tank.

[CSE (Mains) 2017 : 10 Marks]

Solution:



Removing the partition is equivalent to allowing the partition to move in the right direction until the ideal gas fills the entire tank. As the right part of insulated tank is evacuated (i.e. there is vacuum), the resisting force in this would be zero. Since the resisting force is zero, the work done by the expanding gas is zero.

As per first law of thermodynamics,

$$\Delta Q = \Delta W + \Delta U$$

$$\Delta U = 0 \qquad [\Delta Q = 0 \text{ for insulated tank and } \Delta W = 0 \text{ for free expansion}]$$

for an ideal gas,

$$u = f(T) \text{ only, so, } T_2 = T_1 = 50°\text{C}$$

As $T_2 = T$, the process is isothermal.

$$P_1V_1 = mRT_1 = P_2V_2$$

$$P_2 = P_1\left(\frac{V_1}{V_2}\right) = P_1 \times \frac{1}{2} = \frac{800}{2} = 400 \text{ kPa}$$

3. Gases and Mixture

Q.3 Define steam, quality and derive expression for specific volume of steam in terms of steam quality,

$v = v_f + xv_{fg}$

[CSE (Mains) 2017 : 10 Marks]

Solution:

Definition of steam : When we provide continuous heat to water then at 100°C temperature and 1 atmospheric pressure, it boils and changes its phase and liquid to vapour. This vapour is known as steam. Steam is a vapour and is used as the working substance in the operation of stem engines and steam turbines.

Quality of steam : The steam in the steam space of a boiler generally contains water mixed with it in the form of a mist (fine water particles). Such a steam is termed as wet steam. The quality of steam as regards its dryness is termed as dryness fraction. Dryness fraction is usually expressed by the symbol x and is often spoken as the quality of wet steam.

$$\text{Dryness fraction, } x = \frac{m_s}{m_s + m}$$

where m_s = mass of dry steam contained in the steam considered.

m = mass of water in suspension in the steam considered.

Example : If dryness fraction of wet steam, $x = 0.7$, then one kg of wet steam contains 0.3 kg of moisture (water) in suspension and 0.7 kg of dry steam.

Specific volume of steam : The specific volume is the volume occupied by the unit mass of steam and it is expressed in m^3/kg. In other way, the value of cubic meter per kg of dry saturated steam (m^3/kg) is known as the specific volume of dry saturated steam.

If the steam is wet, having a dryness fraction of x, one kg of wet steam will consist of x kg of dry (pure steam) and $(1 - x)$ kg of water held in suspension. Let V be the total volume of liquid vapour mixture of quality x, V_f the volume of the saturated liquid, and V_g the volume of the saturated vapour, the corresponding masses being m, m_f and m_g, respectively.

Now,
$$m = m_f + m_g$$
and
$$V = V_f + V_g$$
$$mv = m_f v_f + m_g v_g \quad \text{(because } V = mv\text{)}$$
$$mv = (m - m_g)\, v_f + m_g v_g$$
$$v = (1 - x)v_f + x v_g = v_f + x(v_g - v_f)$$
$$v = v_f + x\, v_{fg}$$

Hence derived

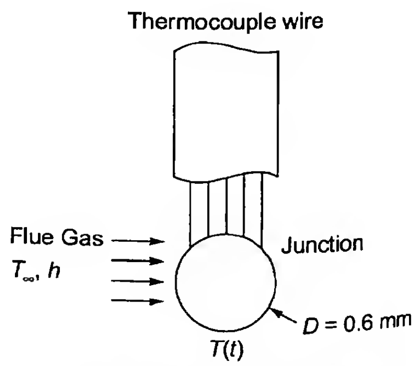
Unit-2 Heat Transfer

1. Conduction

Q.1 A flue gas stream is to be monitored for its temperature using a thermocouple. The thermocouple design needs to be evaluated in terms of its time response to accurately predict the measured temperature. The thermocouple junction can be approximated as a sphere of diameter 0.6 mm, density of the bead material (ρ) 8500 kg/m³, thermal conductivity(k) is 30 W/mK; specific heat (c) is 0.3 kJ/kgK. The convective heat transfer coefficient (h) between the junction and flue gas is 300 W/m²K. Determine the time required to read 90% of the initial temperature difference. Neglect radiation effect and change in thermophysical properties with temperature.

[CSE (Mains) 2017 : 20 Marks]

Solution:



Schematic of thermocouple

Characteristic length, $L = \dfrac{V}{A}$

$$L = \frac{1}{6}\pi D^3 \times \frac{1}{\pi D^2} = \frac{D}{6}$$

Biot number, $\text{Bi} = \dfrac{hL}{k} = \dfrac{300 \times 0.6 \times 10^{-3}}{6 \times 30}$

$$\text{Bi} = 0.001$$

Since Bi < 0.1, lumped system analysis can be used.

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = (1 - 0.9) = 0.1 = \exp\left[-\frac{hAt}{\rho CV}\right]$$

$$\frac{hAt}{\rho CV} = \ln 10$$

$$t = \left[\frac{\rho CV}{hA}\right]\ln 10 = \frac{8500 \times 0.3 \times 10^3 \times 0.6 \times 10^{-3}}{6 \times 300} \times 2.3026$$

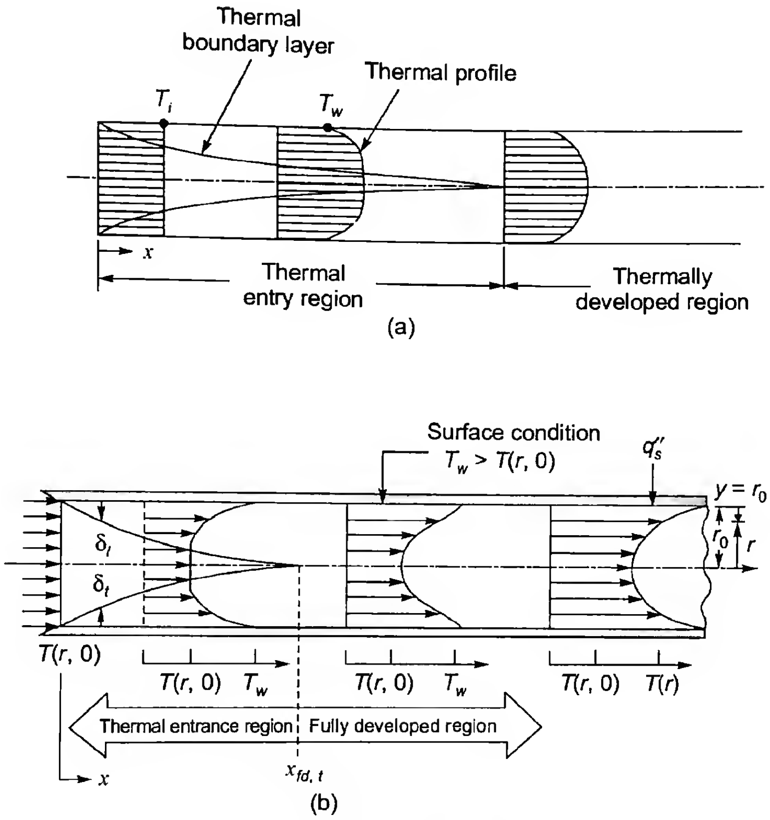
Time (t) = 1.9572 seconds

2 Free and Forced Convection

Q.2 Explain clearly what is "thermally developed zone" in case laminar flow through a tube both for (i) constant wall temperature case (ii) constant heat flux case.

[CSE (Mains) 2017 : 10 Marks]

Solution:



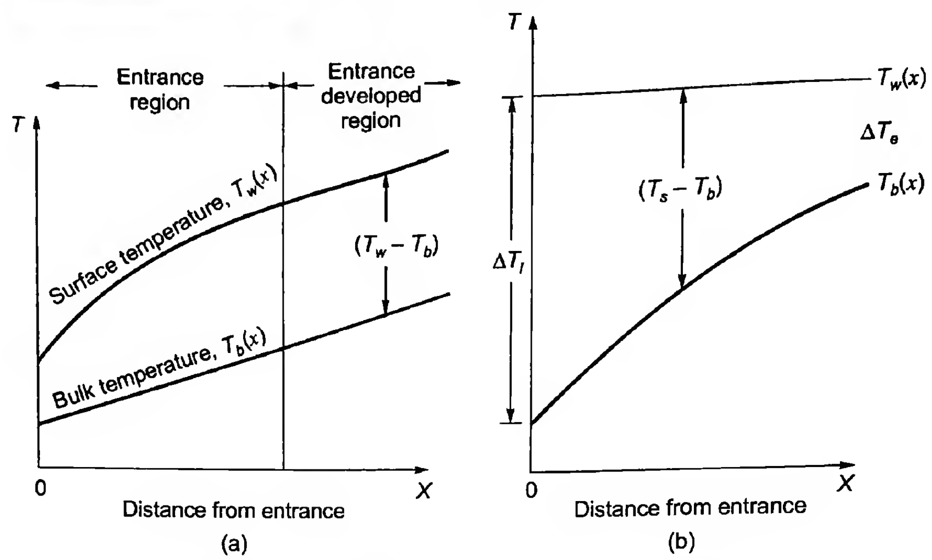
Development of thermal boundary layer in a tube (a) when $T > T_w$ and (b) when $T_w > T$

Let us consider that a fluid at a uniform temperature enters a circular tube with its wall at a different temperature. The fluid particles in the layer in contact with the surface of the tube will assume the tube surface or wall temperature T_W.

This will initiate convection heat transfer in the tube followed by development of the thermal boundary layer along the tube. The thickness of this thermal boundary layer reaches the tube center and thus fills the entire tube.

The region of flow over which the thermal boundary layer develops and reaches the tube centre is called the thermal entry region. The region beyond the thermal entry region in which the temperature profile remains unchanged is called the thermally developed region/zone.

The dimensionless temperature profile $\left(\dfrac{T - T_W}{T_C - T_W}\right)$ does not also change upstream of thermal entry length. The cone in which the flow is both hydrodynamically and thermally developed is called the fully developed region. The shape of the fully developed temperature profile $T(r, x)$ a uniform heat flux is maintained. For both surface conditions, however, the amount by which fluid temperature exceed the entrance temperature increases with increasing x.



Variation of average bulk temperature of a fluid in a pipe for
(a) constant heat flux and (b) constant wall temperature

Nusselt number for fully developed laminar flow in a tube is given as

$$Nu_d = \frac{hD}{k} = \frac{48}{11} = 4.364 \text{ (for constant heat flux)}$$

$$Nu_d = \frac{hD}{k} = 3.66 \text{ (for } T_w = \text{constant)}$$

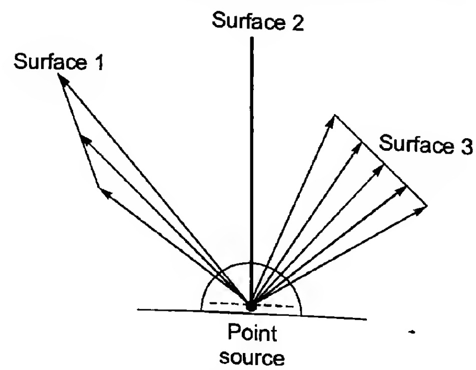
3. Radiation

Q.3 **What is the shape factor in case of radiative heat exchange? Discuss the four (4) basic shape factor laws.**

[CSE (Mains) 2017 : 10 Marks]

Solution:

Radiation heat transfer between surfaces depends on the orientation of the surfaces relative to each other as well as their radiation properties and temperatures as illustrated in figure shown below:



To account for the effects of orientation on radiation heat transfer between two surfaces, we define a new parameter called the view factor, which is a purely geometric quantity and is independent of the surface properties and temperature. It is also called the shape factor, configuration factor, and angle factor.

The four basic shape factor laws are as follows:

Reciprocity Theorem

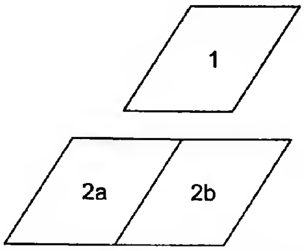
- $A_1F_{1-2} = A_2F_{2-1}$
- It indicates that the net radiant interchange may be evaluated by computing one ways configuration factor from either surface to the other.

Shape Factor of Some Important Surfaces

- Plane surface = 0 ($F_{11} = 0$)
- Convex surface = 0 ($F_{11} = 0$)
- Concave surface ≠ 0 ($F_{11} \neq 0$)

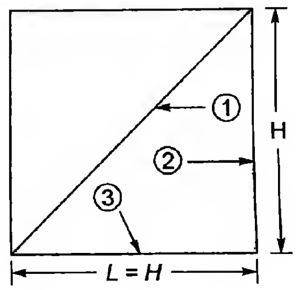
Summation Rule

This rule says that the shape factor from a surface (1) to another (2) can be expressed as a sum of the shape factors from (1) to (2a), and (1) to (2b). Using this rule allows us to break up complicated geometry into smaller pieces for which the individual shape factors can be found. The illustration of summation rule is shown in following figure:



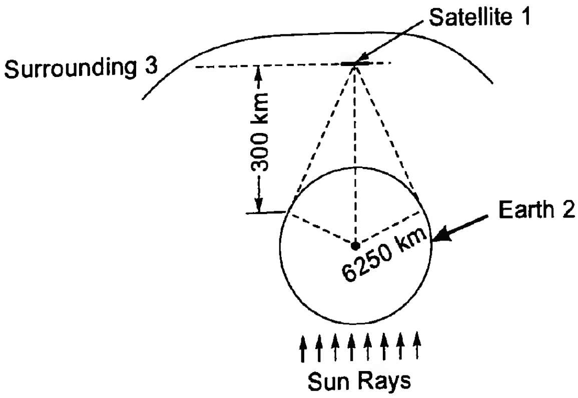
Summation rule for shape factor in radiation heat exchange

Symmetry rule. If the geometry involves symmetry, then the determination of the view factor is simplified. The presence of symmetry can be judged by inspection while keeping the definition of the view factor in mind. Identical surfaces 2 and 3 that are oriented in an identical manner with respect to surface 1, as shown in figure, will intercept identical amounts of radiation leaving surface 1. That is, $F_{1\text{-}2} = F_{1\text{-}3}$.



Therefore, the symmetry rule states that two (or more) surfaces that posses symmetry about a third surface will have identical view factors with respect to the third surface.

Q.4 A small disc-shaped earth satellite, I m in diameter circles the earth (radius 6250 km) at a distance of 300 km from the surface. The flat surface of the disc is oriented tangential to the earth's surface. The satellite surface has an emissivity of 0.3 and is at – 18°C. Calculate the net rate at which energy is leaving the satellite. Assume that: (i) The average earth surface temperature is 27°C and the earth is black body, (ii) The satellite is in shadow of the earth and (iii) The part of the satellite surrounding not occupied by the earth is black and at 0 K, (iv) Stefan-Boltzmann constant (σ) = 5.67 × 10^{-8} W/m²K⁴.



[CSE (Mains) 2017 : 20 Marks]

Solution:

The rate of emission of radiant energy from the satellite surface is

$$Q_1 = 2\epsilon_1 A_1 \sigma_b T_1^4$$

The factor 2 above accounts for two surfaces of satellite

$$Q_1 = 2 \times 0.3 \times \frac{\pi}{4} \times 1^2 \times 5.67 \times 10^{-8} \times (273 - 18)^4 = 112.976 \text{ Watt}$$

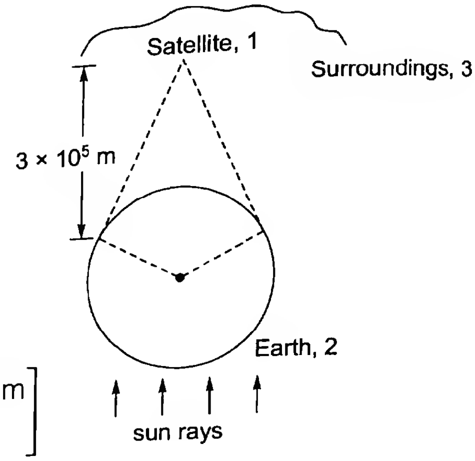
The earth and the surroundings are state to be black and as such the above calculated radiant energy would be completely absorbed by these surfaces. The satellite would not receive back any of the radiation emitted by it.

The earth at temperature T_2 emits radiation equal to $(A_2 F_{21} \sigma_b T_2^4)$. Upon raching the satellite a portion $(\alpha_1 A_2 F_{21} \sigma_b T_2^4)$ would be absorbed by the satellite from Kirchoff's law, absorptivity α_1 of the satellite equals its emissivity $\in_1$. Therefore the rate at which the satellite receives and absorbs energy coming from the earth is

$$Q_2 = \in_1 A_2 F_{21} \sigma_b T_2^4$$

from reciprocity theorem, $A_1 F_{12} = A_2 F_{21}$, therefore

$$Q_2 = \in_1 A_1 F_{12} \sigma_b T_2^4$$



Since the satellite is small in relation to the earth's surface, its radiation shape factor would be

$$F_{12} = \sin^2\alpha = \left(\frac{R}{R+L}\right)^2$$

(Refer text book for its derivation)

$$\begin{bmatrix} R = 6250 \text{ km} = 6.25 \times 10^6 \text{m} \\ L = 300 \text{ km} = 3 \times 10^5 \text{m} \end{bmatrix}$$

$$F_{12} = \left(\frac{6.25 \times 10^6}{6.25 \times 10^6 + 0.3 \times 10^6}\right)^2 = 0.91$$

$$Q_2 = 0.3 \times \frac{\pi}{4} (1)^2 \times 0.91 \times 5.67 \times 10^{-8} \times (300)^4 = 98.47 \text{ W}$$

Since the surroundings are at 0 K, they do not emit any radiations. Net rate at which energy leaves the satellite surface is

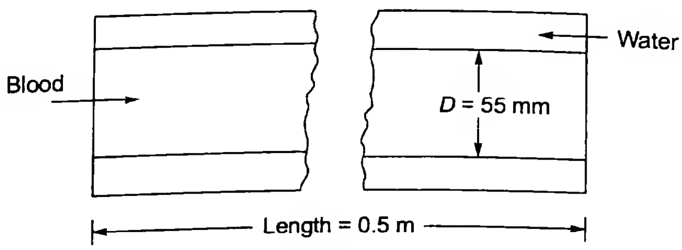
$$Q = Q_1 - Q_2 = 112.976 - 98.47 = 14.5 \text{ W}$$

4. Heat Exchanger

Q.5 In an open heart surgery, under hypothermic conditions, the patient's blood is cooled before surgery and rewarmed afterwards. It is proposed that a concentric tube counterflow heat exchanger of length 0.5 m is to be used for this purpose, with a thin walled inner tube having diameter of 55 mm. If water at 60°C and 0.1 kg/s is used to heat the blood entering the heat exchanger at 18°C at a flow rate of 0.01 kg/s, what is the temperature of the blood leaving the heat exchanger? One may assume, overall heat transfer coefficient (U) = 500 W/m²K, specific heat of blood and water are respectively $C_{P\,\text{blood}}$ = 3.5 kJ/kgK, $C_{P\,\text{water}}$ = 4.187 kJ/kgK.

[CSE (Mains) 2017 : 20 Marks]

Solution:



Hot Fluid	Cold Fluid
Water	Blood
mass flow rate, $\dot{m}_h = 0.1$ kg/s	$\dot{m}_c = 0.01$ kg/s
Specific heat, $C_h = 4.187$ kJ/kg K	$C_c = 3.5$ kJ/kg K
$T_{hi} = 60°C$	$T_{ci} = 18°C$

$$\text{Heat capacity of water, } C_h = \dot{m}_h C_h$$

$$C_{max} = C_h = 0.1 \times 4.187 = 0.4187 \text{ kW/kg-K}$$

$$\text{Heat capacity of blood, } C_c = \dot{m}_c C_c$$

$$C_{min} = 0.01 \times 3.5 = 0.035 \text{ kW/kg-K}$$

$$\text{Capacity ratio, } C = \frac{C_c}{C_h} = \frac{C_{min}}{C_{max}} = \frac{0.035}{0.4187} = 0.0836$$

$$\text{NTU} = \frac{UA}{C_{min}} = \frac{500 \times \pi \times 0.055 \times 0.5}{0.035 \times 10^3} = 1.2342$$

Effectiveness for counterflow heat exchanger is given by

$$\in = \frac{1 - \exp[-\text{NTU}(1-C)]}{1 - C\exp[-\text{NTU}(1-C)]}$$

$$\text{NTU}(1 - C) = 1.2342(1 - 0.0836) = 1.131$$

$$\in = \frac{1 - \exp[-1.131]}{1 - 0.0836\exp[-1.131]} = \frac{1 - 0.3227}{1 - 0.0836 \times 0.3227} = 0.696$$

Effectiveness is also defined by

$$\in = \frac{\text{Actual Heat Transfer}}{\text{Maximum Heat Transfer}}$$

$$\in = \frac{q}{q_{max}} = \frac{C_c(T_{co} - T_{ci})}{C_{min}(T_{hi} - T_{ci})}$$

$$q = \in C_{min}(T_{hi} - T_{ci})$$

$$= 0.696 \times 0.035 \times 10^3(60 - 18) = 1023.12 \text{ W}$$

$$\text{Actual heat transfer, } q = C_c(T_{co} - T_{ci})$$

$$T_{co} = T_{ci} + \frac{q}{C_c}$$

$$T_{co} = 18 + \frac{1023.12}{35} = 47.232°C$$

Q.6 Write down the assumptions to analyze a counterflow heat exchanger using LMTD (Log mean temperature difference) method and also write down the expression for LMTD in a counterflow heat exchanger with the help of terminal temperatures.

[CSE (Mains) 2017 : 10 Marks]

Solution:

Assumptions to analyze a counterflow heat exchanger using LMTD method are as under:

(i) The overall heat transfer coefficient U is constant throughout the heat exchanger.

(ii) The specific heats and mass flow rates of both the fluids are constant. This also implies that heat capacities (a product of mass and specific heat) of the fluids are constant over the entire length of flow path.

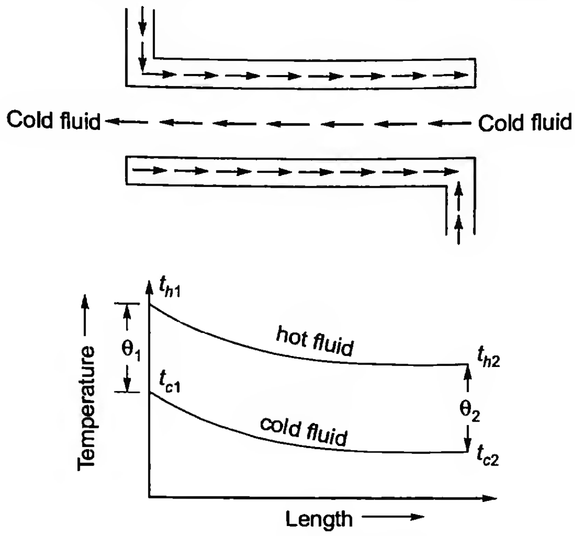
(iii) The exchanger is perfectly insulated and so the heat loss to the surroundings is negligible.

(iv) The temperature at any cross-section of the stream is uniform, i.e., at any c/s of heat exchanger, each of the fluid can be characterised by a single temperature.

(v) There is no conduction of heat along the tubes of heat exchanger.

(vi) The KE and PE changes are negligible.

(vii) There is no partial phase change in the systems. The analysis would thus be applicable for sensible heat changes and for the cases when condensation or vaporisation is isothermal over the entire flow path.



Temperature changes of fluids during counterflow arrangements

$$\text{LMTD} = \frac{\theta_1 - \theta_2}{\log_e\left(\frac{\theta_1}{\theta_2}\right)} \quad \leftarrow \text{ Expression}$$

Unit-3 Internal Combustion Engine

1. Basics of I.C. Engines and Air Standard Cycles

Q.1 A single cylinder 4 stroke SI engine is producing 100 kW power at an overall efficiency of 20%. Engine uses 4 fuel ratio of 0.01. Determine how many m³/hr of air is used if air density is 1.2 kg/m³. The fuel vapour density is 4 times that of air. How many m³/hr of mixture is required? Calorific value of fuel is 42000 kJ/kg. [CSE (Mains) 2017 : 20 Marks]

Solution:

$$\text{Overall efficiency, } \eta_0 = \frac{BP}{\dot{m}_f \times CV}$$

$$\dot{m}_f = \frac{BP}{\eta_0 \times CV} = \frac{180}{0.2 \times 42000} = 0.012 \text{ kg/s}$$

$$\text{Air density, } \rho_a = 1.2 \text{ kg/m}^3$$

$$\text{Fuel vapour density, } \rho_f = 4 \times 1.2 = 4.8 \text{ kg/m}^3$$

$$\text{Fuel mass flow rate, } \dot{m}_f = 0.012 \times 3600 = 43.2 \text{ kg/h}$$

$$\text{F/A ratio} = 0.07$$

$$\dot{m}_a \text{ (Air flow rate)} = \frac{\dot{m}_f}{0.07} = 617.143 \text{ kg/h}$$

$$\text{Volumetric flow rate of air} = \frac{\text{Mass flow rate}}{\text{Density}}$$

$$= \frac{617.43}{1.2} = 514.525 \text{ m}^3/\text{hr}$$

$$\text{Volumetric flow rate of fuel} = \frac{\dot{m}_f}{\rho_f} = \frac{43.2}{4.8} = 9 \text{ m}^3/\text{hr}$$

$$\text{Volumetric flow of fuel mixture} = \dot{V}_a + \dot{V}_f = 514.525 + 9 = 523.525 \text{ m}^3/\text{hr}$$

Q.2 **A six cylinder 4-stroke diesel engine has a bore of 60 mm and a crank radius of 32 mm. The compression ratio is 9 : 1 and engine volumetric efficiency is 90%. Determine: (i) Stroke length, (ii) Mean volume per cylinder, (iii) Swept volume per cylinder, (iv) Clearance volume per cylinder, (v) Cubic capacity of the engine, (vi) Actual volume of air aspirated per stroke in each cylinder.**

[CSE (Mains) 2017 : 20 Marks]

Solution:

$$\text{Stroke length, } L = 2 \times \text{crank radius}$$

$$= 2 \times 32 = 64 \text{ mm} \qquad \textbf{Ans.(i)}$$

$$\text{Mean piston speed, } \bar{S}_p = 2LN$$

$$= 2 \times 64 \times 10^{-3} \times \frac{1000}{60}$$

$$= \frac{128}{60} = 2.13 \text{ m/s} \qquad \textbf{Ans.(ii)}$$

$$\text{Swept volume per cylinder, } V_s = \frac{\pi}{4} D^2 L = \frac{\pi}{4} \times 6^2 \times 6.4$$

$$= 180.956 \, (\text{cm})^3 \approx 181 \text{ cc} \qquad \textbf{Ans.(iii)}$$

$$\text{Compression ratio, } r = \frac{V_s + V_c}{V_c} = \frac{V_s}{V_c} + 1$$

$$\text{Clearance volume, } V_c = \frac{V_s}{r-1}$$

$$= \frac{181}{9-1} = 22.625 \text{ cc} \qquad \textbf{Ans.(iv)}$$

$$\text{Cubic capacity of the engine} = \text{Number of cylinder} \times \text{Swept volume}$$

$$= 6 \times 181 = 1086 \text{ cc} \qquad \textbf{Ans.(v)}$$

$$\text{Volumetric efficiency} = \frac{\text{Actual volume flow rate of air}}{\text{Volume flow rate of air corresponding to displacement volume}}$$

$$\dot{V}_a = \eta_v \times \frac{\pi}{4} D^2 L \times \frac{N}{2}$$

$$= 0.9 \times \frac{\pi}{4} \times (0.06)^2 \times 0.064 \times \frac{1000}{2 \times 60}$$

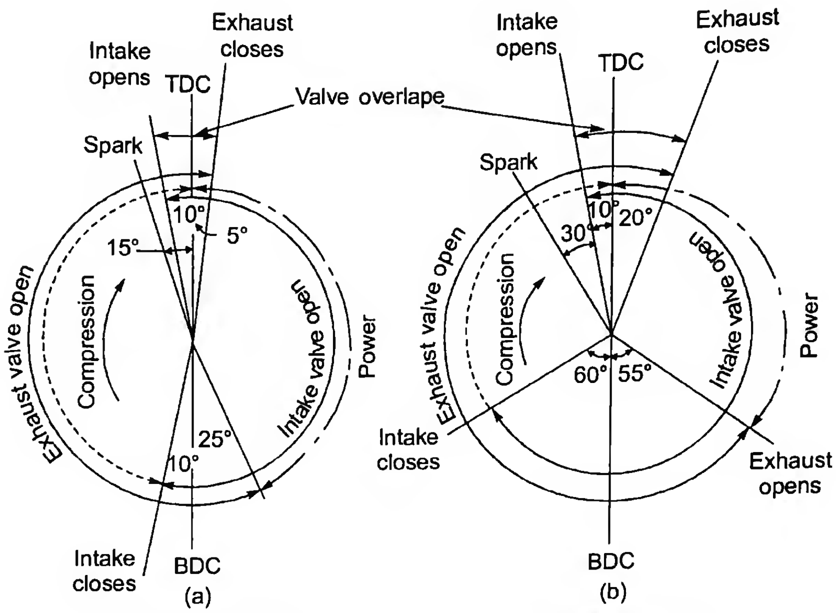
$$\dot{V}_a = 1.357 \times 10^{-3} \text{ m}^3/\text{s or } 4.8858 \text{ m}^3/\text{hr} \qquad \textbf{Ans.(vi)}$$

2. Combustion in S.I. and C.I. Engines

Q.1 Draw valve timing diagram of 4-stroke high speed and low speed SI internal combustion engine.

[CSE (Mains) 2017 : 10 Marks]

Solution:

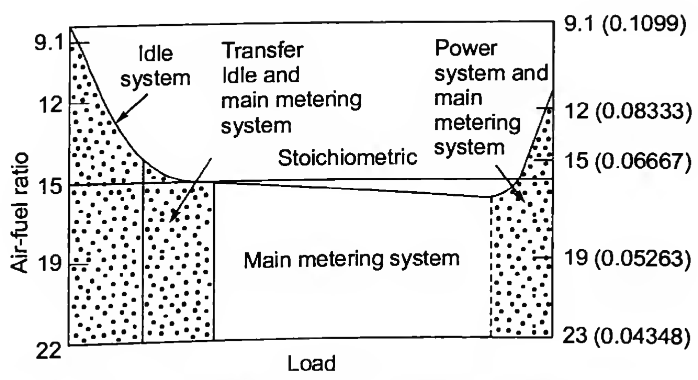


Low speed engine **High speed engine**

Q.2 The air-fuel ratio of an SI engine varies from no-load to full load condition. Write air-fuel ratio requirement for an engine under following conditions with reason: (i) Idling condition, (ii) Cruising condition, (iii) High load condition, (iv) Cold-start condition

[CSE (Mains) 2017 : 10 Marks]

Solution:



Mixture requirements of automotive S.I. engine

1. **Idling and low speed (From no-load to about 20% of rated power):**
Idling refers to no power demand. During idling air supply is throttled and residual gases make up a large fraction of the charge at the end of the suction period. In addition, during valve overlap period some exhaust gases are drawn back into the cylinder. The result is that a chemically correct (stoichiometric) mixture of air and fuel (=15 : 1) would be so diluted by residual gases that combustion would be erratic or impossible. A rich mixture, therefore, must be supplied during idling (say A/F ratio 11:1 or 12:1). The richness should gradually change to slightly lean for the second range as shown above.

2. **Cruising or normal power (from about 25% to about 75% of rated power):** In the normal power range the main consideration is fuel economy. Because mixture of fuel and air is never completely homogeneous the stoichiometric mixture of fuel and air will not bum completely and some fuel will be wasted. For this reason an excess of air, say 10% above theoretically correct (≈ 16.5: l), is supplied in order to ensure complete burning of the fuel.
3. **Maximum power (from 75% to 100% of rated power):** Maximum power is obtained when all the air supplied is fully utilized. As the mixture is not completely homogeneous a rich mixture must be supplied to assure utilization of air (though this would mean wasting some fuel, which would pass in exhaust in unburned state), The air-fuel ratio for maximum power is about 13:1.
4. When the engine is started from cold, its speed and temperature are low and as such much of 'heavy ends' (The hydrocarbons with high vapour pressures and low boiling points are called 'light ends' and those which are less volatile are called 'heavy ends') supplied by the carburettor do not vaporise and remain in liquid form. Further vaporised fuel may recondense on coming in contact with cold cylinder walls and piston head. Thus, even when the F/A ratio at the carburettor is well within the normal combustion limits or petrol-air mixtures, the ratio of the 'evaporated fuel' to air in the cylinder may be too lean to ignite. Consequently it is necessary to supply a rich mixture during starting, as much as 5 to 10 times the normal amount of petrol (A / F ratio 3 : 1 to 15 : 1 or F/A ratio 0.3 to 0.7), in order that 'light ends' are available for proper ignition. With the warming up of the engine there is an increase in the amount of evaporated fuel and hence the mixture ratio should be progressively made leaner, too rich evaporated F/A ratio is avoided.

3. Fuel and Emission Control

Q.1 Discuss experimental determination of calorific value of solid fuel with a neat diagram.

[CSE (Mains) 2017 : 10 Marks]

Solution:

Bomb Calorimeter for determination of Calorific Value of solid fuel : Calorific value (CV) of a fuel is a characteristic of fuel which is defined as the energy liberated per kg of fuel burnt.

- It is used to measure the calorific value (CV) of solid as well as liquid fuel. But to determine the CV of gas, one need to choose Junker's calorimeter.
- A calorimeter contains thick walled cylindrical vessel and it consists of the lid which supports two electrodes which are in contact with fuse and fuel sample of known weight.
- The lid also contains oxygen inlet valve through which high-pressure oxygen gas (at about 25 to 30 atm) is supplied.
- Entire lid with fuel sample is now held in a copper calorimeter containing known weight of water. A mechanical stirrer is provided to stirred well for uniform heating of water.
- A thermometer is also provided to measure the change in temperature of water due to combustion of fuel in Lid.

Procedure of bomb calorimeter experiment

- A known quantity of fuel sample is added in the crucible.
- Start the stirrer and note down the initial temperature of water.
- Start current through the crucible and let fuel sample to burn in presence of oxygen.
- Heat released during combustion of fuel is taken by water and hence temperature of water rises.
- Note final steady state temperature of water.

Higher Calorific Value of fuel $= (m_1 + m_2) \times (T_c + T_1 - T_2) \times C_w / m_f$

where

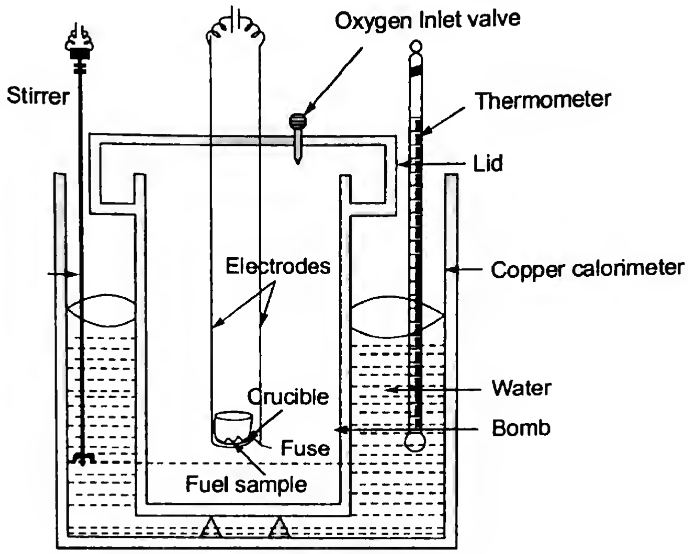
m_1 and m_2 are the mass of water in the copper calorimeter and water equivalent of bomb calorimeter respectively.

m_f is the mass of fuel sample whose calorific value is to he determined.

T_1 and T_2 are the final and initial temperature of the water sample. T_c is temperature correction for radiation losses.

C_w is specific heat of water.

Water equivalent of bomb calorimeter can be found out for particular bomb calorimeter by first by doing an experiment based on known fuel sample whose calorific value is already known. It depends on the manufacturer of the Bomb calorimeter.



Bomb Calorimeter

Practical applications of Bomb Calorimeter:

Bomb Calorimeter is used for the measurement of the calorific value of fuel oils, gasoline or petrol, coke, coal, combustion waste, foodstuffs and building materials etc.

A bomb calorimeter is also used for energy balance study in ecology and study of Nano-material, ceramics, zeolite. The bomb calorimeter is helpful to study the thermodynamics of common combustible materials.

Q.2 A sample of fuel was found to have the following percentage analysis by weight: C 80; H2 16; and ash etc. 4. Determine the minimum weight and volume of air required to burn 1 kg of this fuel. Density of O^2 is 1.429 kg/m^3.

[CSE (Mains) 2017 : 10 Marks]

Solution:

We have to find the oxygen required to burn the two combustibles (carbon and hydrogen). The chemical reactions are:

(i)
$$C + O_2 = CO_2$$
$$12 + 32 = 44$$
$$\text{or } 1 + 2.67 = 3.67$$

(ii)
$$2H_2 + O_2 = 2H_2O$$
$$4 + 32 = 36$$
$$\text{or } 1 + 8 = 9$$

Above equation shows that 1 kg of C requires 2.67 of O_2 to burn it, and 1 kg of H_2 requires 8 kg of O_2 to burn it.

Mass of C in 1 kg of fuel = 0.80 kg (as given)

$\therefore$ O_2 required to burn 0.80 kg of C = $0.80 \times 2.67 = 2.136$ kg

O_2 required to burn 0.16 kg of hydrogen = $0.16 \times 8 = 1.28$ kg

Total O_2 required = $2.136 + 01.28 = 3.416$ kg/kg of fuel

Minimum weight of air required to burn 1 kg of fuel

$$= 3.416 \times \frac{100}{23} = 14.85 \text{ kg}$$

$$\text{Volume of } O_2 \text{ required to burn 1 kg of fuel} = \frac{3.416}{1.429} = 2.39 \text{ m}^3$$

$$\text{So, minimum volume of air required} = 2.39 \times \frac{100}{21} = 11.383 \text{ m}^3\text{/kg of fuel}$$

4. Performance and Testing of I.C. Engines

Q.3 The engine test on a single cylinder four stroke diesel engine has following observations:

Test duration = 1 hr
Bore × Stroke = 0.3 m × 0.45 m
Fuel consumption = 11.4 kg
Calorific value of fuel = 42 MJ/kg
Indicated mean effective pressure = 6 bar
Net load on brake = 1500 N
Engine rpm = 300 rpm
Brake drum diameter = 1.8 m
Brake rope diameter = 20 mm
Quantity of the jacket cooling water = 600 kg
Temperature rise of cooling water = 55°C
Quantity of exhaust measured = 290 kg
Exhaust gas temperature = 420°C
Specific heat of exhaust gas = 1.03 kJ/kgK
Ambient temperature = 20°C

Estimate: (i) The indicated power, (ii) The brake power, (iii) The indicated thermal efficiency, (vi) Draw up an energy balance sheet.

[CSE (Mains) 2017 : 20 Marks]

Solution:

$$\text{Indicated power, IP} = \frac{P_{im} LAN}{2 \times 60000}$$

$$= \frac{6 \times 10^5 \times 0.45 \times \frac{\pi}{4} \times 0.3^2 \times 300}{2 \times 60000} = 47.713 \text{ kW}$$ **Ans.(i)**

$$\text{Brake power} = \frac{(W - S)\pi DN}{60 \times 10^3} \quad [D = 1800 + 20 = 1820 \text{ mm}]$$

$$= \frac{1500 \times \pi \times 1.82 \times 300}{60000} = 42.883 \text{ kW}$$ **Ans.(ii)**

$$\text{Indicated thermal efficiency, } \eta_{ith} = \frac{\text{IP}}{\dot{m}_f \times CV} = \frac{47.713}{\frac{11.4}{3600} \times 42 \times 10^3} = 0.3587 \text{ or } 35.87\%$$ **Ans.(iii)**

Energy Balance Sheet (on minute basis)

$$\text{Heat input} = \frac{11.4}{60} \times 42 \times 10^3 = 7980 \text{ kJ/min}$$

$$\text{Heat equivalent of BP} = 42.883 \times 60 = 2572.98 \text{ kJ/min}$$

$$\text{Heat lost to cooling water} = m_w c_{pw}(t_{w2} - t_{w1}) = \frac{600}{60} \times 4.18 \times 55 = 2299 \text{ kJ/min}$$

$$\text{Quantity of exhaust gas, } m_g = (m_f + m_a) = 290 \text{ kg}$$

$$\text{Heat carried away by dry exhaust gases} = m_g C_{pg}(t_g - t_a)$$

$$= \frac{290}{60} \times 1.03 \times (420 - 20) = 1991.33 \text{ kJ/min}$$

Heat Input (per minute)	(kJ)	Heat expenditure (per minute)	(kJ)	(%)
Heat supplied by fuel	7980	1. Heat equivalent to BP	2573	32.24%
		2. Heat lost to cooling medium	2299	28.8%
		3. Heat lost to exhaust	1191.33	24.95%
		4. Unaccounted loss	1116.67	13.99%
		Net	7980	100%

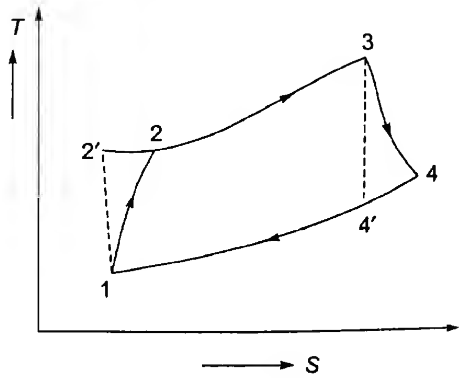
Unit-4 Steam Engineering

1. Gas Turbines

Q.1 The efficiencies of the compressor and turbine of a gas turbine are 70% and 71%, respectively. The heat added in the combustion chamber per kg of air is 476.35 kJ/kg. Find a suitable pressure ratio such that the work ratio is 0.054. Also find the corresponding temperature ratio. The inlet total temperature of air is 300 K.

[CSE (Mains) 2017 : 20 Marks]

Solution:



$$\text{Work ratio} = \frac{\text{Net work } (W_N)}{\text{Turbine work } (W_T)} = \frac{W_T - W_C}{W_T} = 1 - \frac{W_C}{W_T}$$

Let $r_c = r_t = r$, $c = r^{\left(\frac{\gamma-1}{\gamma}\right)}$ and $t = \frac{T_{max}}{T_{min}} = \frac{T_3}{T_1}$

$$\frac{W_N}{W_T} = 1 - \frac{\left(\frac{C_p T_1}{\eta_c}\right)\left(r^{\frac{\gamma-1}{\gamma}} - 1\right)}{\eta_T C_p T_3 \left(1 - \frac{1}{r^{\frac{\gamma-1}{\gamma}}}\right)}$$

$$\text{Work ratio} = 1 - \frac{r^{\frac{\gamma-1}{\gamma}}}{\eta_c \eta_T}\frac{T_1}{T_3} = 1 - \frac{c}{t}\frac{1}{\eta_c \eta_T}$$

$$0.0544 = 1 - \frac{c}{t}\frac{1}{0.70 \times 0.71}$$

$$\frac{c}{t} = (1 - 0.0544) \times 0.70 \times 0.71 = 0.4699632 \approx 0.47$$

Heat added in combustion chamber, $Q = 476.35$ kJ/kg

Assuming specific heat of combustion gases as $C_p = 1.147$ kJ/kg k

$$Q = C_p(T_3 - T_2) = 476.35 \qquad \text{[Heat addition]}$$

$$T_2 = T_1 + \frac{T_1}{\eta_c}(c-1)$$

$$Q = C_p\left(T_3 - T_1 - \frac{T_1}{\eta_c}c + \frac{T_1}{\eta_c}\right)$$

$$Q = C_p T_1\left(\frac{T_3}{T_1} - 1 - \frac{c}{\eta_c} + \frac{1}{\eta_c}\right)$$

$$\frac{Q}{C_p} = T_1\left(t - 1 - \frac{0.47t}{\eta_c} + \frac{1}{\eta_c}\right)$$

$$\frac{476.35}{1.147} = 300\left(t - 1 - \frac{0.47}{0.70}t + \frac{1}{0.70}\right)$$

$$415.3 = 300\,(t - 1 - 0.6714t + 1.4286)$$

$$1.3843 = 0.3286 + 0.4286$$

$$t = \frac{1.3843 - 0.4286}{0.3286} \approx 2.91$$

$$\frac{T_3}{T_1} = t = 2.91$$

$$\frac{c}{t} = 0.47 \quad \text{So, } c = 2.91 \times 0.47 = 1.3677$$

$$(r_p)^{\frac{\gamma-1}{\gamma}} = 1.3677$$

$$\text{Pressure ratio, } r_p = (1.3677)^{\frac{1.4}{0.4}} = 2.992 \approx 3$$

2. Compressors

Q.2 The rotor of an axial flow fan has a mean diameter of 30 cm. It runs at 1470 rpm. Its velocity triangles at entry and exit are described by the following data: Peripheral velocity components of the absolute velocities at entry and exit are:

$C_{y_1} = \frac{1}{3}u, C_{y_2} = \frac{2}{3}u$ where, C = fluid velocity, u = peripheral speed

(i) Draw the inlet and exit velocity triangles for the rotor and prove that the work is given by $W_c = \frac{1}{3}u^2$.

(ii) Calculate the pressure rise, take a constant density of air, $\rho = 1.25$ kg/m³.

[CSE (Mains) 2017 : 20 Marks]

Solution:

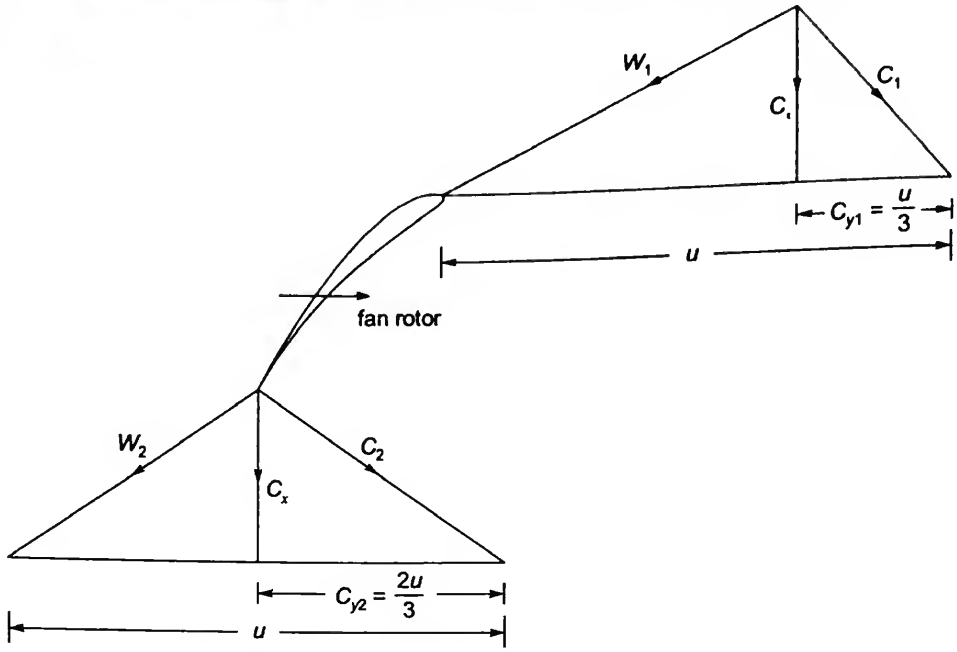
The rate of work done is given by

$$\text{Work} = \text{Torque} \times \text{angular velocity of the rotor}$$

$$W_C = \dot{m}(\omega r_2 C_{\theta 2} - \omega r_1 C_{\theta 1})$$

$$W_C = \dot{m}(u_2 C_{\theta 2} - u_1 C_{\theta 1})$$

$$\text{Specific work, } w_C = \frac{W_c}{\dot{m}} = u_2 C_{\theta 2} - u_1 C_{\theta 1} \qquad \ldots(i)$$



Velocity triangles at the entry and exit of a fan rotor

$$\text{Mean speed of fan rotor, } u = u_1 = u_2 = \frac{\pi \times \text{mean diameter} \times N}{60}$$

$$= \frac{\pi \times 0.3 \times 1470}{60} = 23.091 \text{ m/s}$$

from equation (i),

$$w_C = u(C_{y2} - C_{y1})$$

$$= u\left(\frac{2u}{3} - \frac{u}{3}\right) = \frac{1}{3}u^2 \qquad \textbf{Ans.(i)}$$

$$\frac{\Delta P}{\rho} = \Delta h = W_c = \frac{1}{3}u^2$$

$$\Delta P = \frac{1}{3}\rho u^2 = \frac{1}{3} \times 1.25 \times (23.091)^2 = 222.16 \text{ N/m}^2 \qquad \textbf{Ans.(ii)}$$

3. Steam Turbines

Q.3 (i) Explain about (I) back pressure turbine (II) by-product power cycle (III) co-generation plant (IV) tri-generation plant.

(ii) Define overall efficiency, boiler efficiency, cycle efficiency, mechanical efficiency and generator efficiency of a Rankine cycle based power plant and also prove that:

$$\eta_{overall} = \eta_{boiler} \times \eta_{cycle} \times \eta_{mechanical} \times \eta_{generator}$$

[CSE (Mains) 2017 : 10 Marks]

Solution:

(i) There are several industries, such as paper mills, textile mills, chemical factories, dying plants, rubber manufacturing plants, sugar factories etc., where saturated steam at the desired temperature is required for heating, drying, etc. For constant temperature hearing (or drying), steam is a very good medium, since isothermal condition can be maintained by allowing saturated steam to condense at that temperature and utilizing the latent heat released for heating purposes. Apart from the process heat, the factory also needs power to drive various machines, for lighting, and for other purposes.

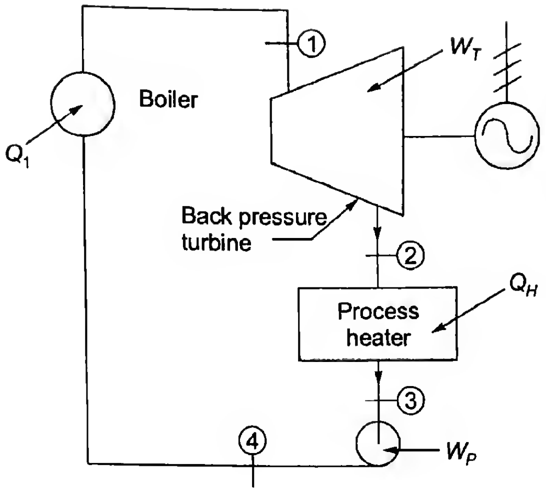


Fig. (a)
Back pressure turbine

Formerly it was the practice to generate steam for power purposes at a moderate pressure and to generate separately saturated steam for process work at a pressure which gave the desired heating temperature. Having two separate units for process heat and power is wasteful, for of the total heat supplied to the steam for power purposes, a greater part will normally be carried away by the cooling water in the condenser. By modifying the initial steam pressure and exhaust pressure, it is possible to generate the required power and make available for process work the required quantity of exhaust steam at the desired temperature. In fig (a), the exhaust steam from the turbine is utilized for process heating, the process heater replacing the condenser of the ordinary Ranking cycle. The pressure at exhaust from the turbine is the saturation pressure corresponding to the temperature desired in the process heater. Such a turbine is called a back pressure turbine. A plant producing both power and process heat is sometimes known as a cogeneration plant. When the process steam is the basic need, and the power is produced incidentally as a by-product, the cycle is sometimes called a by-product power cycle. Figure (b) shows the T-s plot of such a cycle. If W_T is the turbine output in kW, Q_H the process heat required in kJ/h, and w is the steam flow rate in kg/h.

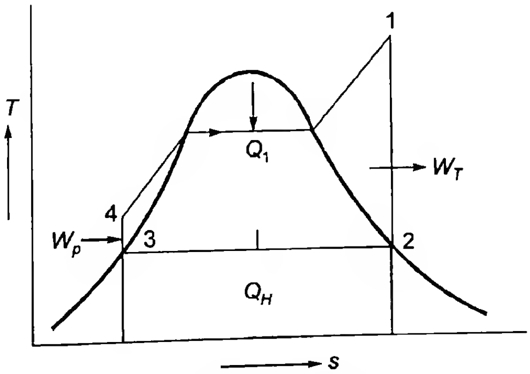


Fig. (b)
By-product power cycle

$$W_T \times 3600 = w(h_1 - h_2)$$

and

$$w(h_2 - h_3) = Q_H$$

∴

$$W_T \times 3600 = \frac{Q_H}{h_2 - h_3}(h_1 - h_2)$$

or

$$Q_H = \frac{W_T \times 3600 \times (h_2 - h_3)}{h_1 - h_2} \text{ kJ/h}$$

Of the total energy input Q_1 (as heat) to the by-product cycle, W_T part of it only in converted into shaft work (or electricity). The remaining energy $(Q_1 - W_T)$, which would otherwise have been a waste, as in the

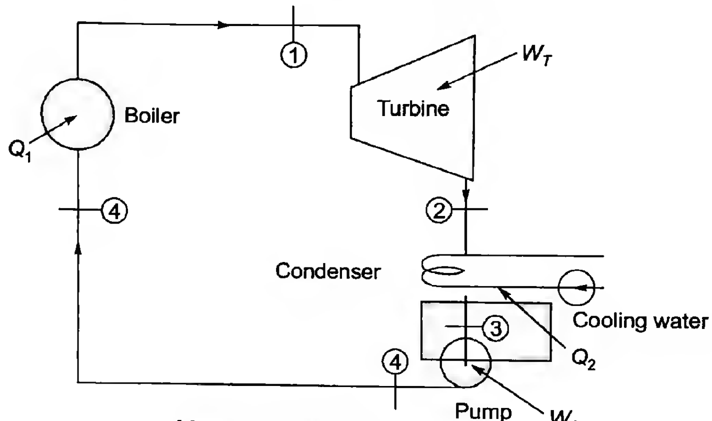
Ranking cycle (by the Second Law), is utilized as process heat.
Fraction of energy (Q_1) utilized in the form of work (W_T), and process heat (Q_H) in a by-product power cycle

$$= \frac{W_T + Q_H}{Q_1}$$

Tri-generation Plant

Tri-generation is the simultaneous production of three forms of energy: electricity, heating and cooling. A trigeneration system can provide power, hot water, space heating and air conditioning from a single system. Generators lose heat as they create electricity. A tri-generation facility captures this heat that would otherwise be lost and uses it to generate both hot and cold water. The chilled water is created by an absorption chiller, which is generated by the excess heat and which operates like a refrigerator. It creates water at sufficiently low temperatures to be used for air conditioning.

(ii)



Ideal Rankine Cycle

Overall or turbine efficiency ($\eta_{overall}$). This efficiency covers internal and external losses; for example, bearings and steam friction, leakage, radiation etc.

$$\eta_{overall} = \frac{\text{Work delivered at the turbine coupling in heat units per kg of steam}}{\text{Total adiabatic heat drop}}$$

'Boiler efficiency' is the ratio of heat actually utilised in generation of steam to the heat supplied by the fuel in the same period.

i.e., Boiler efficiency $= \dfrac{m_a(h - h_{f1})}{C}$

where, m_a = Mass of water actually evaporated into steam per kg of fuel at the working pressure, and
C = Calorific value of the fuel in kJ/kg.

If the boiler, economiser, and superheater are considered as a single unit, then the boiler efficiency is termed as overall efficiency of the boiler plant.

For 1 kg fluid

The S.F.E.E. for the boiler (control volume) gives

$$h_4 + Q_1 = h_1$$
$$\therefore \quad Q_1 = h_1 - h_4$$

The S.F.E.E. for the turbine as the control volume gives

$$h_1 = W_T + h_2$$
$$\therefore \quad W_T = h_1 - h_2$$

Similarly, the S.F.E.E. for the condenser is

$$h_2 = Q_2 + h_3$$
$$\therefore \quad Q_2 = h_2 - h_3$$

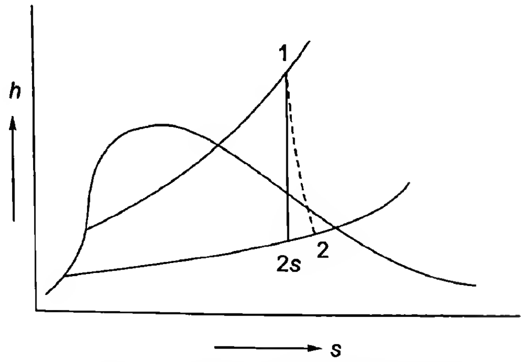
and the S.F.E.E. for the pump gives

$$h_3 + W_P = h_4$$

$$\therefore \quad W_P = h_4 - h_3$$

The efficiency of the Ranking cycle is then given by

$$\eta = \frac{W_{net}}{Q_1} = \frac{W_T - W_P}{Q_1} = \frac{(h_1 - h_2) - (h_4 - h_3)}{h_1 - h_4}$$



Internal efficiency of a steam turbine

The internal efficiency of turbine is defined as

$$\eta_{internal} = \frac{\text{Internal output}}{\text{Ideal output}} = \frac{h_1 - h_2}{h_1 - h_{2s}}$$

Work output available at the shaft is less than the internal output because of the external losses in the bearings, etc.

$\therefore$ Brake output or shaft output = Internal output – External losses

= Ideal output – Internal and External losses

= (kW × 3600 kJ/h)

The brake efficiency of turbine is defined as

$$\eta_{brake} = \frac{\text{Brake output}}{\text{Ideal output}} = \frac{\text{kW} \times 3600}{w_s(h_1 - h_{2s})}$$

The mechanical efficiency of turbine is defined as

$$\eta_{mech} = \frac{\text{Brake output}}{\text{Internal output}} = \frac{\text{kW} \times 3600}{w_s(h_1 - h_2)}$$

$$\therefore \quad \eta_{brake} = \eta_{internal} \times \eta_{mech}$$

While the internal efficiency takes into consideration the internal losses, and the mechanical efficiency considers only the external losses, the brake efficiency takes into account both the internal and external losses (with respect to turbine casing).

The generator (or alternator) efficiency is defined as

$$\eta_{generator} = \frac{\text{Output at generator terminals}}{\text{Brake output of turbine}}$$

The boiler efficiency is defined as

$$\eta_{boiler} = \frac{\text{Energy utilized}}{\text{Energy supplied}} = \frac{w_s(h_1 - h_4)}{w_f \times \text{C.V.}}$$

where w_f is the fuel burning rate in the boiler (kg/h) and C.V. is the calorific value of the fuel (kJ/kg), i.e. the heat energy released by the complete combustion of unit mass of fuel.

The power plant is an energy converter from fuel to electricity, and the overall efficiency of the plant is defined as

$$\eta_{overall} = \eta_{plant} = \frac{\text{kW} \times 3600}{w_f \times \text{C.V.}}$$

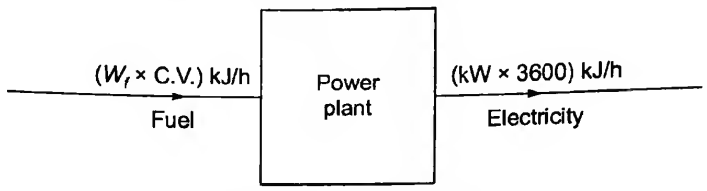
This may be expressed as

$$\eta_{overall} = \frac{\text{kW} \times 3600}{w_f \times \text{C.V.}} = \frac{w(h_1 - h_4)}{w_f \times \text{C.V.}} \times \frac{w(h_1 - h_2)}{w_s(h_1 - h_4)} \times \frac{\text{Brake output}}{w_s(h_1 - h_2)} \times \frac{\text{kW} \times 3600}{\text{Brake output}}$$

or

$$\eta_{overall} = \eta_{boiler} \times \eta_{cycle} \times \eta_{turbine\,(mech)} \times \eta_{generator}$$

where pump work has been neglected in the expression for cycle efficiency.

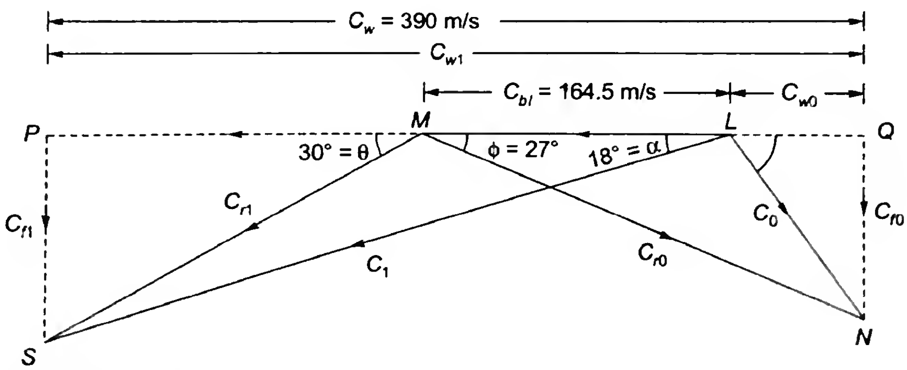


Power plant-an energy converter from fuel to electricity

Q.4 The mean diameter of the blades of an impulse turbine with a single row wheel is 105 cm and the speed is 3000 rpm. The nozzle angle is 18°, the ratio of blade speed to steam speed is 0.42 and the ratio of the relative velocity at outlet from the blades to that at inlet is 0.84. The outlet angle of the blade is to be made 3° less than the inlet angle. The steam flow is 8 kg per sec. Draw the velocity diagram for the blades and estimate the (i) resultant thrust on the blades (ii) tangential thrust on the blades (iii) axial-thrust on the blades (iv) power developed in blades and (v) blade efficiency.

[CSE (Mains) 2017 : 20 Marks]

Solution:



Mean diameter of the blades, D = 105 cm

Speed, N = 3000 rpm

$$\text{Speed ratio, } \rho = \frac{\text{blade speed}}{\text{steam speed}} = 0.42$$

$$\text{Ratio of relative velocity, } K = \frac{C_{r0}}{Cr_1} = 0.84$$

Outlet blade angle, $\phi = \theta - 3°$

Steam flow rate, $\dot{m}$ = 8 kg/s

$$\text{Blade speed, } C_{bl} = \frac{\pi DN}{60} = \frac{3.14 \times 105 \times 10^{-2} \times 3000}{60} = 164.5 \text{ m/s}$$

$$C_1 = \frac{C_{bl}}{\rho} = \frac{164.5}{0.42} = 392 \text{ m/s}$$

$$C_{w1} = C_1 \cos 18° = 392 \cos 18° = 372.81 \text{ m/s (Refer } \Delta LPS)$$

Again from ΔLPS, sin 18° = 121.135 m/s

From cosine rule, $C_{r1}^2 = C_1^2 + C_{bl}^2 - 2C_1C_{bl}\cos 18°$

$= (392)^2 + (164.5)^2 - 2 \times 392 \times 164.5 \cos 18°$

$C_{r1} = 241$ m/s

$C_{r0} = 241 \times 0.84 = 202.44$ m/s

from ΔMPS, $\sin\theta = \dfrac{C_{f1}}{C_{r1}} = \dfrac{121.135}{241} \approx 0.5$

$\theta = 30°$ [This can be measured geometrically also.]

$\phi = \theta - 3° = 27°$

Applying cosing rule for ΔLMN: $C_0^2 = C_{r0}^2 + C_{bl}^2 - 2C_{r0}C_{bl}\cos 27°$

$= (202.44)^2 + (164.5)^2 - 2 \times 202.44 \times 164.5 \cos 27°$

$C_0 = 93.27$ m/s

from ΔMNQ : $\sin\phi = \dfrac{C_{f0}}{C_{r0}}$

$C_{f0} = C_{r0}\sin\phi = 202.44 \sin 27° = 91.9$ m/s

from ΔLNQ; $C_{w0} = \left[C_0^2 - C_{f0}^2\right]^{1/2}$

$= \sqrt{93.27^2 - 91.9^2} = 15.927$ m/s

$C_w = C_{w1} + C_{w0} = 372.81 + 15.927 = 388.74$ m/s [Analytical]

By exact geometrical measurement, $C_w = 390$ m/s

Tangential thrust, $F_t = \dot{m}(C_{w1} + C_{w0})$

$= 8 \times 388.74 = 3109.92 \approx 3110$ N **Ans.(ii)**

Axial thrust, $F_a = \dot{m}(C_{f1} - C_{f0})$

$= 8(121.135 - 91.9)$

$F_a = 233.88\text{ N} \approx 234\text{N}$ **Ans.(iii)**

Resultant thrust, $F_r = \sqrt{(3110)^2 + (234)^2} = 3118.79$ N **Ans.(i)**

Power developed, $P = \dfrac{\dot{m}(C_{w1} + C_{w0})C_{bl}}{1000}$

$= \dfrac{8 \times 38874 \times 164.5}{1000} = 511.582$ kW **Ans.(iv)**

Blading efficiency $= \dfrac{2C_{bl}(C_{w1} + C_{w0})}{C_1^2}$

$= \dfrac{2 \times 164.5 \times 388.74}{(392)^2} = 83.23\%$ **Ans.(v)**

4. Boilers, Condensers and Accessories

Q.5 Classify different types of boilers and discuss factors important for the boiler selection.

[CSE (Mains) 2017 : 10 Marks]

Solution:

Classification of Boilers:

The boilers may be classified as follows :

1. Horizontal, Vertical or Inclined
   If the axis of the boiler is horizontal, the boiler is called as horizontal, if the axis is vertical, it is called vertical boiler and if the axis is inclined it is known as inclined boiler. The parts of a horizontal boiler can be inspected and repaired easily but it occupies more space. The vertical boiler occupies less floor area.
2. **Fire Tube and Water Tube:** In the fire tube boilers, the hot gases are inside the tubes and the water surrounds the tubes. Examples: Cochran, Lancashire and Locomotive boilers.
   In the water tube boilers, the water is inside the tubes and hot gases surround them. Examples: Babcock and Wilcox, Stirling, Yarrow boiler etc.
3. **Externally Fired and Internally Fired:** The boiler is known as externally fired if the fire is outside the shell. Examples: Babcock and Wilcox boiler, Stirling boiler etc. In case of internally fired boilers, the furnace is located inside the boiler shell. Examples : Cochran, Lancashire boiler etc.
4. **Forced Circulation and Natural Circulation:** In forced circulation type of boilers, the circulation of water is done by a forced pump. Examples : Velox, Lamont, Benson boiler etc. In natural circulation type of boilers, circulation of water in the boiler takes place due to natural convention currents produced by the application of heat. Examples : Lancashire, Babcock and Wilcox boiler etc.
5. **High Pressure and Low Pressure Boilers:** The boilers which produce steam at pressures of 80 bar and above are called high pressure boilers. Examples : Babcock and Wilcox, Velox, Lamont, Benson boilers. The boilers which produce steam at pressure below 80 bar are called low pressure boilers. Examples : Cochran, Cornish, Lancashire and Locomotive boilers.
6. Stationary and Portable
   Primarily, the boilers are classified as either stationary (land) or mobile (marine and locomotive).
   - Stationary boilers are used for power plant-steam, for central station utility power plants, for plant process steam etc.
   - Mobile boilers or portable boilers include locomotive type, and other small units for temporary use at sites (just as in small coal-field pits).
7. **Single Tube and Multi-tube Boilers:** The fire tube boilers are classified as single tube and multi-tube boilers, depending upon whether the fire tube is one or more than one. The examples of the former type are cornish, simple vertical boiler and rest of the boilers are multi-tube boilers.

Selection of a Boiler: While selecting a boiler the following factors should be considered :

1. The working pressure and quality of steam required (i.e., whether wet or dry or superheated).
2. Steam generation rate
3. Floor area available.
4. Accessibility for repair and inspection.
5. Comparative initial cost.
6. Erection facilities.
7. The portable load factor.
8. The fuel and water available.
9. Operating and maintenance costs.

5. Compressible Flow

Q.6 What is the difference between 'normal' and 'oblique' shock? State the significance of each.

[CSE (Mains) 2017 : 10 Marks]

Solution:

Difference between Normal and Oblique Shock Waves:

Normal Shock Wave	Oblique Shock Wave
Occurs in supersonic flow ($M_1 > 1$).	Occurs in supersonic flow ($M_1 > 1$, No Difference).
Normal to the Flow direction.	Inclined at some other angle.
Flow direction does not change (Flow Deflection angle = 0).	Flow direction changes (Flow Deflection angle $\neq$ 0)
The downstream of the shock wave flow is always subsonic (M_2<1).	The downstream of the shock wave flow may be subsonic or supersonic (M_2 < 1 or M_2 > 1).
Shock strength is high.	Shock strength is low.

Significance of Normal Shock Wave

If the shock wave is perpendicular to the flow direction, it is called a normal shock. A normal shock occurs in front of a supersonic object if the flow is turned by a large amount and the shock cannot remain attached to the body. The detached shock occurs for both wedges and cones. A normal shock is also present in most supersonic inlets. Across the normal shock the flow changes from supersonic to subsonic conditions. Since gas turbine engines operate under subsonic conditions, it is necessary to introduce a normal shock in the inlet compression system. Normal shocks also are generated in shock tubes. A shock tube is a high velocity wind tunnel in which the temperature jump across the normal shock is used to simulate the high heating environment of spacecraft re-entry.

Significance of Oblique Shock Wave

When a shock wave is inclined to the flow direction it is called an oblique shock. Oblique shocks are generated by the nose and by the leading edge of the wing and tail of a supersonic aircraft. Oblique shocks are also generated at the trailing edges of the aircraft as the flow is brought back to free stream conditions. Oblique shocks also occur downstream of a nozzle if the expanded pressure is different from free stream conditions. In high speed inlets, oblique shocks are used to compress the air going into the engine. The air pressure is increased without using any rotating machinery.

Q.7 **A supersonic wind tunnel settling chamber expands air through a nozzle from a pressure of 10 bar to 4 bar in the test section. Calculate the stagnation temperature to be maintained in the settling chamber to obtain a velocity of 500 m/s in the test section. Take $C_{p,\,air}$ = 1.025 kJ/kgK and $C_{V,\,air}$ = 0.735 kJ/kgK.**

[CSE (Mains) 2017 : 10 Marks]

Solution:

From energy equation, $$h_0 = h + \frac{1}{2}C^2$$

where h is enthalpy and c is flow velocity

$$T_0 = T + \frac{C^2}{2C_P}$$

Assuming isentropic flow, $$\frac{T}{T_0} = \left(\frac{P}{P_0}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_0 = \frac{C^2}{2C_p\left\{1-\left(\frac{P}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\right\}}$$

$$\gamma = \frac{C_P}{C_V} = \frac{1.025}{0.735} = 1.394$$

$$\frac{\gamma-1}{\gamma} = \frac{1.394-1}{1.394} = 0.283$$

$$T_0 = \frac{(500)^2}{2\times 1025(1-0.4^{0.283})}$$

$$T_0 = 533.89 \text{ K}$$

Unit-5 Refrigeration and Air-Conditioning

1. Introduction and Basic Concepts

Q.1 Explain harmful effects of R-12 and R-22 refrigerant. Write their chemical formula and NBP temperature. Also suggest new ecofriendly substitutes of these two with chemical composition.

[CSE (Mains) 2017 : 10 Marks]

Solution:

Harmful Effect of R-12 Refrigerant: The earth's ozone layer in the upper atmosphere (stratosphere) is needed for the absorption of harmful ultraviolet rays from the sun. These UV rays can cause skin cancer. R-12 is dichlorodifluoromethane and comes under CFCs category of refrigerant. In 1985, as per scientific observations, it was found that there is a gaping hole above Antarctic in the ozone layer which protects earth from UV rays. CFC-12 refrigerant has been linked to the depletion of this ozone layer. CFCs are having varying degrees of ozone depletion potential (ODP). In addition, they act as greenhouse gases. Hence, CFCs have global warming potential (GWP) as well. According to an international agreement (montreal protocol), the use of fully halogenated CFCs (no hydrogen in the molecule) are considered to high ODP and R-12 falls in this category. The problem with CFC refrigerant is mainly that a single atom of Cl released from CFC reacts taking out 100,000 O_3 (ozone) molecules. That's why use of CFC-12 refrigerant is detrimental to the environment.

Harmful effect of R-22 Refrigerant: R-22 is Hydro-chlorofluorocarbons refrigerant which contain hydrogen atom alongwith chlorine atom. HCFCs have much lower ODP and GWP as compared to R-12 refrigerant. ODP of R-22 is only 5% of that of R-12. R-22 is considered to be transitional refrigerant and will have to be ultimately phased out by 2030 AD.

Refrigerant	R-12	R-22
Chemical formula	$C\,Cl_2\,F_2$	$CHClF_2$
NBP temperature	–29.8°C	–40.8°C

Refrigerant used before year 2000	Substitute Refrigerant	
	Short Term	Long Term
R-12	R-134a	R-134a
R-22	R-22 (upto 2030 Ad)	HFC 134 a, R 407C, R410A
	R-134a	Other blends of R-32, R-134a and other

Chemical composition of R-134a : $CF_3\,CH_2F$
Chemical composition of R-407c : HFC based
Chemical composition of R-410A : HFC based
Chemical composition of R-32 : $CH_2\,F_2$

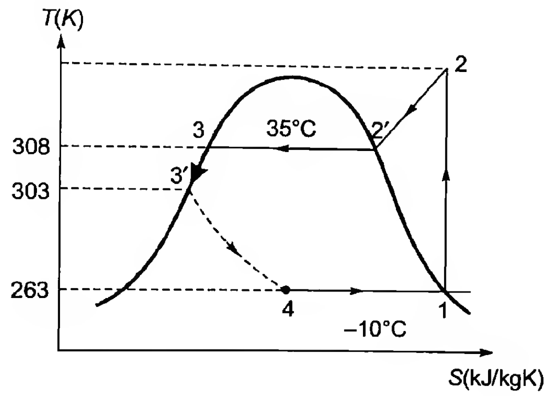
2. Vapour Compression Systems

Q.2 A single stage, single acting vapour compression refrigeration system uses R-134a. Condenser and evaporator temperatures are 35°C and – 10°C and refrigerant is undercooled by 5°C. Clearance volume per swept volume is 0.03 and swept volume is 269.4 cm³. Compressor speed efficiency are 2800 rpm and 80% respectively. Expansion index is 1.12. Determine (i) Compressor exit temperature (ii) Enthalpy of refrigerant at compressor exit (iii) Enthalpy at the exit of the subcooler (iv) Volumetric efficiency of compressor (v) Refrigerant mass flow rate. Specific heat of vapour and liquid at condenser pressure are 1.1 kJ/kgK and 1.458 kJ/kgK respectively. Assume suction vapour dry saturated and isentropic compression.

Pressure (bar)	t°C	V_g, (m³/kg)	Enthalpy (kJ/kg)		Entropy (kJ/kg-K)	
			h_f	h_g	S_f	S_g
2.104	–10	0.0994	186.7	392.4	0.9512	1.733
8.870	35	–	249.1	417.6	1.1680	1.715

[CSE (Mains) 2017 : 20 Marks]

Solution:



$\frac{V_c}{V_s} = 0.03$, $N = 2800$ rpm, $V_s = 269.4$ cc, $n = 1.12$, $\eta_c = 0.8$, $c_{pg} = 1.1$ kJ/kgK, Cpl = 1.458 kJ/kgK

For isentropic compression, $S_1 = S_2$

$$S_1 = S_{2'} + C_{pg}\ln\left(\frac{T_2}{T_{2'}}\right)$$

$$1.733 = 1.715 + 1.1\ln\left(\frac{T_2}{T_{2'}}\right)$$

$$\ln\left(\frac{T_2}{T_{2'}}\right) = \frac{1.733 - 1.715}{1.1} = 0.016364$$

$$T_2 = 308\exp(0.016364)$$
$$= 308 \times 1.0165 = 313\text{ K}$$ **Ans.(i)**

$$h_2 = h_{2'} + C_{pg}(T_2 - T_{2'})$$

$$= 417.6 + 1.1\,(313 - 308) = 423.19 \text{ kJ/kgK} \qquad \textbf{Ans.(ii)}$$

$$h_3 - h_{3'} = C_{pl}(T_3 - T_{3'}) \text{ (for sub-cooling process)}$$

$$h_{3'} = h_3 - C_{pl}(T_3 - T_{3'}) = 417.6 - 1.458\,(308 - 303)$$

$$= 410.31 \text{ kJ/kgK} \qquad \textbf{Ans.(iii)}$$

$$\text{Volumetric efficiency, } \eta_{vol} = 1 + C - C\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}} = 1 + 0.03 - 0.03\left(\frac{8.87}{2.014}\right)^{\frac{1}{1.12}}$$

$$= 1 + 0.03 - 0.03 \times 3.7573$$

$$= 0.9173 \text{ or } 91.73\% \qquad \textbf{Ans.(iv)}$$

Volume of refrigerant admitted at compressor inlet = swept volume × η_{vol}

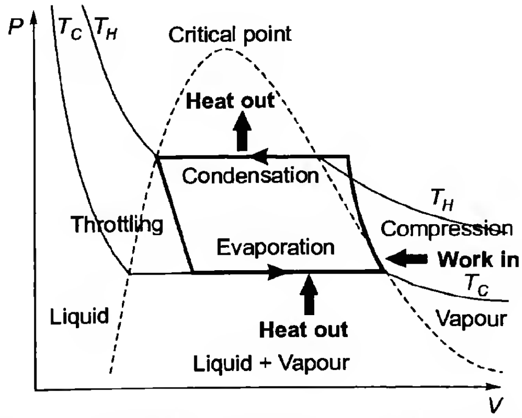
$$= 269.4 \times 10^{-6} \times 0.9173 = 247.12 \times 10^{-6} \text{ m}^3\text{/cycle}$$

$$\text{Refrigerant mass flow rate} = \frac{247.12 \times 10^{-6} \times 2800}{0.0994} = 6.961 \text{ kg/min} \qquad \textbf{Ans.(v)}$$

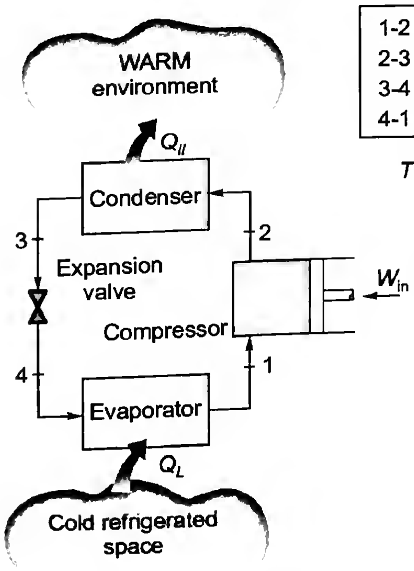
Q.3 **Show processes of saturated vapour compression refrigeration cycle on p-v, t-s and p-h diagram and mark graphically on t-s diagram refrigerating effect, heat rejection by condenser and compressor work.**

[CSE (Mains) 2017 : 10 Marks]

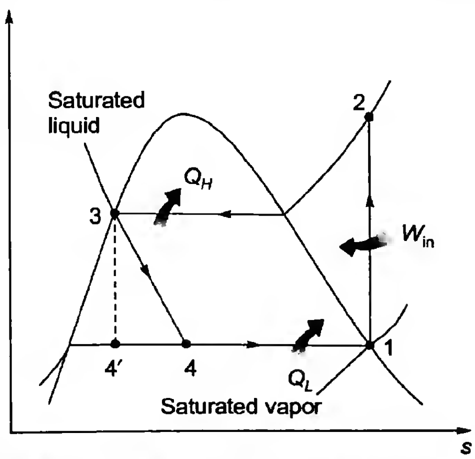
Solution:



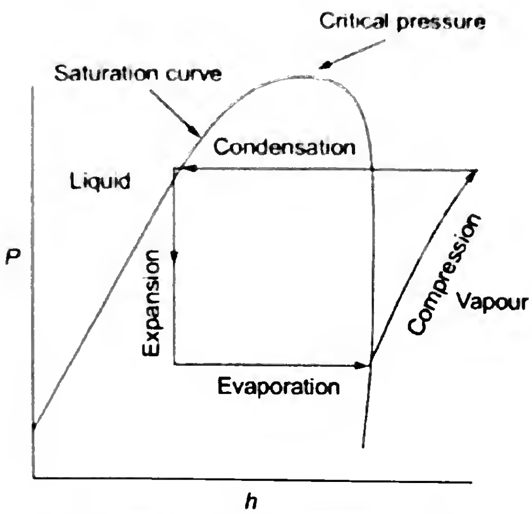
Standard Vapour Compression Refrigeration Cycle (VCRS) on p-v diagram



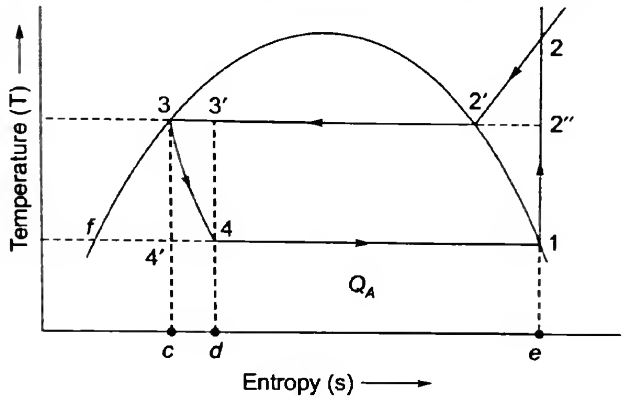
1-2	Isentropic compression in a compressor
2-3	Constant-pressure heat rejection in a condenser
3-4	Throttling in an expansion device
4-1	Constant-pressure heat absorption in an evaporator



Standard Vapour Compression Refrigeration Cycle (VCRS) on T-S diagram



Standard Vapour Compression Refrigeration Cycle (VCRS) on p-h diagram



Refrigerating Effect, Q_A = area 1-4-d-e

Heat rejected, Q_R = area 2-2′-3-c-e

Work done, $W = Q_R - Q_A$ = 1-2-2′-3-c-d-4-1

3. Psychrometry and Air Conditioning Processes

Q.4 Differentiate clearly between ventilation and infiltration. Discuss the methods of estimation of infiltrated air.

[CSE (Mains) 2017 : 10 Marks]

Solution:

Ventilation Air can be natural or mechanical. In modern commercial buildings, the term ventilation refers to mechanical ventilation. It is the intentional controlled introduction of outdoor air into an enclosed occupied space. Ventilation is provided using mechanical systems such as fans. The entry of outdoor air through an open door or window is considered infiltration and not ventilation. The total air supplied to a space consisting of outdoor air and indoor recirculation air is not ventilation air either. It is referred to as Supply Air.

Infiltration Air is the unintentional and uncontrolled entry of outdoor air into an enclosed space. Infiltration occurs through cracks in the building envelope and due to pressure differences between inside and outside. The outdoor air entering through open doors and windows is considered infiltration although the purpose of opening the door or window might be ventilation. Infiltration occurs mainly in winter when the air outside is colder and heavier than the air inside. It depends on wind velocity, wind direction and the air-tightness of the building envelope. In the case of high-rise buildings the stack effect also causes infiltration.

Calculating Average Air Infiltration: Air infiltration is typically measured in terms of the number of times the entire volume of air in a building is replaced each hour. This unit is called "air changes per hour" $\dot{N}$. The actual number of air changes per hour in a building varies continuously over time with drivers of infiltration such as wind velocity. Instantaneous measurements of infiltration can be made by releasing a tracer gas in a house and recording the rate of decay in the concentration of the gas. However, this method is somewhat impractical for wide spread use. Two methods for estimating the average number of air changes per hour are described below.

ASHRAE Method: The ASHRAE method provides a simple way to estimate air infiltration based on the tightness of construction, seasonal variations in wind speed, and the indoor/outdoor air temperature difference that drives the stack effect.

$$\dot{N} = a + b|T_{ia} - T_{oa}|$$

Time of year	Construction	a	b
Winter	Tight	0.280	0.00630
	Medium	0.408	0.00873
	Loose	0.483	0.01224
Summer	Tight	0.210	0.00720
	Medium	0.310	0.00840
	Loose	0.310	0.01400

Blower Door Method: The blower door method estimates the average air infiltration by depressurizing a house with a blower and measuring the quantity of air that must be removed from the house to maintain a constant pressure difference between indoor and outdoor air. Leaky houses will require more air to maintain a constant pressure difference than tight houses. To use the blower door method, install a blower door in a door way and seal the door against the door jam to minimize leakage. Next, depressurize house until

$$\Delta P = P_{ia} - P_{oa} = 50 \text{ Pascals}$$

The air flow required to maintain a 50 Pa pressure difference is called $\dot{V}_{50}$ or "CFM50." Use the following relation to convert from $\dot{V}_{50}$(CFM50) to $\dot{N}_{50}$(ÄCH50)

$$\dot{N}_{50} = \dot{V}_{50}\left[\frac{ft^3}{\text{min}}\right] \times \frac{1}{V_{\text{house}}[\text{ft}^3]} \times 60\left[\frac{\text{min}}{\text{hr}}\right] \quad \left(\text{or ACH}_{50} = \frac{\text{CFM}_{50} \times 60}{V[ft^3]}\right)$$

The "Princeton" Method to estimate the average $\bar{\dot{N}}$ from $\dot{N}_{50}$ is :

$$\bar{\dot{N}} = \frac{\dot{N}_{50}}{20} \quad \left(\text{or ACH} = \frac{\text{ACH}_{50}}{20}\right)$$

The "Sherman" Method to estimate the average $\bar{\dot{N}}$ from $\dot{N}_{50}$ is:

$$\bar{\dot{N}} = \frac{\dot{N}_{50}}{C \times H \times S \times L} \quad \left(\text{or ACH} = \frac{\text{ACHa}_{50}}{C \times H \times S \times L}\right)$$

where the following coefficients can be estimated from the graph and tables that follow.

C = leakage infiltration ratio ; 26 (hot climate) < C < 14 (cold climate)

H = height correction factor

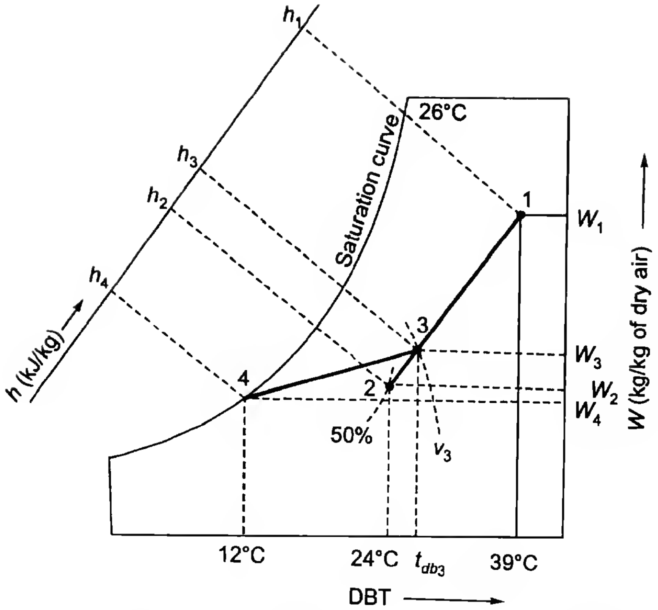
S = wind shielding correction factor

L = leakiness correction factor

Q.5 In an air-conditioning plant, an air handling unit supplies at total of 4000 m³/min of dry air which comprises by mass 20% of fresh air at 39°C DBT and 26°C WBT and 80% re-circulated air at 24°C DBT and 50% RH. The air leaves the cooling coil at 12°C saturated. Using Psychrometric chart calculate (i) Total cooling load and (ii) Room heat gain. Also show the process on Psychrometric chart.

[CSE (Mains) 2017 : 20 Marks]

Solution:



Given : $V_3 = 4000$ m$_3$/min, $t_{db1} = 39°C$, $t_{wb1} = 26°C$, $t_{db2} = 24°C$, $\phi_2 = 50\%$ RH, $t_{db4} = ADP = 12°C$

Following are the steps for showing processes on Psychrometric chart.

1. Locate point 2 at the intersection of 39°C DBT and 26°C WBT lines.
2. Locate point 2 at the intersection of 24°C DBT and 50% RH lines.
3. Join the points 1 and 2.
4. Locate point 3, which represents 20% of fresh air and 80% of recirculated air, on the line 1-2, such that length 2-3 = 0.2 × length 2-1
5. Locate point 4 on the saturation curve where it meets vertical line drawn at 12°C DBT.
6. Joint the point 3 and 4.

Using psychrometric chart, we find :

$$h_1 = 80.6 \text{ kJ/kg of dry air}$$
$$h_2 = 48 \text{ kJ/kg of dry air}$$
$$h_3 = 54.4 \text{ kJ/kg of dry air}$$
$$h_4 = 34.1 \text{ kJ/kg of dry air}$$
$$v_{s3} \text{ (specific volume)} = 0.865 \text{ m}^3\text{/kg of dry air}$$
$$t_{db3} = 27°C \text{ and}$$
$$W_3 = 0.0106 \text{ kg/kg of dry air}$$

$$\text{Mass of air entering the coil, } m_{a3} = \frac{V_3}{v_3} = \frac{4000}{0.865} = 4624.3 \text{ kg/min}$$

(i) $$\text{Total cooling load} = m_{a3}(h_3 - h_4) = 4624.3\,(54.4 - 34.1) = 93873.3 \text{ kJ/min}$$
$$= \frac{93873.3 \times 60}{14000} = 402.3 \text{ tonnes}$$

(ii) Since the total mass of air (m_{a3} = 4624.3 kg/min) comprises 20% of fresh air, therefore, mass of fresh air supplied at point 1,

$$m_{a1} = 0.2 \times 4624.3 = 924.86 \text{ kg/min, and}$$
$$\text{fresh air load} = m_{a1}(h_1 - h_2) = 924.86\,(80.6 - 48) = 30150.4 \text{ kJ/min}$$
$$= \frac{30150.4 \times 60}{14000} = 129.2 \text{ tonnes}$$
$$\text{Room heat gain} = 402.3 - 129.2 = 273.1 \text{ tonnes}$$

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